THE MONOPOLY BENCHMARK ON TWO-SIDED MARKETS

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The literature on the effects of market concentration in platform industries or two-sided markets often compares the competitive outcome against a benchmark. This benchmark is either the "joint management" solution in which one decision maker runs all platforms or a "pure" monopoly with just one platform. Literature has not generally discussed which benchmark is the appropriate one, i.e. how many platforms the monopolist will operate. In this paper we show that the optimal number of platforms depends on whether agents multi- or singlehome, whether the network externalities are positive or negative, and in some cases on the properties of the demand functions. (JEL: D42, D43, K20, L12, L13, L51)

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1. Introduction

In the last decade, an enormous amount of research has been conducted on "two-sided markets" or "platform industries", starting with the seminal articles of Caillaud and Jullien (2003) and Rochet and Tirole (2003). The term "two-sided markets" refers to all industries where a platform enables interaction between two distinct groups of agents. Many contributions are interested in the impact of market concentration on prices, quantities, and welfare. One approach to study such effects is to compare the competitive outcome against the monopoly benchmark. In doing so, some contributions assume the monopoly to be a "joint management" of the existing platforms, i.e. they assume that all platforms continue to operate and are controlled by a single decision maker who maximizes joint profits (e.g. Ambrus and Reisinger (2006); Anderson and Coate (2005); Chandra and Collard-Wexler (2009); Weyl (2010)). Other authors assume that a monopolist operates just one platform (e.g. Chaudhri (1998); McCabe and Snyder (2007)). However, in most cases, the authors do not discuss which benchmark is the appropriate one.

From the perspective of economic theory, this problem is equivalent to the question of how many platforms a monopolistic platform provider optimally chooses. To our knowledge, this question has so far only been analyzed by Ambrus and Argenziano (2009). They consider a situation with purely positive network externalities and singlehoming agents. In their model, the monopolist can choose between opening one or two homogeneous platforms. Consistent with our results, they find that a monopolist will always prefer to operate just one platform, if agents on each market side are homogeneous (their Theorem 2). They add that if agents are heterogeneous, there might be an incentive to operate a second platform in order to implement second-degree price discrimination.

Other reasons why a monopolist may find it more profitable to run just a single platform, instead of multiple ones (or vice versa) could be the well-known ones from traditional markets, e.g. the presence (or absence) of economies of scale or high fixed costs. However, on two-sided markets, the monopolist must also take into account the impact of the number of platforms on the magnitude of the indirect network effects, when deciding how many platforms to operate. To understand the economic intuition, consider the case of singlehoming agents on both market sides, i.e. agents join at most one platform, and positive externalities that is, both market sides benefit from a high number of agents on the other market side. Then, because of singlehoming, operating two platforms would mean that the "joint manager" faces cannibalization effects in the sense that platforms steal members from each other, making both of them less attractive to agents than a large, unified platform. Agents would experience a higher gross utility on a unified platform that translates into a higher willingness to pay, which results in increasing profits. As a consequence, in this case the "joint manager", i.e. the monopolist, would decide to operate just one platform, instead of two platforms. Real-world examples are manifold and can be found across many different industries. For instance, in 1999 eBay bought German rival Alando, and closed it. Since 2001, one of the major German cinema operators, Cinestar, took over movie theaters of two rivals in Chemnitz, Germany, just to close these locations shortly after.

Of course, there are also many examples of platforms coexisting after merger. In 2009, eBay bought South Korean rival Gmarket, and both platforms coexist. In Rostock, Germany, Cinestar currently operates three movie theaters. After entering the market in 1991 buying two locations, they opened a third one in 1996. NASCAR operates multiple racing series. Next to their top-level series Sprint Cup, the Nationwide series fields similar cars and some drivers even compete in both series simultaneously, so that both series are almost homogeneous.[1] The basic argument also applies to differentiated platforms, although product heterogeneity relaxes competition (or cannibalization), and hence, heterogeneous platforms might well coexist, where homogeneous ones would not.

The present paper contributes to the literature by analyzing the choice of the optimal number of platforms on a monopolistic two-sided market in more detail. In a rather general homogeneous two-sided market setting that allows for negative network externalities as well as for multihoming agents, we will compare a one-platform monopoly (OPM) with a two-platform monopoly (TPM).[2],[3] Profits will, ceteris paribus, differ in OPM and TPM for any given number of agents on each market side. For example, if the OPM profit exceeds the TPM profit, the monopolist will rationally decide to close one platform. In this case, using a TPM as a benchmark to study competitive effects would be of theoretical interest only, because a TPM would be a fictitious case. For practical considerations, e.g. when assessing the impact of potential mergers, the OPM would be the appropriate benchmark.

