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SEARCH WITH HOMOGENEOUS AND HETEROGENEOUS AGENTS*

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Two types of agents want to pair. They meet randomly, and if each is acceptable to each other pairing takes place. One type's tastes are uniform over the other type while the other type's tastes are not. We assume nontransferable utility and study whether the outcome is more efficient when the type heterogeneous in tastes searches or vice versa. (JEL: C78, D4)

1. Introduction

In the search, random matching and directed search literature it is often just postulated that one of two types of agents who want to pair up searches and the other type waits. Sometimes it is interesting to ask whether it makes more sense for one party to search and the other party to wait. This question seems particularly important when there is heterogeneity. Should the more heterogeneous agents search for less heterogeneous ones or vice versa? We answer this question in a model where the parties are men and women. There is idiosyncratic heterogeneity on one side only; all the men regard all the women the same, while a man's value to a woman is different across women. However, there is no unambiguously best man but one woman may regard a man as very desirable while another woman may regard the same man as not very worthy. On top of this we assume nontransferable utility. This is basically the same model as in Burdett and Wright (1996).

The question about who searches in an environment where meetings feature frictions has been answered for the homogeneous population in Kultti, Miettunen, Takalo and Virrankoski (2009) and Kultti, Miettunen and Virrankoski (2006). The robust result of these articles is that the more numerous party should search.

This article is organised as follows. In section 2 we introduce the model and in section 3 we conduct the analysis. To get more clear cut re-

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sults, in section 4 we focus on the special case where heterogeneity is uniformly distributed. Concluding remarks are in section 5.

2. The model

Consider a model with M men and W women. All the women look alike to the men and if a man manages to pair with a woman he gets utility 1. The men do not look the same to the women but their quality is distributed according to a distribution function F with support on interval $[\underline{z},\bar{z}]$. The quality of a man is not an intrinsic feature but rather an idiosyncratic feature of a woman; ex ante all the men are symmetric and upon meeting a man a woman's evaluation of him is determined by a random draw from distribution F. Thus, a man may be regarded of low quality by some woman but of high quality by some other woman. The draws across women are independend. This kind of heterogeneity is studied in Burdett and Wright (1996). We also adopt their assumption about nontransferable utility; it is not possible to provide any transfers to help pairing but the only choice the men and women have is whether to pair up or not. Pairing requires the consent of both parties.

2.1. Men look for women

We assume that the parties meet according to an urn-ball-matching model where women wait (urns) and men search (balls) or are randomly distributed over the women. The economy proceeds in discrete time to infinity, and when agents pair up they are replaced by identical agents so that the ratio of men to women remains the same. Let $\theta = \frac{M}{W}$. Next we calculate the expected life time utilities of men and women assuming that both have the same discount factor δ. We evaluate the value functions at the end of a period assuming that women's strategy is to accept any man whose quality exceeds a certain limit, i.e. women accept a man of quality z if and only if $z \geq \tilde{z}$.

(0)
$$V_w = \delta \begin{Bmatrix} \sum_{k=0}^{\infty} e^{-\theta} \frac{\theta^k}{k!} F^k(\tilde{z}) V_w \\ + \sum_{k=0}^{\infty} e^{-\theta} \frac{\theta^k}{k!} \int_{\tilde{z}}^{\bar{z}} zkF^{k-1}(z)\,dF(z) \end{Bmatrix}.$$

Above the first term lists the cases when a woman meets any number k of men. She does not accept any of them if all of them are below her treshold $\tilde{z}$ which happens with probability $F^k(\tilde{z})$ where it is assumed that the men's qualities are independent. When a woman does not accept anyone she goes on waiting for the meetings that take place next period and the value of this option is V_w. The second term lists the cases where a woman meets k men and accepts the best of them. The highest quality man can be any of the k men and this quality can be any $z \in [\tilde{z}, \bar{z}]$.This happens with probability $dF(z)$. In order to be the highest value all the other $k - 1$ men's qualities have to be lower and this happens with probability $F^{k-1}(z)$. To get the expected value one must integrate over all the possible qualities.

