THE PENSION SCHEME NEED NOT BE PAY-AS-YOU-GO: AN OVERLAPPING GENERATIONS APPROACH*

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A relevant question in pension scheme research is: Should a country gradually unload its pension funds in order to, for example, counter some of the negative effects of the aging population and thus to prevent pension contribution rate from rising too much? As both Diamond (1965) and Samuelson (1975) have emphasized, ignoring transitional welfare effects is not a good idea and can potentially lead to wrong policy conclusions. Still many choose to concentrate solely on steady state effects. In this paper I illustrate the transitional and steady state effects of moving from a mixed pension scheme to a pay-as-you-go scheme and I show that, given a set of simplifying assumptions, this may not be a wise policy. On the contrary, a country should gradually switch over to a fully funded scheme. (JEL: C68, H54, H55)

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1. Introduction

For many countries, a relevant question in pension scheme research is whether they should gradually unload their pension funds in order to counter some of the negative effects of the aging population and, thus, to prevent pension contribution rate from rising or replacement rate from decreasing too much. As both Diamond (1965) and Samuelson (1975) have emphasized, an analysis of a transition from one system to another should take transitional welfare effects into account and it does not suffice to compare only the corresponding steady states. Yet, exploration of the transitional effects is surprisingly rare in the literature.

In this paper I illustrate both the transitional and steady state effects of moving from a mixed pension scheme[1] to a pay-as-you-go scheme.[2] I show that, given a set of simplifying assumptions, this may not be a wise policy. On the contrary, a country should gradually switch over to a fully funded scheme. This conclusion is backed up by, for example, the Nobel laureate Franco Modigliani: "In a word, we need to abandon the pay-as-you-go system, which is a wasteful and inefficient system, and replace it with a fully funded system."[3] In addition, a reform of this size is not unheard of or unrealistic. Many countries, Chile in 1981 being the most prominent example, have made the political decision of moving from a PAYG to a funded system.

The subject of optimal pension system is particularly topical in the case of Finland as the population is ageing and there is a lively public debate on whether or not to start unloading the pension funds in the near future. Consequently, the analysis of this paper concentrates on Finland. In a bigger picture, however, the analysis carried out in this paper can be generalized to any similar country with a mixed pension system.

One essential question is how a social security system affects saving behavior. In general, there are three effects at play; wealth substitution effect, retirement effect and bequest effect. Wealth substitution effect is referred to when an individual saves less than she would have without social security system in place. Agent knows that a part of her saving is done via the system and therefore saves less on her own. The aggregate response of saving depends on the details of the social security system in place. With a pay-as-you-go system, aggregate saving is reduced meaning less physical capital and, thus, reduction of output in the long run. The reduction is due to the different return on saving in different pension schemes (real interest rate versus the population growth rate). The retirement effect means that individuals might retire earlier with a social security in place. However, if the retirement period lengthens, as it has because of longevity, an individual needs to save more to maintain the preferred level of consumption. This effect increases saving. The bequest effect refers to a desire to leave bequests for the young which also increases saving.

In a seminal study, Feldstein (1974) concluded that the wealth substitution effect dominates the retirement and bequest effects, thus, the PAYG significantly lessens capital accumulation in the United States. Samwick (2000) finds that countries which operate PAYG seem to have lower saving rates, especially if the PAYG covers a large portion of the population. Feldstein (1996) estimates that the social security program in the US reduces the overall saving by nearly 60% of their potential. There are also opposite conclusions (c.f. Leimer and Lesnoy 1982), but in general the consensus seems to be that PAYG pension scheme has negative effect on saving (Hurd 1990). In addition, according to for example Maddison (1992), "There is a general positive relationship between the faster postwar growth in output per head and the acceleration in saving rates..." If this was indeed the case, one

[1] In a mixed system, a part of contributions are funded and the rest are given directly to beneficiaries as pensions.
[2] Pay-as-you-go is abbreviated PAYG from hereon.
[3] In an interview from 1999 published in "Inside the Economist's Mind: Conversations with Eminent Economists." eds. Samuelson and Barnett 2007.

might ask why moving from a mixed system to a PAYG would be a good idea.

Sometimes it's easy to get lost in details and not to see the big picture. In this paper I simulate a (numerical) dynamic general equilibrium model in which I neglect the small details and concentrate on the big picture. The usual way of illustrating the differences between funded pension scheme and pay-as-you-go is to use the Diamond-Samuelson overlapping generations framework. The two systems in the model diverge only in the way in which pensions are funded. In the funded system the mandatory pension contributions, x_t, are collected from the young generation in a lump-sum manner. The contributions are then collectively invested and returned with interest in the next period, $(1 + r_{t+1})x_t$. The idea is to collectively save for old age. In the PAYG scheme, on the other hand, pension contributions, x_t, are used to finance pensions for the old generation in the same period. This translates to each young individual paying x_t in contributions and each old person receiving $(1 + n)x_t$ where n is the population growth rate (or economy's growth rate, if you will). This immediately displays the problems with a declining n. Also, PAYG is not neutral to saving decisions as long as $n \neq r$. The funded scheme, on the other hand, tends to be neutral to aggregate saving. The young people will fully offset the savings which the social security fund does on their behalf and therefore the aggregate saving is not distorted. The PAYG, on the other hand, tends to diminish aggregate saving compared to a world without the system in place, given that $n < r_{t+1}$. This situation is referred to as dynamic efficiency as in Malinvaud (1953), or, from another point of view, the Aaron condition (Aaron 1966). The policy conclusion within the traditional Diamond-Samuelson overlapping generations framework is straightforward; a dynamically efficient economy should increase saving in the long run and the pension scheme should be shifted towards a fully funded system. The opposite applies in a dynamically inefficient economy. The conclusion is not as straightforward in a more complex model, for example, in a model with open economy or endogenous growth. Also, taking the transition path into account is of crucial importance as is shown later in this paper.

The most cited article, in which dynamic efficiency is empirically assessed, is the one by Abel, Mankiw, Summers and Zeckhauser (1988). The authors conclude that "In the United States, profit has exceeded investment in every year since 1929. This finding leads us to conclude that the United States economy is dynamically efficient." They reach the same conclusion for the United Kingdom, France, Germany, Italy, Canada and Japan between 1960 and 1984. The underlying logic is such that the capital sector is constantly contributing to the level of consumption instead of being a drain which would mean that investments exceed profits. Also Barbie, Hagedorn and Kaul (2004) argue that the US economy does not overaccumulate capital between 1890–1999. From a more theoretical side, d'Albis and Decreuse (2007) argue that strong intergenerational altruism and high life expectancy prevent the occurrence of inefficient equilibria. They run a simulation exercise and conclude that the actual life expectancy is sufficiently low to guarantee that the US economy is dynamically efficient. The same general conclusion is reached, with a more minimal inspection, by Modigliani, Ceprini and Muralidhar (2000).