Indeed, we find that for most cases, one profit function is unambiguously above or below the other. In other cases, we determine sufficient conditions the inverse demand functions must satisfy in order to draw clear-cut conclusions. However, the model of homogeneous platforms is highly stylized and its main purpose is to illustrate the mechanisms involved in a monopolist‘s choice of the number of platforms in the simplest possible setting.

Turning to the case of heterogeneous platforms, we build on the widely used model of Armstrong (2006) to study whether our results hold for heterogeneous platforms as well. This model has been used a lot in the literature, for instance recently by Weyl (2010), who generalizes the model with regard to the distribution of agents and compares the competitive outcome with a TPM. We find that in the specific setting of the Armstrong model, OPM profits exceed TPM profits.

The paper is structured as follows: In Section 2, we present a rather general model for homogeneous platforms, and we discuss whether a monopolist establishes an OPM or a TPM depending on whether each market side multi- or singlehomes, whether the externalities are positive or negative, and in some cases on the properties of the demand functions. Section 3 illustrates our results using a slightly modified version of Armstrong’s singlehoming model Armstrong (2006), in which we will also study the case of heterogeneous platforms. Our conclusions are summarized in Section 4.

2. Homogeneous Platforms

Consider a two-sided market in which N_i denotes the (price-dependent) total number of agents on market side i, where $i=1,2$. Prices p_i represent the prices charged to agents on market side i, and are determined by the inverse demand functions $p_i\left(q_1^i, q_2^i\right)$, where $q_1^i, q_2^i > 0$ represents the relevant number of agents from the perspective of market side *i*. The relevant number of agents may or may not be equal to the total number of agents, depending on the market structure and other factors to be discussed later. Inverse demand functions are assumed to be strictly decreasing in their own argument that is $\partial p_i / \partial q_i^i < 0$, $i=1,2$.

In case of an OPM the profit function of the monopolist is defined as

$$\Pi_{OPM}\left(N_1, N_2\right) = N_1 \cdot p_1\left(N_1, N_2\right) + N_2 \cdot p_2\left(N_1, N_2\right).^4$$

1 However, to some extent it remains a matter of interpretation whether some platforms, e.g. movie theaters located in the same town, should be considered identical, and in some cases it may be difficult to draw the line between homogeneous and heterogeneous platforms.

2 As a “two-platform” monopoly is sufficient to illustrate our point, we abstain from generalizing the model to an “n-platform” monopoly. The exposition would become more complex without adding to our argument.

3 Whether the platform operator on a TPM is actually a monopolist or two platform operators with “joint management” or colluding oligopolists, is irrelevant as long as decision making is as if a monopolist owns both platforms.

4 As we will discuss in the conclusions, the impact of costs is straightforward.

Since there is just one platform, the relevant number of agents is the total number of agents that is $q_1^i = N_1$ and $q_2^i = N_2$. However, in this case the total number of agents is also equal to the number of agents on the platform n_i, that is, $N_i = n_i$. Therefore, OPM profit can also be denoted as

$$\Pi_{OPM}(n_1, n_2) = n_1 \cdot p_1(n_1, n_2) + n_2 \cdot p_2(n_1, n_2).$$

Now assume, in contrast, that there are two perfectly homogeneous platforms on the market, which are jointly managed (TPM). Since these platforms are perfectly homogeneous, agents do not have any intrinsic preference for either one. We therefore focus on perfect symmetry that is both platforms are identical twins in terms of prices and patronage. Hence, each platform earns a profit defined by

$$\Pi_{TPM/2} = n_1 \cdot p_1(q_1^1, q_2^1) + n_2 \cdot p_2(q_1^2, q_2^2)$$

and because of symmetry, joint profits are

$$\begin{aligned}\Pi_{TPM} &= 2 \cdot \Pi_{TPM/2} \\ &= 2 \cdot \left(n_1 \cdot p_1(q_1^1, q_2^1) + n_2 \cdot p_2(q_1^2, q_2^2)\right) \\ &= N_1 \cdot p_1(q_1^1, q_2^1) + N_2 \cdot p_2(q_1^2, q_2^2)\end{aligned}$$

The relevant number of agents depends on whether agents are allowed to multihome or restricted to singlehoming. Throughout the paper, we will use the following assumption:

Assumption 1. *Whether agents multihome or singlehome is an exogenous constraint.*

This assumption simplifies our analysis substantially, because multihoming agents will always join all or none of the available platforms. They are neither restricted by income nor do they voluntarily singlehome for other reasons.[5]

For the TPM, the relevant number of agents (q_1^i, q_2^i) in $p_i(q_1^i, q_2^i)$ will be defined in the subsequent analysis, where we consider three cases: First, we analyze the case of two-sided multihoming (Case 1), which will serve as a benchmark. Then, we study the case where one market side is allowed to multihome, whereas the other side singlehomes (Case 2) as well as a situation where both market sides are restricted to singlehoming (Case 3).