To get a more convenient form we partially integrate the expression under the latter sum

(1)
$$\int_{\tilde{z}}^{\bar{z}} zkF^{k-1}(z)\,dF(z) = \bar{z} - \tilde{z}F^k(\tilde{z}) - \int_{\tilde{z}}^{\bar{z}} F^k(z)\,dz.$$

Inserting this into the first equation, and changing the order of summation and integration then yields

(2)
$$V_w = \delta \begin{Bmatrix} e^{-\theta(1-F(\tilde{z}))} V_w + \bar{z} - \tilde{z} e^{-\theta(1-F(\tilde{z}))} \\ - \int_{\tilde{z}}^{\bar{z}} e^{-\theta(1-F(z))} dz \end{Bmatrix}.$$

Next we evaluate the expected utility of a man

(3)
$$V_m = \delta \begin{Bmatrix} F(\tilde{z}) V_m + \sum_{k=1}^{\infty} e^{-\theta} \frac{\theta^k}{k!} \int_{\tilde{z}}^{\bar{z}} \left(1 - F^k(z)\right) dF(z) V_m \\ + \sum_{k=0}^{\infty} e^{-\theta} \frac{\theta^k}{k!} \int_{\tilde{z}}^{\bar{z}} F^k(z)\,dF(z) \end{Bmatrix}.$$

The first term is the probability that the woman finds the man unacceptable times the expected utility from search. The second term lists the cases where a man is not selected by the woman, even though he is good enough, as there are even better men available. Thus the man continues search next period, the value of which option is V_m. The third term lists the cases where a man, upon meeting a woman with k other men, is selected by the woman. This happens if the man is of highest quality amongst the $k+1$ men, and his quality is above $\tilde{z}$. The probability for this event is $F^k(z)dF(z)$, and then we must integrate over all possible values to get the expected value.

Manipulating a little yields

(4)
$$V_m = \delta\left\{\frac{\theta - 1 + e^{-\theta(1-F(\tilde{z}))}}{\theta}V_m + \frac{1-e^{-\theta(1-F(\tilde{z}))}}{\theta}\right\}.$$

2.2. Women look for men

An alternative market structure is such that men wait for women to contact them. In this case the Poisson-parameter governing the meeting probabilities is $\phi = \theta^{-1} = \frac{W}{M}$. The analysis proceeds analogously to the previous case. The women's acceptance treshold is not necessarily the same and we denote it by $\hat{z}$. Let the woman under study use acceptance treshold y. The expected utility of a woman is given by

(5)
$$U_w = \delta\left\{\begin{array}{l} F(y)U_w \\ +\sum_{k=0}^{\infty} e^{-\phi}\frac{\phi^k}{k!}\sum_{j=0}^{k}\binom{k}{j}(1-F(\hat{z}))^j F^{k-j}(\hat{z})\frac{1}{j+1}\int_y^{\bar{z}} z dF(z) \\ +\sum_{k=1}^{\infty} e^{-\phi}\frac{\phi^k}{k!}\sum_{j=1}^{k}\binom{k}{j}(1-F(\hat{z}))^j F^{k-j}(\hat{z})\frac{j}{j+1}(1-F(y))U_w \end{array}\right\}$$

Above the first term lists the case where the woman does not match because the man is not of high enough quality, and this happens with probability *F(y)* regardless of how many other women meet the man. The second term lists the cases where the woman pairs up with a man of sufficient quality; when there are k other women this requires going through all the cases where any number of these other women also regard the man as sufficiently good. If there are j such women the probability that the woman under study matches with the man is $\frac{1}{j+1}$.