The matter of dynamic efficiency is, after all this evidence, still a bit ambiguous and quite subtle issue to say the least. Be it this way or that way, many policy recommendations implicitly assume that the economy is dynamically efficient. The results presented in this paper also depend on this observation. The reader is advised to familiarize herself with, for example, Weil (2008), Abel et al. (1988) or Blake (ch. 4, 2006) for a deeper discussion of dynamic efficiency.

The paper is organized as follows. The next section introduces the framework used to investigate the problem at hand, that is, the comparison of different pension schemes. The third section discusses the calibration of the model and its steady state properties. The fourth section studies differences between the two pension systems and discusses the policy implications of this analysis. The fifth section concludes.

2. The Framework

The model used here is an extended version of the Diamond-Samuelson overlapping generations framework. Again, in the words of Franco Modigliani: "[...] the notion of parsimoniousness is a useful notion, the notion that one should try to construct models that are not too big, models that are more compact in size." Thus, while my model intends to be as simple and tractable as possible, it tries to capture the relevant features of the Finnish pension system. The model includes five cohorts, a mixed pension system and endogenous labour supply.

2.1. Demographics

There is a large number of identical agents (workers) born each period. There are five cohorts.[4] The first three cohorts constitute the labour force. Each cohort encloses approximately 12 age groups of one year. Individuals fare through their lives deterministically. Individual that is born in the beginning of period t works for three periods, t, $t+1$ and $t+2$. Last two periods of her life, $t+3$ and $t+4$, she spends enjoying leisure, that is, she is fully retired.[5] Death is certain, thus, the particular individual no longer exists at $t+5$.

Formally, I write the demographics in the following way:

(1)
$$POP_t = N_t^1 + N_t^2 + N_t^3 + N_t^4 + N_t^5,$$
$$POP_{t+1} = (1+n)(1+g)POP_t$$

N_t^i is the size of a cohort i at time t, POP_t is the (technology adjusted) total population at time t, n is the constant population growth rate and g is the Harrod-neutral technical progress.

2.2. Individuals

The decision problem of an individual is to choose a sequence of consumption, leisure, and asset holdings, given the factor prices, a sequence of exogenous variables for demographic developments and pension contributions, that maximize the discounted value of lifetime utility subject to her constraints. There is perfect foresight. The problem of an age-i individual born at t is written as

(2)
$$V_t^i = \max_{s_{t+i-1}^i, l_{t+i-1}^i} \sum_{i=1}^{5} \beta^{i-1} \times \left(\frac{(c_{t+i-1}^i)^{1-\sigma^c}}{1-\sigma^c} + \gamma^i \frac{(1-l_{t+i-1}^i)^{1-\sigma^l}}{1-\sigma^l} \right)$$

s.t.

(3)
$$c_{t+i-1}^i + s_{t+i-1}^i = w_{t+i-1}\epsilon^i l_{t+i-1}^i + (1+r_{t+i-1})s_{t+i-2}^{i-1} + p_{t+i-1}^i - x_{t+i-1}^i,$$

$$c_t^i \geq 0 \ \forall \ t, i,$$

$$l_t^i \geq 0 \ \forall \ t, i.$$

The periodic utility function is characterized by separating isoelastic preferences to keep the analysis as simple as possible. c_{t+i-1}^i denotes consumption, s_{t+i-1}^i saving, l_{t+i-1}^i labour supply[6], p_{t+i-1}^i pension income and

[4] *Multi-cohort structure better captures the life-cycle aspects of the problem at hand than the usual two-cohort representation.*

[5] *Also smoother retirement was tried, that is, individual was allowed to be both working and to be retired in the fourth period. The results of this article remain in principal the same.*

[6] *Individuals are endowed with one unit of time, hence* $(1-l_{t+i-1}^i)$ *denotes leisure.*

x^i_{t+i-1} is the pension contribution. As usual, w_{t+i-1} and r_{t+i-1} denote wages and real interest rate, respectively. γ^i is a positive parameter that denotes the relative weight given to utility from leisure and $1/\sigma^c$ and $1/\sigma^l$ denote the intertemporal elasticity of substitution with respect to consumption and leisure, respectively. The parameter ϵ^i is an age specific efficiency index of an age-i individual. Individuals are born with zero assets $(s^0_t = 0 \; \forall \; t)$. There is no saving in the last period of life because individuals know that they die after retirement period $(s^5_t = 0 \; \forall \; t)$. In other words, I do not consider the bequest effect of saving. Also, retirement is exogenous so that individuals do not work in the fourth or fifth period of their lives $(l^4_t = l^5_t = 0 \; \forall \; t)$. For all c^i_{t+i-1} and $1 - l^i_{t+i-1}$, the usual Inada conditions hold with the assumed form of utility function (equation (2)).

Pension contributions, x^i_t, are determined in the following way:

(4) $$x^i_t = \begin{cases} \tau^p w_t \epsilon^i l^i_t, & \text{for } i = 1,2,3 \\ 0, & \text{for } i = 4,5 \end{cases}$$

where τ^p is the pension contribution rate. The pension contribution distorts the labour supply decision, thus, the steady state solution of the model is different with and without a pension scheme. This would not be the case with lump-sum contribution scheme. Pension contributions are partly funded and partly used in a PAYG scheme. The total paid out pension benefits, B_t, are determined by:

(5)
$$B_t = \mu^f_{t-1}(1 + r_t)(N^1_{t-1}x^1_{t-1} + N^2_{t-1}x^2_{t-1} + N^3_{t-1}x^3_{t-1})$$
$$+(1 - \mu^f_t)(N^1_t x^1_t + N^2_t x^2_t + N^3_t x^3_t)$$

where μ^f_t is an exogenous policy variable that describes the share of pension contributions that are funded. If $\mu^f_t = 0$, funding rate is zero and the scheme can be characterized as PAYG. If $\mu^f_t = 1$, funding rate is one and the scheme is a fully funded pension scheme. While $0 < \mu^f_t < 1$, the pension scheme is a mixed one, in other words, part of the funding comes from the previous period (with interest) and the rest is collected from the workers of the present period. The first line of (5) is the total contributions paid out from a pension fund at time t. Thus, the maturity of the investment in the pension fund is one period. The second line of (5) is the pay-as-you-go part of the pension system. The pension scheme is described in Figure 1.