2.1. Case 1

We first consider a benchmark case where both market sides multihome, even though we recognize that this case is not very realistic in the case of homogeneous platforms. If agents on market side i are allowed to multihome, they will join a platform, if their net benefit is positive. By Assumption 1, the decision to join a platform is independent of the decision to join the other platform at the same time, which implies that $q_i^i = n_i$. Therefore, the corresponding inverse demand functions are $p_i(n_1, n_2)$. Since both platforms are perfectly homogeneous, an agent will ceteris paribus always join either both or none of the platforms so that total demand on the market is $2 \cdot n_i$. The implications of this case are straightforward: If at (n_1, n_2) OPM profit is positive, TPM profit strictly exceeds OPM profit. In particular, we have $\Pi_{TPM}(n_1, n_2) = 2 \cdot \Pi_{OPM}(n_1, n_2)$.

This result is not surprising given Assumption 1. Since by Assumption 1 all agents who join the single platform in an OPM, also join a second, identical platform, if available, the monopolist is able to double her profits by offering a second platform.[6]

Relaxing Assumption 1 would result in TPM profit being less than twice the OPM profit. However, the economic intuition of our findings continues to hold: If agents on both market sides multihome, a monopolistic

5 Independence of income is a common assumption in the literature. For instance, Rochet and Tirole (2003) assume a demand function Di(pi), in which demand of market side i only depends on the price charged to market side i, but not on income. The model of Armstrong (2006) that will be used in subsequent sections, also excludes income effects.

6 In fact, in the simplified setting considered here, the monopolist could realize infinite profits by offering infinitely many platforms. This prediction is admittedly unrealistic, however, due to the high fixed costs often associated with platform provision.

platform operator has an incentive to operate more than just one platform, because it generates additional revenues. Different from the case of singlehoming agents, which we will discuss later, platforms are not rival in patronage; they do not cannibalize.

2.2. Case 2

A lot of applications require that one market side is restricted to singlehoming, while the other one is allowed to multihome. Armstrong (2006) coined the term "competitive bottlenecks" for this case and mentions several examples that are reflected by this setting. One of the most intuitive examples is the newspaper industry, because most (time-constrained) people might read just one newspaper (i.e. they singlehome), while advertisers are usually simultaneously placing ads in several newspapers (i.e. they multihome). The same holds for the movie theater industry. Another relevant real world example for this framework is supermarkets, because consumers usually tend to regularly visit just one specific super-market, whereas suppliers offer their products in several supermarkets at the same time.

If agents on market side i are restricted to singlehoming, they face the trade-off which platform to join. Since the decision to join a platform depends on the decision to join the other platform, the inverse demand function depends on the total number of agents on market side i, reflecting rivalry for singlehoming agents, i.e. $q_i^i = N_i$. Independent of single- and multihoming, the relevant number of agents from the other market side, q_{-i}^{ι}, is always the number of agents that join the considered platform, i.e. $q_{-i}^i = n_{-i}$. Hence, the corresponding inverse demand functions are $p_i(n_i, n_{-i})$ and $p_{-i}(N_{-i}, n_i)$. In our setting, without loss of generality, we will assume market side 1 to be the multihoming one, and side 2 to be the singlehoming one.

Proposition 1. *On a competitive bottleneck two-sided market with homogeneous platforms, TPM profit strictly exceeds OPM profit if* $\partial p_1 / \partial q_2^1 < 0$. *If* $\partial p_1 / \partial q_2^1 > 0$, *TPM profit strictly exceeds OPM profit if and only if* $p_1(n_1, 0.5 \cdot N_2) > 0.5 \cdot p_1(n_1, N_2) \, \forall (n_1, N_2)$.

Proof. See the Appendix A.

To understand the economic intuition behind Proposition 1, let us first remember that the agents' willingness to pay depends on the number of agents from the other market side that are on the same platform. Taking into account that under TPM those multihoming agents, who join the one platform, also join the other platform, singlehoming agents are exposed to the same magnitude of the externality in TPM and OPM for any given n_1. This implies that revenue from the singlehoming market side 2 does not depend on whether there are one or two platforms. Multihoming agents, on the other hand, are under TPM only exposed to the network effects caused by half of the total number of agents from side 2. If multihoming agents are negatively affected by the presence of singlehoming agents, i.e. $\partial p_1 / \partial q_2^1 < 0$, the magnitude of this negative effect is lower under TPM than under OPM, and therefore the willingness to pay of side 1 agents is higher for the TPM , which translates into TPM profit exceeding OPM profit.