For these reasons there is the binomial-type second sum. Finally, to get the expected value we have to integrate over all acceptable quality levels. The last sum lists the cases where the woman does not match with the man even though the man would be of sufficient quality. This happens if some of the other women who regard the man as good enough gets to match with him. To get a nice form for the woman's utility we first note that the binomial-type sum in the second term can be expressed as

(6)
$$\frac{1-F^{k+1}(\hat{z})}{(k+1)(1-F(\hat{z}))}\int_y^{\bar{z}} z dF(z)$$

and that the binomial-type sum in the third term can be expressed as

(7)
$$(1-F(y))\left[1-\frac{1-F^{k+1}(\hat{z})}{(k+1)(1-F(\hat{z}))}\right]U_w.$$

Inserting these into expression (5) and manipulating a little yields

(8)
$$U_w = \delta\left\{\begin{array}{l} \frac{\phi(1-F(\hat{z})) - (1-F(y))\left(1-e^{-\phi(1-F(\hat{z}))}\right)}{\phi(1-F(\hat{z}))}U_w \\ +\frac{1-e^{-\phi(1-F(\hat{z}))}}{\phi(1-F(\hat{z}))}\int_y^{\bar{Z}} z dF(z) \end{array}\right\}.$$

The men's expected utility is given by

(9)
$$U_m = \delta\left\{\begin{array}{l} \sum_{k=0}^{\infty} e^{-\phi}\frac{\phi^k}{k!}F^k(\hat{z})U_m \\ +\sum_{k=1}^{\infty} e^{-\phi}\frac{\phi^k}{k!}\left(1-F^k(\hat{z})\right) \end{array}\right\}.$$

The first term lists the cases when a man is not selected, and this happens when no woman turns up or when all the women who turn up regard the man as of insufficient quality. The second term lists the cases where the man gets to pair up with a woman, and this happens with the residual probability. A little manipulation yields the following simple expression

$$(10)\quad U_m = \delta\left\{e^{-\phi(1-F(\hat{z}))}U_m + 1 - e^{-\phi(1-F(\hat{z}))}\right\}.$$

3. Analysis

Let us first determine the women's acceptance levels. When men search for women the women are assumed to accept any man $z \geq \tilde{z}$. In equilibrium a woman must be indifferent between accepting a man $z = \tilde{z}$ and continuing to wait for other men which means that $\tilde{z} = V_w$. Notice that a woman's decision does not depend on what acceptance level other women adopt, and the optimal level for one woman is optimal for every woman. Solving for V_w and manipulating a little yields the following condition

$$(11)\quad \tilde{z} - \delta\left(\bar{z} - \int_{\tilde{z}}^{\bar{z}} e^{-\theta(1-F(z))}dz\right) = 0.$$

Let us denote the left hand side of (11) by function $f(\tilde{z})$. Evaluating the function at the lower bound gives expression

$$(12)\quad f(\underline{z}) = \underline{z} - \delta\left(\bar{z} - \int_{\underline{z}}^{\bar{z}} e^{-\theta(1-F(z))}dz\right)$$

which is clearly negative for sufficiently large values of the discount factor. Evaluating the function at the upper bound gives expression

$$(13)\quad f(\bar{z}) = \bar{z}(1-\delta)$$

which is clearly positive. As f is a continuous function there exists a value that satisfies $\tilde{z} = V_w$. Further, as the derivative of f

$$(14)\quad f'(z) = 1 - \delta e^{-\theta(1-F(z))}$$

is everywhere positive this equilibrium value is unique.