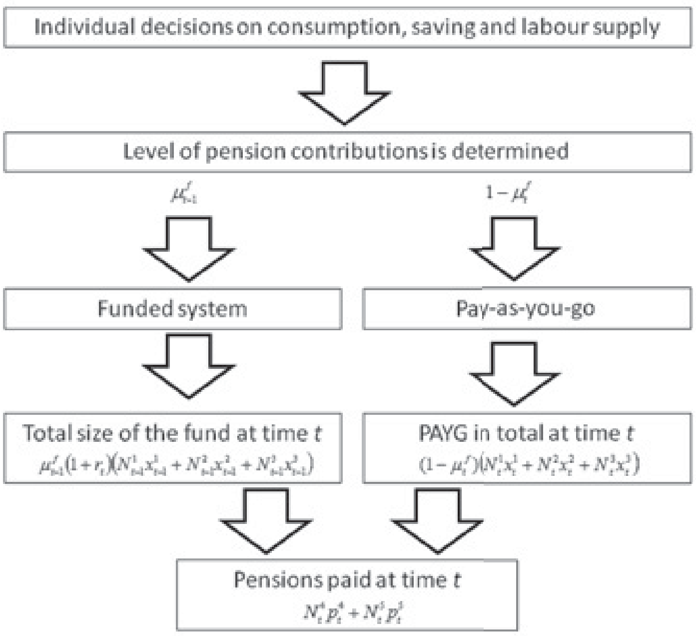

Figure 1. Pension system in the model

Pension benefits are given by:

(6) $$p_t^i = \begin{cases} 0 \text{ for } i = 1,2,3 \\ \mu_t^{rr} \frac{\sum_{j=1}^{3} w_t \epsilon^j l_t^j}{3}, \text{ for } i = 4,5 \end{cases}$$

where μ_t^{rr} is the replacement rate, that is, the percentage amount of mean gross wage income that is given to the retirees, and it adjusts endogenously to the changes in the pension scheme.

Maximization problem gives out the following first-order conditions for an individual born at *t*:

(7) $$\frac{1}{(c_{t+i-1}^i)^{\sigma^c}} = \frac{\beta(1+r_{t+i})}{(c_{t+i}^{i+1})^{\sigma^c}} \text{ for } i = 1,2,3,4,$$

(8) $$\frac{(1-\tau^p) w_{t+i-1} \epsilon^i}{(c_{t+i-1}^i)^{\sigma^c}} = \frac{\gamma^i}{(1-l_{t+i-1}^i)^{\sigma^l}} \text{ for } i = 1,2,3,$$

Equation (7) is the Euler equation which determines the optimal consumption path and (8) gives out the intratemporal condition for labour supply.

2.3. The Production Sector

Firms operate in a competitive market and maximize profits with respect to capital and labour. Every period the firms will use capital until marginal product equals the rental rate and employ labour until marginal product of labour equals the wage rate. Constant returns to scale are assumed. Production function is given by:

(9) $$Y_t = (K_t)^{\alpha} (L_t)^{1-\alpha},$$

where Y_t, K_t, and L_t are output, capital and effective labour supply, respectively.[7] Labour share of output is given by $(1 - \alpha)$.

According to, for example, Antras (2004), the aggregate elasticity of substitution is between 0.5 and 1, thus, I assert that the Cobb-Douglas form is a reasonable specification.

[7] *Capital letters denote aggregate variables from here onwards.*

Wage rate and interest rate are determined by the following equations:

(10) $$w_t = (1-\alpha)\left(\frac{K_t}{L_t}\right)^{\alpha},$$

(11) $$r_t = \alpha \left(\frac{K_t}{L_t}\right)^{\alpha-1} - \delta,$$

and δ denotes the depreciation rate of capital. The Inada conditions apply. The resource constraint of the economy holds at all times, thus, the next period capital is given by:

(12) $$K_{t+1} = Y_t - C_t + (1-\delta) K_t.$$

3. Properties of the Model

3.1. Calibration

I calibrate the model to match the long-run features of the Finnish economy. Individuals are assumed to be born as workers at the age of 21 and live for 60 years. A period in the model corresponds to 12 years and individuals live for 5 periods. This fits the data quite well. In 2012, the life expectancy at birth was a little over 80 years and the expected duration of working life[8] in Finland in 2011 was a little over 37 years.

3.1.1. Preferences

I set the yearly discount factor, β, to 0.97, thus, I have $0.97^{12} \approx 0.6938$. This value produces empirically reasonable capital-output ratio of approximately 3. The risk aversion parameter or, if you will, the inverse intertemporal elasticity of substitution with respect to consumption, σ^c, is calibrated to be unity, thus, I assume logarithmic preferences with respect to consumption. According to Rogerson (2007), the estimates of the inverse elasticity of substitution with respect to labour, σ^l, are in the range between 1 and 3. The inverse of elasticity with respect to labour is calibrated to be equal to 2.

[8] *Eurostat calculates the duration of working life indicator as the number of years a person aged 15 is expected to be active in the labour market throughout his or her life.*

I calibrate the cohort-specific parameter, γ^i, that measures the relative weight of leisure to consumption in the utility function, so that the working hours of the Finnish economy are replicated. The working hours of an average individual as a function of age is an inverse U-shaped curve so that 15–24, 25–44 and 45–64 year olds work, on average, 9%, 24% and 21% of their active time, respectively.[9] Using this information and the assumption that the average value of a group is also the median value, I use standard interpolation methods to find that $\bar{l}^1 = 0.169$, $\bar{l}^2 = 0.250$, and $\bar{l}^3 = 0.236$, where the upper bar denotes a steady state value and l^i is cohort i's labour supply. The emerging γ^1, γ^2 and γ^3 are reported in Table 1.

3.1.2. Technology

The share of capital in the production function, α, is set to 1/3. Capital depreciation rate is set to 0.08 which means that approximately 63% of capital depreciates in 12 years. The age specific efficiency index is taken from Auerbach and Kotlikoff (1987) who use the formula:

(8)

$$\epsilon(exp) = 4.47 + 0.033exp - 0.00067exp^2,$$

where *exp* is the work experience of an individual in years. The index is given in Table 1. I am implicitly assuming, that wages represent efficiency. Empirically wages keep on rising until the very last years of working career, thus, I assume that efficiency, within the model framework, is rising as well.

Table 1. Labour supply parameters

Generation (i)	1	2	3
Efficiency index (ϵ^i)	4.656	4.851	4.853
Appreciation of leisure (γ^i)	4.358	3.059	2.626

[9] *Source: Statistics Finland, Time use survey*

The exogenous index of the level of technology, also known as Harrod-neutral technical progress, is set to grow 1% a year. This means that $g = (1 + 0.01)^{12} - 1 = 0.1268$.