If the multihoming agents' willingness to pay is positively affected by the number of singlehoming agents, i.e. if $\partial p_1 / \partial q_2^1 > 0$, there are two effects in opposite directions: Firstly, if there are two platforms, multihoming agents join both of them, and hence, the platform operator can multiply revenues by offering more platforms (see Case 1). Secondly, because multihoming agents are exposed to an externality of lower magnitude in the TPM, their willingness to pay is lower than in the OPM. Proposition 1 states that if the first effect is stronger than the second effect, a TPM is more profitable than an OPM, while the OPM is more profitable if the second effect dominates, that is, if network effects are strong enough.

2.3. Case 3

Now we assume that both market sides singlehome, which implies that the inverse demand functions are $p_i(N_i, n_{-i})$, $i = 1,2$. Proposition 2 summarizes the corresponding implications.

Proposition 2. *If agents on either side of the market are restricted to singlehoming, TPM profit strictly exceeds OPM profit if and only if* $\partial p_i / \partial q^i_{-i} < 0$.

Proof. See the Appendix B.

Proposition 2 describes the effect of rivalry in patronage on both market sides. When discussing the economic intuition of Proposition 1, we stated that there are two opposing effects, if the indirect network externality is positive for both market sides, i.e. if $\partial p_i / \partial q^i_{-i} < 0$. The first effect originated in the ability of side 1 agents to multihome. This effect, of course, vanishes if agents are restricted to singlehoming. The second effect was a downward-shift in the willingness to pay due to the lower magnitude of the externality under TPM, because on each platform there is only half the number of agents from the other side. Hence, different from the discussion of Proposition 1, revenues from market side 1 are strictly lower for the TPM than for the OPM. Another difference is that this argument now holds for both market sides, while in Case 2 market side 2 was unaffected, because market side 1 was able to multihome. Therefore, given positive externalities, OPM profit strictly exceeds TPM profit. Analogously, if network effects are negative for both market sides, i.e. if $\partial p_i / \partial q^i_{-i} < 0$, agents on each market side are exposed to a negative externality that is smaller in magnitude in TPM than in OPM, which translates to TPM profits strictly exceeding OPM profits.

In case of mixed indirect network effects, i.e. $\partial p_i / \partial q^i_{-i} > 0, \partial p_{-i} / \partial q^{-i}_i < 0$, a simple comparison of general profit functions does not suffice to draw conclusions. Whether TPM or OPM yields higher profits for a given (N_1, N_2), depends on the price difference between the market forms, weighted by the corresponding N_i. Assume, for instance, that market side 1 experiences positive externalities and market side 2 experiences negative externalities. Then, if

$$N_1 \cdot [p_1(N_1, N_2) - p_1(N_1, 0.5 \cdot N_2)]$$

$$> N_2 \cdot [p_2(0.5 \cdot N_1, N_2) - p_2(N_1, N_2)]$$

it follows that OPM profit exceeds TPM profit. This illustrates the key message of our paper: Whether a monopolist in a two-sided market will choose one or two platforms, depends on the specific situation studied. It depends not only on whether agents are restricted to singlehoming or are allowed to multihome, as well as on the direction and magnitude of the indirect externality, but also on size (represented by N_i), and willingness to pay (represented by $p_i(q^i_1, q^i_2)$) of each market side.

So far, we only studied general profit functions as a whole, and not their optimal values. The reason is simply the fact that with regard to the message of our paper, we do not gain from comparing first-order conditions of a general environment. In specific settings with specific functional forms, it might however be possible and helpful to determine optimal solutions and compare them in OPM and TPM, as we will demonstrate using the model of Armstrong (2006) in the next section.