The case where women look for men is dealt with analogously. In equilibrium the woman's acceptance level has to be optimal, i.e. solving for U_w, taking the derivative with respect to y and evaluating it at $y = \hat{z}$ must be zero. Doing this yields the following condition, which is the same condition that would result from evaluating U_w at $y = \hat{z}$ and imposing $\hat{z} = U_w$,

$$(15)\quad \hat{z}(1-F(\hat{z}))\left[\phi(1-\delta) + \delta\left(1 - e^{-\phi\left(1-F^k(\hat{z})\right)}\right)\right] - \delta\left(1 - e^{-\phi\left(1-F^k(\hat{z})\right)}\right)\int_{\hat{z}}^{\bar{z}} z dF(z) = 0.$$

Let us denote the left hand side of (15) by function $g(\hat{z})$. Evaluating it at the lower bound gives expression

$$(16)\quad g(\underline{z}) = \underline{z}\left[\phi(1-\delta) + \delta\left(1-e^{-\phi}\right)\right] - \delta\left(1-e^{-\phi}\right)\int_{\underline{z}}^{\bar{z}} z dF(z)$$

which is clearly negative for large enough values of the discount factor. Evaluating the function at the upper bound gives expression

$$(17)\quad g(\bar{z}) = 0.$$

To figure out how function g behaves we next determine its derivative

$$(18)\quad \begin{aligned} g'(\hat{z}) = {} & \left(1-F(\hat{z}) - \hat{z}f(\hat{z})\right)\left[\phi(1-\delta) + \delta\left(1-e^{-\phi\left(1-F^k(\hat{z})\right)}\right)\right] \\ & - \hat{z}(1-F(\hat{z}))\delta e^{-\phi\left(1-F^k(\hat{z})\right)}\phi f(\hat{z}) \\ & + \delta e^{-\phi\left(1-F^k(\hat{z})\right)}\phi f(\hat{z})\int_{\underline{z}}^{\bar{z}} z dF(z) \\ & + \delta\left(1-e^{-\phi\left(1-F^k(\hat{z})\right)}\hat{z} f(\hat{z})\right). \end{aligned}$$

Evaluating the derivative at the upper bound yields $-\bar{z}f(\bar{z})\phi(1-\delta)$ which is negative. Thus, there exists a value $\hat{z}$ such that $\hat{z} = U_w$.

We cannot show uniqueness, and based on Burdett and Wright (1996) it is unlikely that there is always a unique equilibrium.

4. Equal numbers

Our aim is to study the efficiency of the market depending on whether the heterogenous or the homogenous agents search. To this end we assume in the sequel that there are equal numbers of men and women, i.e. $\theta = \phi = 1$. As mentioned in the introduction it is well known that when the numbers of the parties differ in the homogenous case it is efficient for the more numerous party to search. Assuming $\theta = 1$ eliminates this effect and also yields simpler formulae.

Now the men's and women's expected utitilities when men search are as follows

$$(19)\quad V_m = \delta \frac{1-e^{-(1-F(\tilde{z}))}}{1-\delta e^{-(1-F(\tilde{z}))}}$$

$$(20)\quad V_w = \delta \frac{\bar{z} - \tilde{z}\, e^{-(1-F(\tilde{z}))} - \int_{\tilde{z}}^{\bar{z}} e^{-(1-F(z))} dz}{1-\delta e^{-(1-F(\tilde{z}))}}.$$

When women search and men wait the utilities are as follows

$$(21)\quad U_m = \delta \frac{1-e^{-(1-F(\hat{z}))}}{1-\delta e^{-(1-F(\hat{z}))}}$$

$$(22)\quad U_w = \delta \frac{\frac{1-e^{-(1-F(\hat{z}))}}{1-F(\hat{z})} \int_{\hat{z}}^{\bar{z}} z dF(z)}{1-\delta e^{-(1-F(\hat{z}))}}.$$

Let us next determine the best acceptance levels from the women's point of view as well as the socially efficient acceptance levels in cases where men search, and then when women search. Note that as men regard all women as equally good they favour as low acceptance levels as possible.