3.1.3. Demographics

Population growth has been quite stable in Finland since the 1970s. The volatility in population growth is mostly due to the World Wars and years right after. Motivated by the most recent decades, I set n to be 0.046 in the benchmark scenario. This translates to yearly population growth of approximately 0.38% which matches the average growth rate in the data in 1970–2011.

3.1.4. Pension Scheme

The average earnings-related contribution rate, including employers and employees, in 2011 was 22.1%. As the contribution rate is the only policy rate, I set the total contribution rate to be {0.221 / (1 + 0.221)} = 0.1810.

Total pension benefits in 2012 were 22.0 billion euros. Of that amount 2.6 billion euros came from the pension funds and the rest, 19.4 billion euros, were taken directly from the pension contributions. Therefore I set μ_t^f to 2.6 / 22.0 = 0.118 and $(1 - \mu_t^f)$ to 0.882.

Table 2. Benchmark calibration

α	β	δ	σ^c	σ^l	n	μ_t^f
0.33	0.6938	0.6823	1	2	0.046	0.118

3.2. Steady State

Inspection of steady states gives us insight into how the model behaves. In steady state all aggregate variables grow at the exogenously given rate which equals (1 + n)(1 + g). Table 3 presents the steady state capital-output ratio K / Y, consumption-output ratio C / Y, the annualized interest rate r, the wage rate w, ratio of total contributions to output *X* / *Y*, ratio of total pension benefits to output *B* / *Y*, aggregate labour supply per capita *l* and the endogenous replacement rate μ^{rr} in the absence of pension scheme, with the benchmark[10] pension scheme and both polar extremes of pension schemes (funded and PAYG).

The capital-output ratio in reported steady states varies from 2.90 with PAYG to 3.34 with competitive market. For comparison, the Finnish capital-output ratio was at its highest[11] in 1993 being 3.4 and at its lowest in 2007 being 2.4. The 1975–2010 mean is 2.9. The capital-output ratios obtained from the model are thus reasonable. All scenarios are dynamically efficient so that there is no overaccumulation of capital in steady state.

I shall first examine the effect of social security in the scenarios. Pension contributions are collected as a percentage of periodic labour income, thus, the labour supply decision is distorted with the social security in operation. The wage income net of social contributions is lower and therefore labour supply is depressed.

The capital-output ratio is also lower with social security compared to competitive economy, translating into lower wage rate and higher real interest rate. In addition, aggregate consumption decreases when social security system is in place, but the consumption-output ratio increases because the aggregate output decreases even more than consumption. There is clearly less aggregate saving with the social security in place, and this means lower capital accumulation and lower capital-output ratio.

The differences between different forms of pension schemes are less significant, but the direction of the change is quite clear. At first glance, the welfare loss[12] seems to be smaller within the fully funded system as the replacement rate is higher and capital accumulation is larger. This stems from the fact that the return of a fully funded pension scheme (r_t) is higher than the return from PAYG scheme (*n*). I will analyze the overall and transitional welfare effects in section 4.

Table 3. Steady states

	No social security	Mixed*	PAYG**	Funded system***
C / Y	0.774	0.803	0.804	0.797
K / Y	3.34	2.91	2.90	3.01
r (annual)	0.038	0.047	0.048	0.045
w	0.352	0.328	0.328	0.334
l	0.770	0.714	0.716	0.704
X / Y	0	0.121	0.121	0.121
B / Y	0	0.127	0.121	0.174
μ^{rr}	0	0.427	0.404	0.578

* $\mu^f = 0.118$
** $\mu^f = 0$
*** $\mu^f = 1$

[10] *Benchmark is the case where 11.8% of the contributions are funded.*
[11] *In 1975–2010 data.*

[12] *Welfare will be more precisely defined in subsection 4.2.*

3.3. Sensitivity Analysis

Table 4 shows the response of the benchmark steady state values (in bold) to changes in utility discount factor β, inverse of intertemporal elasticity of consumption σ^c, inverse of elasticity with respect to labour σ^l and contribution rate τ^p with respect to consumption-output ratio C/Y, investment to output ratio I/Y, capital to output ratio K/Y, time allocated to labour l, annual real interest rate r, wage rate w and replacement rate μ^{rr}.

Increasing β makes agents more patient (see equation (7)). They save more and consume less relative to output[13] and, as a result, they work more. Due to working more and accumulating more capital, production increases, interest rate decreases and wage rate increases. The final outcome is that aggregate consumption increases, consumption-output ratio decreases, because output increases more than consumption, capital level increases and so does the capital-output ratio.

Increasing σ^c decreases the intertemporal elasticity of consumption (again see (7)) and makes the agents care more about consumption smoothing. Agents become more risk averse. Individuals are less willing to delay consumption. Consumption increases in all cohorts and as a result the right hand side of (8) decreases and the agent wants to supply more labour in order to equate the equation (8). Labour supply increases significantly, saving (and thus capital) decreases a little and the result is higher output, but clearly lower capital-output ratio.

If σ^l increases, individuals appreciate leisure more than before. Labour supply decreases and, by equation (8), also consumption must decrease. All in all, people are basically poorer because they work considerably less. The great ratios, however, remain approximately the same.

Increasing the contribution rate, τ^p, induces agents to appreciate leisure more as labour supply decision is more distorted than before. Labour supply decreases. At the same time, agents realize that the government save more on their behalf than before and adjust their own saving rate accordingly. In the aggregate, saving decreases, capital accumulation decreases and as a result, output decreases. Youngest workers are worse off, because they bear most of the costs of increased contribution rate, but already the second cohort is better off compared to utility level in benchmark steady state.

4. Policy experiments

4.1. Transition Path

Figure 2 shows the evolution of capital-output ratio, labour supply, interest rate and replacement rate when economy is transitioning from the original steady state with a mixed

Table 4. Sensitivity analysis

Sensitivity to:		β		σ^c		σ^l		τ^p	
		0.96^{12}	0.98^{12}	0.5	2	1	3	0.15	0.21
C/Y	**0.803**	0.832	0.771	0.795	0.822	0.798	0.801	0.799	0.808
K/Y	**2.91**	2.49	3.40	3.03	2.63	2.99	2.94	2.98	2.85
I/Y	**0.197**	0.168	0.230	0.205	0.178	0.202	0.199	0.201	0.192
r	**0.047**	0.058	0.037	0.045	0.054	0.046	0.047	0.046	0.049
w	**0.328**	0.304	0.355	0.335	0.312	0.333	0.330	0.332	0.325
l	**0.714**	0.698	0.735	0.345	1.154	0.701	0.620	0.724	0.705
μ^{rr}	**0.427**	0.436	0.419	0.409	0.440	0.396	0.427	0.351	0.498