3. Heterogeneous Platforms

In this section, we extend our analysis to the case of heterogeneous platforms. The introduction of platform heterogeneity ceteris paribus reduces price competition between platforms and hence potentially reduces cannibalization effects under TPM, which is particularly relevant for the two-sided singlehoming case. In this context, we would expect that the establishment of a second platform ceteris paribus becomes more attractive for the monopolistic platform provider, which potentially affects our findings from Proposition 2. With respect to platform heterogeneity, the probably most widely used framework in the literature builds on the Hotelling (1929) model. We build our analysis on Armstrong (2006), which is a seminal paper that studies heterogeneous platforms in a Hotelling-type setting. Note that our analysis focuses on Armstrong's two-sided singlehoming framework, while leaving aside the "competitive bottleneck" case.[7]

Armstrong's two-sided singlehoming model assumes specific (symmetric) functional forms for each market side's demand function that are derived from an additive-separable utility function. While such a specific formulation has of course some disadvantages[8], it is not only tractable analytically, but also allows us to derive both OPM and TPM demand functions as coming from the same utility function. Note, however, that Armstrong's OPM model is not directly comparable to his setting of duopolistic competition, because his OPM model is not built into a Hotelling setting, while his duopoly model is. In order to directly compare OPM and TPM, we therefore assume that in the OPM case the platform is located in the middle of the Hotelling line that is at location 0.5. We consider two cases of the TPM: To show that our results from the previous section transfer to Armstrong's model, we first assume that both platforms in the TPM are also located at 0.5 that is, platforms are homogeneous. Afterwards, we study the effects of platforms being located at points 0 and 1 on the Hotelling line, as in Armstrong (2006).

Under OPM assume that the utility of agents on market side i, U_i, can be described by the utility function

(1) $U_i = \alpha_i n_j - p_i - |0.5 - x| t_i \quad i, j = 1, 2 \quad i \neq j$,

where x denotes the agent's preferred location on the Hotelling line. Parameter α_i represents the constant marginal indirect network effect of one agent of market side j being present on the same platform, and t_i is the transportation cost parameter. We assume a reservation utility $\underline{u_i}$, which represents the minimum utility an agent requires to join the platform. Setting (1) equal to $\underline{u_i}$, while respecting the absolute value sign, and solving for x yields the locations of those agents, who just obtain their reservation utility, and are hence indifferent between joining and not joining the platform. The marginal agent on the left hand side (right hand side) of the platform is located at

$$x_i^l = \frac{1}{2} - \frac{\alpha_i n_j - p_i - \underline{u_i}}{t_i}$$

$$x_i^r = \frac{1}{2} + \frac{\alpha_i n_j - p_i - \underline{u_i}}{t_i}$$

Following Armstrong, we assume that agents are uniformly distributed on the Hotelling line, which yields the demand function

(2) $$n_i = \frac{2\left(\alpha_i n_j - p_i - \underline{u_i}\right)}{t_i},$$

From (2), we can easily obtain the inverse demand function as,

$$p_i = \alpha_i n_j - \frac{n_i t_i}{2} - \underline{u_i}$$

and hence OPM profit is given by

(3) $$\Pi_{OPM} = \sum_{\substack{i,j=1 \\ i\neq j}}^{2} n_i \left(\alpha_i n_j - \underline{u_i} - \frac{n_i t_i}{2} \right) = \sum_{\substack{i,j=1 \\ i\neq j}}^{2} N_i \left(\alpha_i N_j - \underline{u_i} - \frac{N_i t_i}{2} \right)$$

This setting can easily be transferred to represent a TPM with homogeneous platforms located at 0.5. As platforms are homogeneous, agents obtain ceteris paribus the same utility on each platform. Since both market sides must singlehome, the market will be equally shared between both platforms, and hence, an agent,

7 *The "competitive bottleneck" case is analyzed in Böhme and Müller (2014). The model presented in this paper is also based on Armstrong's setting, but additionally allows the monopolist to choose her optimal platform locations along the unit line. That paper, however, does not analyze the monopolist's choice of the number of platforms.*

8 *For a discussion, see Armstrong (2006, p. 675).*

who joins a platform, is exposed to the indirect network effect of half the number of agents in the market, which yields demand

$$(4)\quad N_i = \frac{\alpha_i n_j - 2\left(p_i + \underline{u_i}\right)}{t_i},$$

inverse demand

$$p_i = \alpha_i n_j - n_i t_i - \underline{u_i}$$

and a total profit of

$$(5)\quad \Pi_{TPM}^{\text{hom}} = \sum_{\substack{i,j=1\\ i\neq j}}^{2} 2n_i\left(\alpha_i n_j - \underline{u_i} - n_i t_i\right) = \sum_{\substack{i,j=1\\ i\neq j}}^{2} N_i\left(\alpha_i \frac{N_j}{2} - \underline{u_i} - \frac{N_i}{2} t_i\right)$$

Subtracting (5) from (3), it is easy to see that for every (N_1,N_2) $\Pi_{OPM} < \Pi_{TPM}^{\text{hom}}$, if $\alpha_i < 0$, $i = 1,2$ and $\Pi_{OPM} > \Pi_{TPM}^{\text{hom}}$, if $\alpha_i > 0$, $i = 1,2$, which is in line with the implications of Proposition 2. If $\alpha_1 > 0$ and $\alpha_2 < 0$, OPM profit strictly exceeds TPM profit, if α_1 is greater than α_2 in absolute terms, while reservation utility levels are sufficiently high. The opposite holds, if α_1 is smaller than α_2 in absolute terms.