Taking the first order condition with respect to $\tilde{z}$ in (20) is easily seen to yield exactly the equilibrium condition (11). This is as one would expect as a woman's decision is not affected by what other women do. Determining the first order condition for (22) with respect to $\hat{z}$ yields the following expression

(23)

$$\int_{\hat{z}}^{\bar{z}} z dF(z) \begin{bmatrix} -e^{-(1-F(\hat{z}))}\left(1-\delta e^{-(1-F(\hat{z}))}\right)\left(1-F(\hat{z})\right) \\ -\left(1-e^{-(1-F(\hat{z}))}\right)\left(1-\delta F(\hat{z}) e^{-(1-F(\hat{z}))}\right) \end{bmatrix}$$
$$-\hat{z}\left(1-e^{-(1-F(\hat{z}))}\right)\left(1-\delta e^{-(1-F(\hat{z}))}\right)\left(1-F(\hat{z})\right) = 0.$$

The $\hat{z}$ determined by this equation is clearly different from that (there may be several) determined by (15). Anyhow, let us see what we can say if we evaluate this expression at $\hat{z}$ given by (15). Substituting, and not requiring equality with zero anymore, yields

(24)

$$\left(1-\delta\right)\int_{\hat{z}}^{\bar{z}} z dF(z) \left[-e^{-(1-F(\hat{z}))}\left(1-F(\hat{z})\right) + 1 - e^{-(1-F(\hat{z}))}\right].$$

To determine the sign of the expression in the brackets let us consider function $f(x) = -e^{-(1-x)}(1-x) + 1 - e^{-(1-x)}$. We immediately see that $f(0) > 0$ and $f(1) = 0$. Its derivative $f'(x) = e^{-(1-x)}(-1+x) < 0$ when $x \in (0,1)$ which means that the expression in the brackets is positive as the distribution function gets values only in the unit interval.

Even if there were several equilibria when women look for men all of them would feature too low an acceptance level from the women's point of view. This is because when women look for men they face competition from other women, and their optimal choice depends on what other women do.

Let us next determine the socially optimal acceptance levels. As men would like to match as fast as possible we know that the social optimum is below the women's optimum. Summing V_m and V_w and taking the first order condition yields

$$(25)\quad -(1-\delta) - \tilde{z} + \delta\left(\bar{z} - \int_{\tilde{z}}^{\bar{z}} e^{-\theta(1-F(z))} dz\right) = 0.$$

Comparing this to the equilibrium condition (11) one immediately sees that at the equilibrium this is negative, and thus the optimal value is lower than the equilibrium value.

Summing U_m and U_w and taking the first order condition yields

(26)

$$\int_{\hat{z}}^{\bar{z}} z dF(z) \begin{bmatrix} e^{-(1-F(\hat{z}))} F(\hat{z})(1-\delta) + 1 - e^{-(1-F(\hat{z}))} \\ -e^{-(1-F(\hat{z}))}\left(1-\delta e^{-(1-F(\hat{z}))}\right) \end{bmatrix}$$
$$+\left(1-F(\hat{z})\right) \begin{bmatrix} -e^{-(1-F(\hat{z}))}\left(1-F(\hat{z})\right)(1-\delta) \\ -\hat{z}\left(1-\delta e^{-(1-F(\hat{z}))}\right)\left(1-e^{-(1-F(\hat{z}))}\right) \end{bmatrix} = 0.$$

Inserting the equilibrium condition here one does not get any definite result about whether the equilibrium is more or less than the social optimum.

4.1. Uniform distribution

To get more clearcut results let us assume that men's quality is uniformly distributed on the interval $[0,2]$. This is suitable in many respects, particularly because then a woman's expected utility when randomly meeting a man equals the utility of the man. One can easily check that this distribution results in a unique equilibrium when women look for men. Let us record the equilibrium conditions as well as the conditions for social optima. The derivations of the expression for welfare are in the appendix.