[13] *Actual individual consumption increases but consumption-output ratio decreases.*

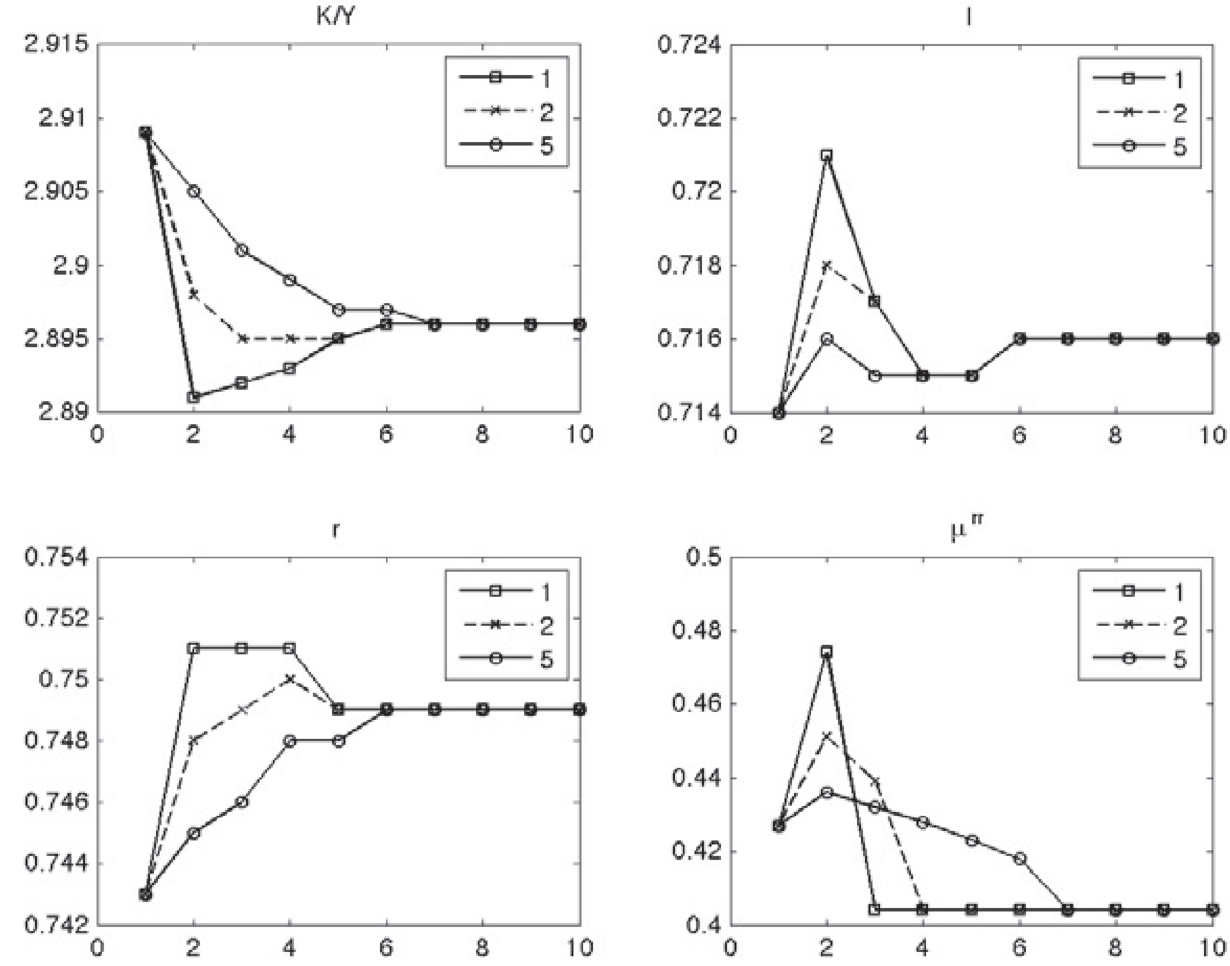

Figure 2. Transition to a full PAYG scheme: transition to a new system takes place in 1, 2 or 5 periods

social security scheme $(\mu_t^f = 0.118)$ to a pure pay-as-you-go scheme $(\mu_t^f = 0)$ with various numbers of transition periods[14]. Figure 3 shows the transition from original steady state to a purely funded system $(\mu_t^f = 1)$. One time unit is approximately 12 years. The regime shift is introduced in period 2.

From Figures 2 and 3 it is obvious that the longer the transition period, the smoother the transition. The bang-bang type of reform introduces huge swing to the system in the first period after which it converges smoothly to the new steady state.

In transition to the PAYG system (Figure 2), at first, there are "excess" funds from the previous period, because part of the previous period's pension contributions were funded to be used in the current period. This induces the replacement rate (μ_t^{rr}) to increase temporarily after which it adjusts to a new, lower level.

In Figure 3 one sees the opposite case from Figure 2. When moving to a fully funded system, there is, at first, a period where the PAYG scheme is no longer functional. Only a small amount of contributions is caried over from the previous period, and, on the other hand, there are no intra-period pension transfers. This, again, induces a decrease in the replacement rate on impact, before it adjusts to a higher (than original) steady state. As the transition progresses and the pension scheme

[14] *Transition takes place in equal sized steps, for example, transition from benchmark economy to fully funded economy in five steps translates to* $\{\mu_t^f, \mu_{t+1}^f, \mu_{t+2}^f, \mu_{t+3}^f, \mu_{t+4}^f, \mu_{t+5}^f \dots\} = \{0.1180, 0.2944, 0.4708, 0.6472, 0.8236, 1, 1, \dots\}$ *where each step size is 0.1764.*