To study platform heterogeneity, we now assume that under TPM one of the platforms is located at point 0 on the Hotelling line, while the other platform is located at point 1. Also note that for the sake of unambiguous results, we do not compare profits for any (q_1,q_2) anymore, but optimal solutions.

By the assumptions made so far, we know that the indifferent agent will be located at point 0.5 on the Hotelling line. A monopolist who operates both platforms will set her price so that the indifferent agent on each market side will just receive their reservation utility. The resulting optimal price is

$$p_{het,i}^{*} = \frac{\alpha_i - t_i}{2} - \underline{u_i},$$

which yields a profit of

$$(6)\quad \Pi_{TPM}^{het*} = \frac{\alpha_1 + \alpha_2 - t_1 - t_2}{2} - \underline{u_1} - \underline{u_2}\ .$$

Armstrong's equilibrium solution implies a utility value of $0.5\cdot\left(\alpha_i + 2\alpha_j - 3t_i\right)$ for the indifferent agent on market side i. We consider this utility level to be the upper bound of reservation utility that is, we assume $\underline{u_i} < 0.5\cdot\left(\alpha_i + 2\alpha_j - 3t_i\right)$. Note that for $\underline{u_i} = 0.5\cdot\left(\alpha_i + 2\alpha_j - 3t_i\right)$, (6) becomes $\Pi_{TPM}^{het*} = t_1 + t_2 - \alpha_1 - \alpha_2$, which is exactly the equilibrium profit, if both platforms compete (Armstrong's equation (13), p. 675).

We show in the Appendix C that OPM profit is given by

$$(7)\quad \Pi_{OPM}^{*} = \frac{t_2\left(\underline{u_1}\right)^2 + 2\underline{u_1}\,\underline{u_2}\left(\alpha_1 + \alpha_2\right) + t_1\left(\underline{u_2}\right)^2}{2\left(t_1 t_2 - \left(\alpha_1 + \alpha_2\right)^2\right)}\ .$$

For simplicity, assume that $\underline{u_1} = \underline{u_2}$ and $t_1 = t_2$. Then for $\alpha_i > 0$, $i = 1,2$, it still holds that $\Pi_{OPM}^{*} > \Pi_{TPM}^{het*}$, as was the case with homogeneous platforms. On the other hand, for $\alpha_i < 0$ it holds that $\Pi_{OPM}^{*} > \Pi_{TPM}^{het*}$. In the case where $\alpha_1 > 0$ and $\alpha_2 < 0$, we obtain ambiguous results. Summarizing our findings, we see that these results are in line with Proposition 2, which implies that platform heterogeneity is per se not sufficient to disprove the results of our paper.

4. Conclusions

Selecting the appropriate benchmark is the basic step when analyzing the effects of market concentration. On two-sided markets it is therefore of essential interest to know, how many platforms a monopolist will operate. She has to trade-off additional revenues that could be gained when operating an additional platform and the loss of revenues due to cannibalization. Determinants in this respect are whether agents single- or multihome on one or on both market sides and whether the indirect network externality is positive or negative. Our analysis follows these situations systematically and shows that in some of them there is an unambiguous tendency towards one (or multiple) platform(s), while in other situations general analysis remains ambiguous. In some cases, ambiguity can be resolved, if details like characteristics of the demand functions and the relative sizes of the market sides are known. We also demonstrated that platform heterogeneity does not resolve the problem per se.

It should be kept in mind that we assume away a number of empirically relevant aspects, which might either strengthen or weaken our results. The effects of these left out aspects are intuitively straightforward and well-known from traditional markets without indirect network effects. One such aspect is binding capacity constraints. In a two-sided market context, the existence of such constraints ceteris paribus favors the existence of two platforms rather than one. Another aspect is the cost function, which might favor either a TPM, if there are low or no fixed costs or increasing marginal costs or an OPM, if there are high fixed costs or decreasing marginal costs.