Men look for women:

(27) $$\tilde{z} - 2\delta e^{-\frac{2-\tilde{z}}{2}} = 0$$

(28) $$-(1-\delta) - \tilde{z}_s + 2\delta e^{-\frac{2-\tilde{z}_s}{2}} = 0.$$

Women look for men:

(29)
$$2\hat{z}\left(1-\delta e^{-\frac{2-\hat{z}}{2}}\right) - \delta\left(1-e^{-\frac{2-\hat{z}}{2}}\right)(2+\hat{z}) = 0$$

(30)
$$\frac{1}{4}\left(4-\hat{z}_s^2\right)\begin{bmatrix} e^{-\frac{2-\hat{z}_s}{2}}\frac{\hat{z}_s}{2}(1-\delta)+1-e^{-\frac{2-\hat{z}_s}{2}} \\ -e^{-\frac{2-\hat{z}_s}{2}}\left(1-\delta e^{-\frac{2-\hat{z}_s}{2}}\right) \end{bmatrix}$$
$$+\frac{2-\hat{z}_s}{2}\begin{bmatrix} -e^{-\frac{2-\hat{z}_s}{2}}\frac{2-\hat{z}_s}{2}(1-\delta) \\ -\hat{z}_s\left(1-e^{-\frac{2-\hat{z}_s}{2}}\right)\left(1-\delta e^{-\frac{2-\hat{z}_s}{2}}\right) \end{bmatrix} = 0.$$

It is easy to see that both (27) and (29) determine a unique treshold value. Inserting these equilibrium conditions into the total utilities, which are got by summing (19) and (20), and (21) and (22), we get the welfare when men search

(31) $$W_{mfw} = \delta\frac{1+e^{-\frac{2-\tilde{z}}{2}}(1-\tilde{z})}{1-\delta e^{-\frac{2-\tilde{z}}{2}}}$$

and when women search

(32)
$$W_{wfm} = \delta\frac{\left(1-e^{-\frac{2-\hat{z}}{2}}\right)(4+\hat{z})}{2\left(1-\delta e^{-\frac{2-\hat{z}}{2}}\right)}.$$

It is hard to analytically determine the relation of the equilibrium tresholds when men search and women search to the socially optimal treshold. Similarly, it is analytically hard to determine whether the society is better-off when men search or when women search.

4.2. Numerical and graphical analysis

As seen in the previous section there is a unique equilibrium value as well as unique values for social optima when men search and women search for a given discount factor. These are plotted in figure 1.

Figure 1 shows that equilibrium threshold values with men searching are the highest at all discount factor values (dark, dotdashed) followed by equilibrium thresholds when women search (light, dashed) and socially optimal thresholds being lower with women searching (light, solid) and the lowest when men do (dark, solid).

As can be seen, the socially optimal threshold values are always lower than private equilibrium ones reflecting the loss of welfare for men and thus in total welfare introduced by higher threshold values. All optimal threshold values converge at zero or two when $\delta \to 0$ or $\delta \to 1$, respectively. At low values of δ socially optimal threshold value is zero regardless of who searches as the value of waiting for an extra period becomes zero. Notably, it stays at zero for small positive values of the discount factor; when men search it stays at zero longer than when women search, and it is also lower for all δ when men search.

To compare welfare results, we plug the given optimal threshold values into the welfare functions (31) and (32) and plot them on a graph as in figure 2.

The welfare is, of course, increasing in the discount factor. Examining the graph closely, we