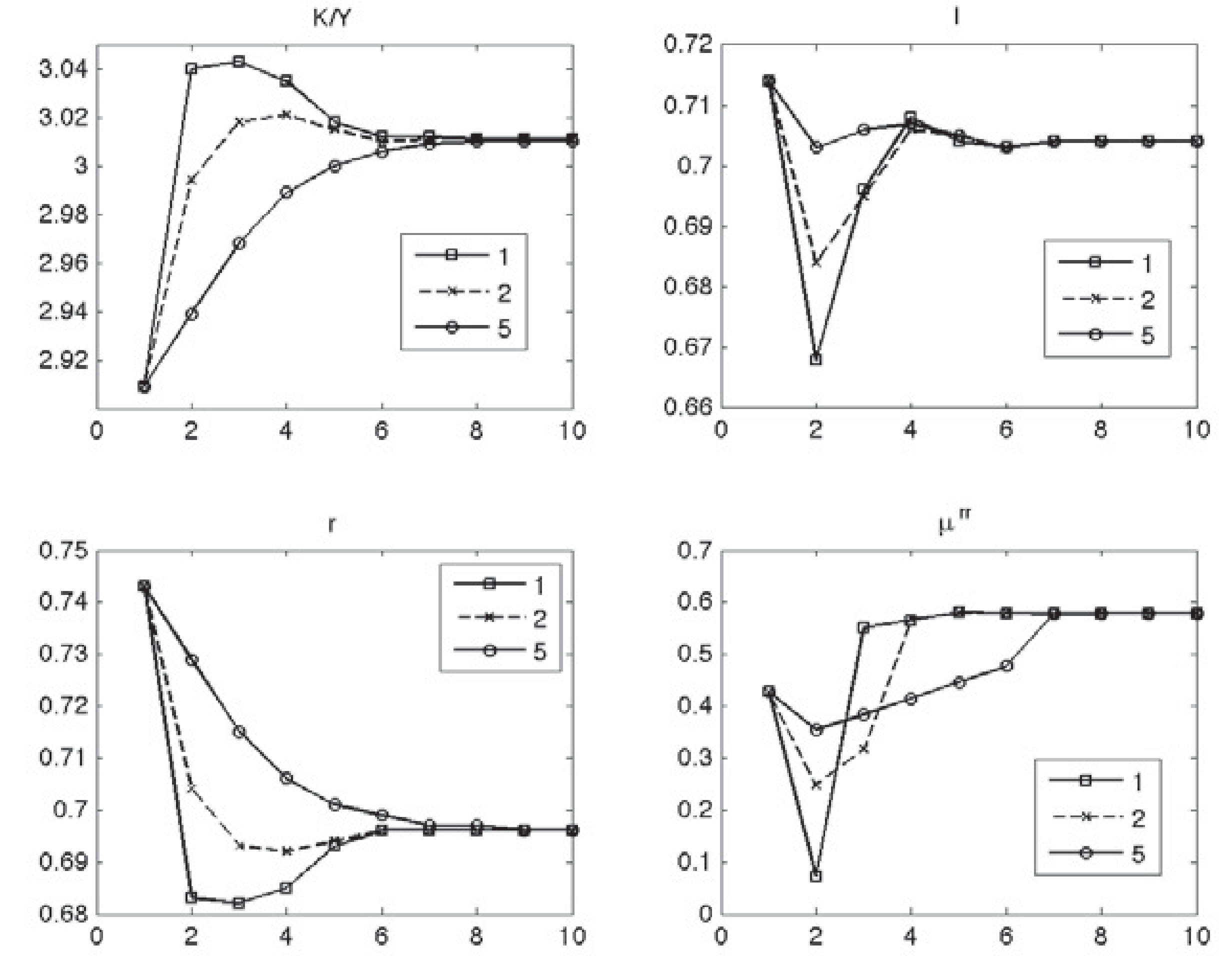

Figure 3. Transition to a fully funded scheme: transition to a new system takes place in 1, 2 or 5 periods

becomes fully funded, the amount of capital in the economy increases, albeit the aggregate private saving decreases. This is a reaction to government saving on individuals behalf via the pension scheme.

4.2. Welfare analysis

Figure 4 presents how total steady state lifetime utility varies as a function of μ_t^f. The more there is funding in the pension scheme, the higher is the lifetime utility of the first cohort. The opposite applies to the second, third, fourth and fifth cohorts. This is mostly due to the fact that, with a fully funded pension scheme, agents' saving decisions are not as distorted as in PAYG and, thus, agents are able to smooth their consumption better. Also the institution (funded scheme) is a lot more efficient in the sense that the agreed pension contribution rate produces a significantly higher replacement rate.

These results give interesting insights regarding the political economy of this framework. Assume that agents vote in the beginning of the period. The median voter in this framework is found in the third cohort implying political pressure on decreasing the funding of the system.Therefore, no matter what the total welfare of the society from this moment on is, the political economy of this model suggests that, as long as politicians try to please the median voter, the funding of the system will be decreased. The society with this sort of demographic structure will end up in the system with $\mu_t^f = 0$.