In the introduction we gave the example of eBay buying a German rival and closing this platform. This was in 1999, when German internet users had smallband access available only,[9] and internet service providers offered two-part tariffs, i.e. users had to pay extra for every minute they spent online. With slow and expensive internet access, users are less likely to use more than one internet auction platform. Therefore, we interpret this example as being a two-sided singlehoming situation with positive network externalities on either side of the market. Our analysis finds that in this situation the OPM solution is more profitable. The case is different for the South Korean eBay example, where the acquisition took place 10 years later. South Korea is among those countries with the highest broadband penetration rate,[10] so using more than one platform, e.g. to compare prices, is easy for buyers, and offering on more than one platform is easy for sellers. We can therefore interpret this as an example for a two-sided multihoming situation for which our analysis finds strong incentives to keep the additional platform. The examples of the movie theaters represent a "competitive bottleneck" situation with mixed network externalities. For such a situation we found that either OPM or TPM can be more profitable. The decisive criterion is by Proposition 1 the intensity of the indirect network effect on the multihoming side, i.e. advertisers' valuation of moviegoers. NASCAR allows fans to multihome by scheduling Nationwide races on Saturdays and Sprint Cup races on Sundays. Sponsors are of course able to participate in both series, so that there is a two-sided multihoming case, and the coexistence of the two series is consistent with our analysis.

Our analysis provides some guidelines in which direction – one or more platforms – a specific model is likely to tend. As a rule of thumb, we can say that factors relaxing competition for patronage make way for additional platforms, while factors that increase cannibalization render a single platform solution more attractive. It is, however, not our intention to give a complete "cooking recipe", which would point out the appropriate benchmark, as there are numerous additional determinants, e.g. economies of scale and fixed costs, which all have to be taken into account. Instead, we emphasize that it is important to be aware of this problem in theory as well as in applied research.

9 Two years later, by the end of 2001, only 2.4% of the population had broadband access to the internet (Source: OECD Broadband Portal on http://www.oecd.org/).

10 In 2008, one third of the population had broadband internet access (Source: OECD Broadband Portal on http://www.oecd.org/).

Appendix A. Proof of Proposition 1

By perfect symmetry on the TPM, we know that the total number of agents on the singlehoming side is equally shared among both platforms as prices and the numbers of agents per platform on the multihoming side will be identical on both platforms. Hence, we know that the number of agents per platform on the singlehoming side is given by $n_2 = 0.5 \cdot N_2$. Multihoming agents will focus on the price and the number of singlehoming agents on *each* platform. Therefore, the TPM profit can be written as

$$\Pi_{TPM}(n_1, n_2) = 2 \cdot n_1 \cdot p_1(n_1, n_2) + 2 \cdot n_2 \cdot p_2(n_1, 2 \cdot n_2).$$

Taking into account that $n_2 = 0.5 \cdot N_2$, we can rewrite the TPM profit as

(A1) $\Pi_{TPM}(n_1, N_2) = 2 \cdot n_1 \cdot p_1(n_1, 0.5 \cdot N_2) + N_2 \cdot p_2(n_1, N_2)$.

Under OPM $n_i = N_i$, hence the OPM profit can be rewritten as

(A2) $\Pi_{OPM}(n_1, N_2) = n_1 \cdot p_1(n_1, N_2) + N_2 \cdot p_2(n_1, N_2)$.

Comparing equations (A1) and (A2), it is easy to see that the second term on the right-hand side of both equations is identical for any given (n_1, N_2), which implies that the profit generated from the singlehoming side is equal under both market structures. Focusing on the profit from the multihoming side, we find that $\Pi_{OPM}(n_1, N_2) > \Pi_{TPM}(n_1, N_2)$, if $p_1(n_1, N_2) > 2 \cdot p_1(n_1, 0.5 \cdot N_2)\ \forall (n_1, N_2)$. The opposite result – that is $\Pi_{OPM}(n_1, N_2) < \Pi_{TPM}(n_1, N_2)$ – holds, if $p_1(n_1, N_2) < 2 \cdot p_1(n_1, 0.5 \cdot N_2)\ \forall (n_1, N_2)$ and $\partial p_1 / \partial q_2^1 > 0$. When $\partial p_1 / \partial q_2^1 < 0$, we can immediately see that $p_1(n_1, N_2) < p_1(n_1, 0.5 \cdot N_2)\ \forall (n_1, N_2)$, and therefore $\Pi_{OPM}(n_1, N_2) < \Pi_{TPM}(n_1, N_2)$. *(q.e.d.)*

Appendix B. Proof of Proposition 2

OPM profit can be written as

(B1) $$\Pi_{OPM}\left(N_1,N_2\right)=N_1\cdot p_1\left(N_1,N_2\right)+N_2\cdot p_2\left(N_1,N_2\right)=\sum_{i=1}^{2}N_i\cdot p_i\left(N_i,N_{-i}\right).$$