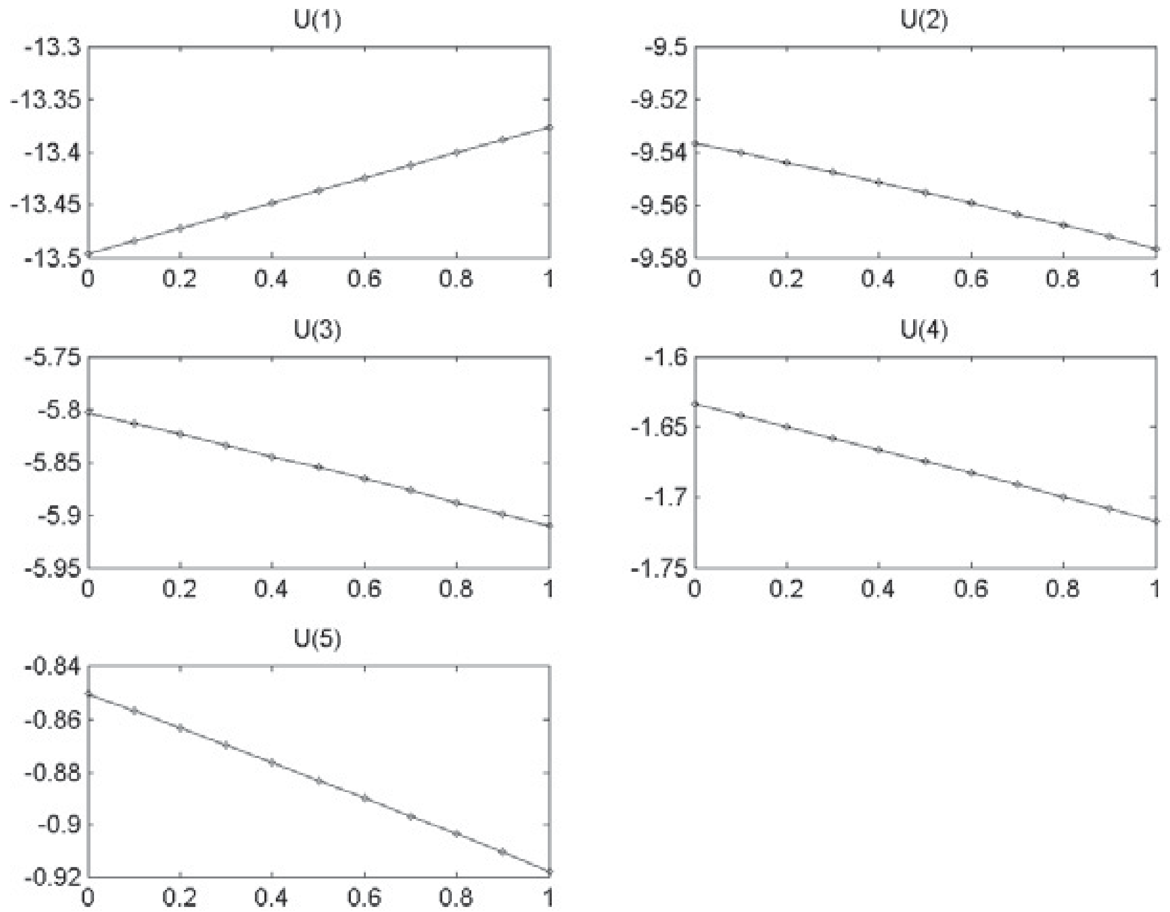

Figure 4. Utility levels in steady state as a function of μ_t^f; *U(i)* is the remaining life time utility of cohort *i*

Still thinking in terms of steady states, Figure 4 implies that there are no possible Pareto improvements to be made by adjusting the funding scheme of the pensions. Whenever funding is increased, the second, third, fourth and fifth cohorts are worse off in the steady state. The opposite applies to the youngest cohort and also to all future, yet unborn, cohorts. One must construct a measure of welfare if one wants to compare different pension schemes from the perspective of the whole society. As a sidenote, it might be possible to accomplish Pareto improvement if one is allowed to use lump-sum transfers in the transition phase. This, too, is left for future research. Next I shall construct a measure of welfare to compare the steady state values with different levels of funding in the pension scheme. After that, I shall turn to analysis of the transition path.

The measure of welfare used in this paper is based on the Hicks compensation principle. I will follow Cooley and Soares (1999) in the construction of this measure. Let s_t^i be the total level of assets of an agent of age i, u_t^i be the compensation given to an agent of age i and S_t is the aggregate state of the economy. The question is, how much must an agent be compensated, in terms of consumption in the first period, so that the lifetime utility level is the same as in the world without social security (competitive economy). Thus, it must be that $V\left(s_t^i + u_t^i, S_t; \mu_t^f\right) = V_t^i$, where V_t^i is the lifetime utility level of agent in the competitive economy. The total discounted welfare cost is:

(14)

$SW = \sum_{t=0}^{\infty} \hat{\beta}^t \varphi^1 u_t^1 + \varphi^2 u_0^2 + \varphi^3 u_0^3 + \varphi^4 u_0^4 + \varphi^5 u_0^5,$,

where $\hat{\beta}$ is the discount factor and φ^i is the population share of cohort $i\,(= N_t^i/POP_t)$. The measure of welfare cost is $SW(1+r)/y$ where r is the real interest rate and y is the real output per capita in the original steady state. The question is how to choose β, the "intergenerational" discount factor. Arrow et al. (2012) provide an answer for how intra- and intergenerational discounting could be made consistent: "This is an easy question: if benefits and costs are to be discounted at a constant exponential rate, the same rate must be used to discount costs and intra- and intergenerational benefits." Thus, I will set $\hat{\beta}$ to equal individual utility discount factor β.

The measure of welfare in different steady states is plotted in Figure 5.[15] This measure is interpreted to be the wellbeing after the transition period is over and the economy has converged into a new steady state.

The measure of welfare is a strictly decreasing function of μ_t^f. This means that agents must be compensated more in the scenario where $\mu_t^f = 0$ compared to scenario where $\mu_t^f = 1$. The conclusion is crystal clear: in steady state, the funded pension scheme is better, because the welfare costs are smaller.

Figure 6 plots the measure of welfare cost as a function of different transition lengths. If the pension reform is introduced in a bang-bang manner, the economy is best off by not reforming at all. If the choice is between funding or PAYG, the economy is better off in moving towards the pay-as-you-go scheme. This is because the "sudden" drop in the pension replacement rate is too large, thus, the welfare loss for the pensioners of the time is bigger than the sum of discounted welfare gains for the first cohort of the now and the future. If the duration of the transition is more moderate, that is to say longer than one period, transition to a funded pension scheme pays off in terms society's wellbeing.

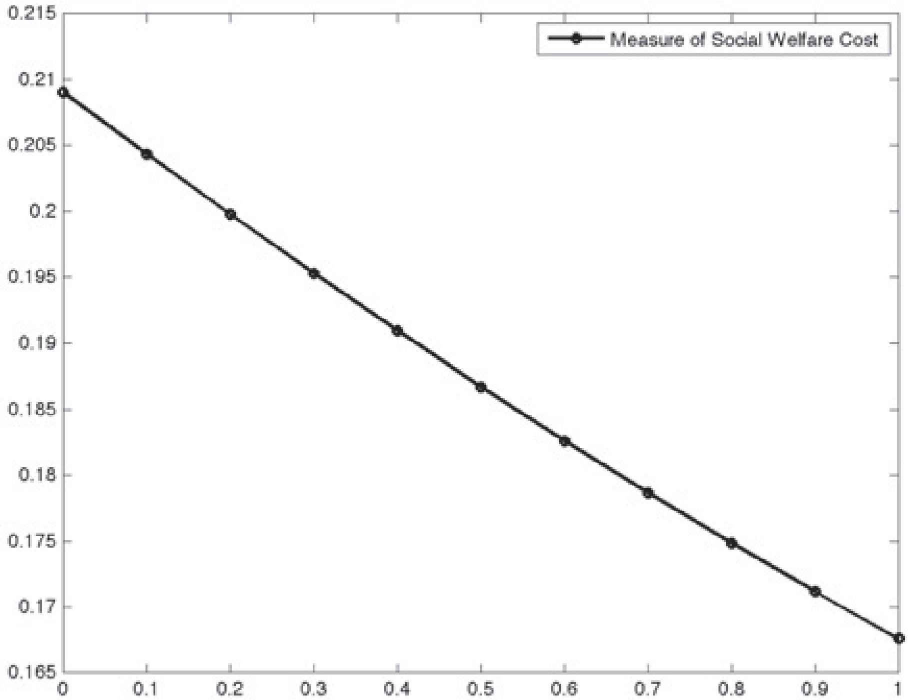

Figure 5. Measure of social welfare cost (y-axis) as a function of (x-axis)

[15] $\sum_{t=0}^{\infty} \hat{\beta}^t \varphi^1 \bar{u}_t^1 + \varphi^2 \bar{u}^2 + \varphi^3 \bar{u}^3 + \varphi^4 \bar{u}^4 + \varphi^5 \bar{u}^5$, where $\bar{u}$ is the steady state value of u^i.

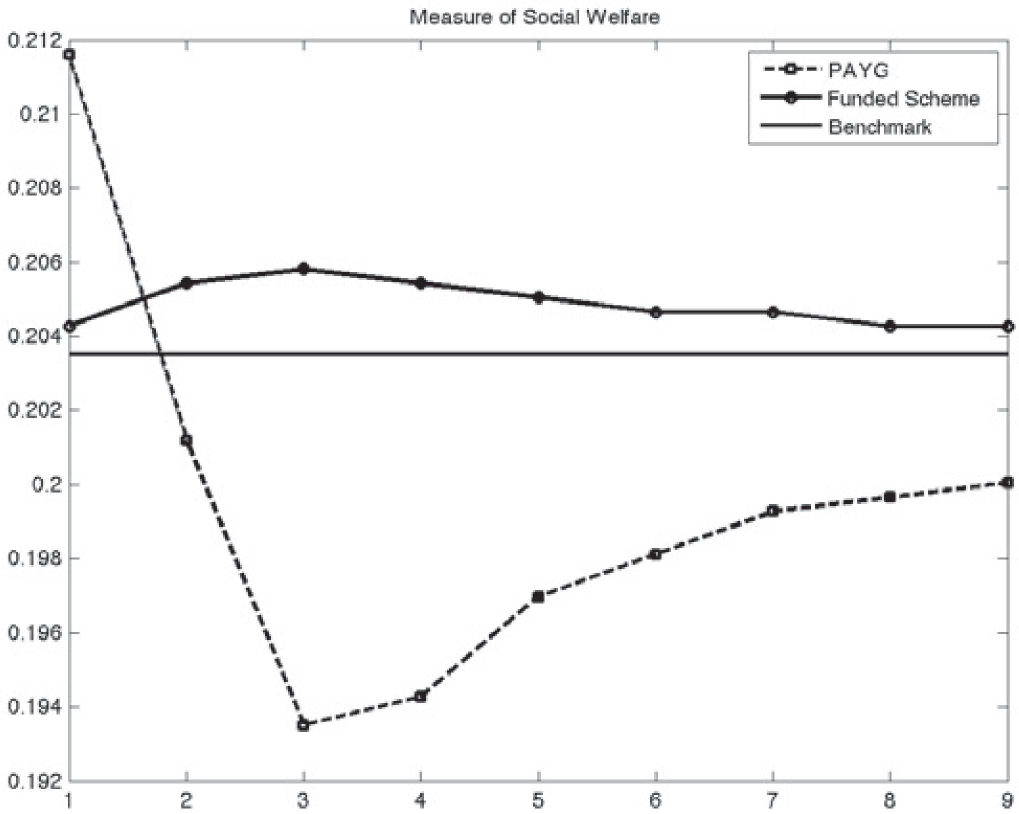

Figure 6. Measure of social welfare cost (y-axis) given different transition periods (x-axis

From Figure 6, we do not only see that reforming the system into a fully funded one is a favourable decision (if it is not carried in haste), but also that the optimal speed of adjustment is three periods, or, about length of an individual's working career. If the reform takes place in t, in period $t + 3$ the pension scheme already provides (almost) steady state benefits, which are higher than the original benchmark benefits, thus, the cohorts born in t or later are the ones that mostly harvest the profit from the reform, and also, they get the biggest weight in the calculation of the measure of social welfare cost $(\varphi^1 > \varphi^i, \forall\, i = \{2,3,4,5\})$.

5. Conclusions

Using an overlapping generations dynamic general equilibrium model, I show that, in steady state, the society is better off having a funded pension scheme instead of a PAYG scheme or a mixed scheme. This result is not new as such and it is demonstrated in many macroeconomic textbooks in a lot more simplified framework, usually with lump sum taxes (cf. Heijdra and van der Ploeg 2002).

In the model used in this paper, there is a richer labour market, several cohorts and a particular pension scheme. Within this framework, a (mixed) pension scheme is described as a linear combination of PAYG and funded pension scheme. This simple extension is, to my knowledge, a novel way of describing the pension scheme.

I show that, in steady state, social welfare is an increasing function of the amount of funding. This is mostly driven by an increase in the saving rate. In addition, the model specification enables to portray a transition from a mixed pension scheme either to a PAYG scheme or to a fully funded scheme. Once I know the transition path, I can compare the welfare implications of a regime shift. Given a certain measure of welfare, I show that transition, if not implemented as a bang-bang solution, to a fully funded system is better than transitioning to a PAYG scheme given my framework.

All the results hinge on the fact that, in the long run, real interest rate outweighs the population growth rate[16], the economy is dynamically efficient and the return on the funded system is greater in steady state compared to a PAYG scheme. This raises another question: is the economy dynamically efficient? According to, for example, Abel et al. (1989) the answer is yes, at least between 1929–1989 in the US. However, as the authors put it *"In an uncertain world, there is no obvious metric for economic growth; nor is there a single rate of return."* Thus in practice, the matter is far more complicated than what is suggested in this article.

Subjects for future research include a richer description for the demographic process. An endogenous population growth and/or immigration flows would be good additions as the pension system critically depends on the changes in the demographic pyramid.

Another addition for the future research would be uncertainty. There are two key features in a pension scheme: time and risk. The risk is that pension benefits will be less than expected when the plan was first started. This risk could be partly accounted for by introducing uncertainty to the return of the pension fund.

A third, interesting and important addition would be the inclusion of a more elaborate public sector. Social security is not only about pension benefits, as in the framework used in this article, but also about the services that the public sector provides. As the population ages, this feature is critical when assessing the long term budget balance of the public sector. A richer public sector would probably not, however, turn around the conclusions of this paper. This is because, as Auerbach and Kotlikoff (1987) demonstrated, PAYG system is equivalent to debt-financed fiscal policy. The analysis in this paper partly takes this fact into account.

A fourth extension left for future research, perhaps the most important one, is to show that a gradual shift from the baseline economy to a fully funded system can (or cannot) be Pareto improving. Modigliani et al. (2000) have already calculated that the US would be able to reduce the social security contribution rate just by transitioning to a fully funded pension scheme. The estimate is a a change from the projected future 18% contribution rate to below 6% without any sacrifices, using the purported surplus of the pension scheme to increase national saving.

A reform to a fully funded pension scheme may seem far-fetched from a pragmatic point of view. However, from an academic or positive point of view, it would not have to be.

[16] Or, actually, the real growth rate of payrolls outweighs the population growth rate, to be more precise.

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