By the assumption of perfect symmetry on the TPM, we know that the total number of agents of each market side will be equally shared between both platforms, i.e. $n_i=0.5\cdot N_i,\ i=1,2$. Hence, singlehoming agents of market side i meet $0.5\cdot N_{-i}$ agents of market side $-i$ on each platform. Therefore, in (N_1,N_2) space TPM profit can be written as

(B2) $$\Pi_{TPM}(N_1,N_2)=N_1\cdot p_1\left(N_1,0.5\cdot N_2\right)+N_2\cdot p_2\left(0.5\cdot N_1,N_2\right)=\sum_{i=1}^{2}N_i\cdot p_i\left(N_i,0.5\cdot N_{-i}\right).$$

Comparing (B1) and (B2), we see that both terms on the right hand side of (B1) strictly exceed their counterparts in (B2), if $\partial p_i/\partial q_{-i}^{i}>0$ and vice versa if $\partial p_i/\partial q_{-i}^{i}<0$. *(q.e.d.)*

Appendix C.

Considering the OPM case where the platform is located at 0.5, the profit can be derived as follows: Given the inverse demand function from equation (3), we know that the OPM profit is determined by

(C1) $$\Pi_{OPM} = \sum_{\substack{i,j=1 \\ i \neq j}}^{2} n_i \left(\alpha_i n_j - \underline{u}_i - \frac{n_i t_i}{2} \right).$$

We maximize profit by taking the derivatives with respect to n_1 and n_2. The corresponding first order conditions are

(C2) $$\frac{\partial \Pi_{OPM}}{\partial n_1} = -n_1 t_1 - \underline{u}_1 + n_2 (\alpha_1 + \alpha_2) = 0,$$

(C3) $$\frac{\partial \Pi_{OPM}}{\partial n_2} = -n_2 t_2 - \underline{u}_2 + n_1 (\alpha_1 + \alpha_2) = 0.$$

With respect to the second order conditions, we find that the Hessian matrix is given by

$$H = \begin{pmatrix} -t_1 & \alpha_1 + \alpha_2 \\ \alpha_1 + \alpha_2 & -t_2 \end{pmatrix}.$$

Hence, we can conclude that the existence of a profit-maximizing solution requires $t_1 > 0$, which holds by assumption, as well as $t_1 t_2 - (\alpha_1 + \alpha_2)^2 > 0 \Leftrightarrow t_1 t_2 > (\alpha_1 + \alpha_2)^2$.

If we simultaneously solve (C2) and (C3) for the profit maximizing number of agents on either side of the market (n_1^*, n_2^*), we find that

(C4) $$n_1^* = -\frac{\underline{u}_2 (\alpha_1 + \alpha_2) + t_2 \underline{u}_1}{t_1 t_2 - (\alpha_1 + \alpha_2)^2}$$

(C5) $$n_2^* = -\frac{\underline{u}_1 (\alpha_1 + \alpha_2) + t_1 \underline{u}_2}{t_1 t_2 - (\alpha_1 + \alpha_2)^2}.$$

Substituting n_1 and n_2 in equation (C1) by (C4) and (C5) yields the profit in the OPM case, which is determined by

(C6) $$\Pi_{OPM}^* = \frac{t_2 (\underline{u}_1)^2 + 2\underline{u}_1 \underline{u}_2 (\alpha_1 + \alpha_2) + t_1 (\underline{u}_2)^2}{2\left(t_1 t_2 - (\alpha_1 + \alpha_2)^2\right)}.$$

Assuming that $u_1 = u_2 = \underline{u}$, it can be shown that for $t_{1,2} > 0, \Pi^*_{OPM} \geq 0, \Pi^{het*}_{TPM} \geq 0, n^*_{1,2} \geq 0$ as well as $t_1 t_2 > (\alpha_1 + \alpha_2)^2$, we have that $\Pi^*_{OPM} > \Pi^{het*}_{TPM}$ if $\alpha_1, \alpha_2 > 0$, while for $t_1 = t_2 > 0, \Pi^*_{OPM} \geq 0, \Pi^{het*}_{TPM} \geq 0, 0 \leq n^*_{1,2} \leq 1$ and $\underline{u_i} \leq 0.5 \cdot (\alpha_i + 2\alpha_j - 3t_i)$, we find that $\Pi^*_{OPM} < \Pi^{het*}_{TPM}$ if $\alpha_1, \alpha_2 < 0$. The corresponding Mathematica code will be provided by the authors upon request.

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