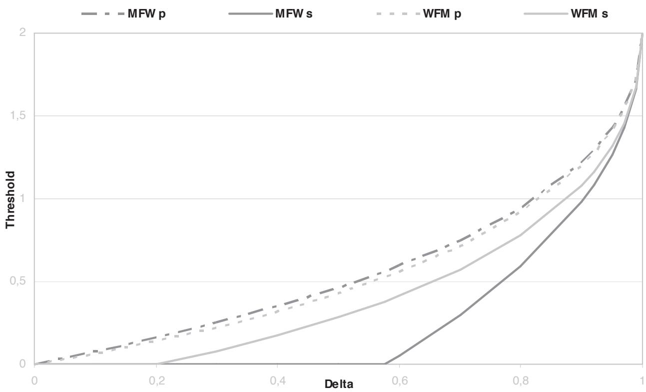

Figure 1. Optimal threshold values

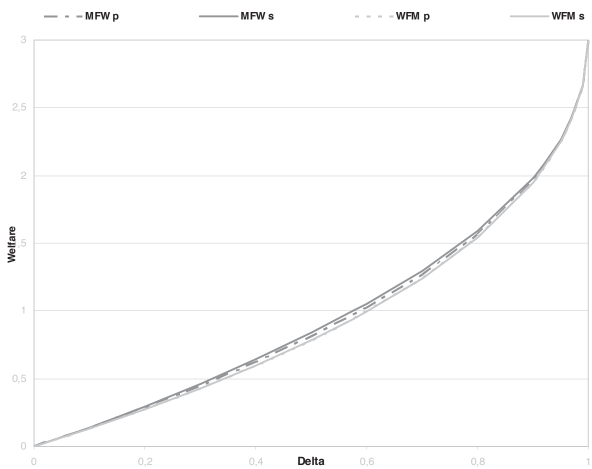

Figure 2. Welfares with socially optimal and equilibrium thresholds for different parties searching

can see that for all $\delta \in (0,1)$ the highest welfare is generated when men search (dark, solid), followed by men searching in the decentralised solution (dark, dotdashed) and the least welfare is generated when women search (light dashed) in the decentralised solution. This is almost at par with welfare generated when women search and threshold is socially optimally chosen (light, solid). At $\delta \to 0$ or $\delta \to 1$ welfares converge at zero and three respectively. In sum, men searching for women (the homogeneous in tastes looking for the heterogeneous) brings higher welfare both in the equilibrium and in social optimum.

One can understand this by considing the case when men search and many men arrive at a woman with more than one of them being above her threshold value (regardless of whether the threshold is privately or socially optimally chosen). Then the woman clearly chooses the man that brings her the most utility. In contrast, when women search, several of them may find a man acceptable, but it is not guaranteed that the woman valuing the man most gets to pair up with him. As a result, the pairing may be inefficient. Of course, the overall efficiency depends also on the equilibrium threshold values; they are higher when men search but still generate higher welfare than when women search. We summarise the results as

Proposition 1. *Most welfare is generated when agents homogeneous in tastes search for the agents with heterogeneous tastes. The optimal threshold values and welfare are increasing in the discount factor and when the discount factor approaches either 0 or 1, then all the threshold values as well as welfares converge to the same value.*

5. Conclusion

We consider a set-up where men and women try to pair up, and where meetings take place in a random manner. The meeting technology is such that the searching agents may end up in a competitive situation as any number of them may meet a single non-searching, or waiting, agent. We assume that one party, the women, is heterogeneous in its tastes, and we study whether the heterogeneous party or the homogeneous party should search. To focus solely on the effect of heterogeneity we assume that utility from one's partner is non-transferable. We find that the agents whose tastes are homogeneous, the men in our model, should be searching as far as social efficiency is considered. This might be a reasonable first guess because the decision maker who does not face direct competition is the waiting agent who actually makes the choice whether to accept one of the potential partners. If there are many the best is, of course, chosen. On the contrary, if it is the agent with homogeneous tastes who makes the choice then any of the acceptable partners has an equal probability to be the chosen one; but it could well be that the chosen agent is not the one who values the chooser the most. What makes our result non-trivial is that the acceptance tresholds of the agents are different depending on whether they search or wait.

Our conjecture is that the result still holds even if both parties are heterogeneous in tastes; the party expressing greater heterogeneity should wait and the other party search. Showing this formally is most likely quite difficult; even in the present simplified setting there are probably several equilibria depending on the distribution of the preferences.

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Appendix

Derivation of the expected life-time utilities of men and women under uniform distribution of tastes.

1. Men search for women: Derivation of the welfare functions which are the sum of the individual expected life-time utilities given below

$$V_m = \delta \frac{1 - e^{-\frac{2-\tilde{z}}{2}}}{1 - \delta e^{-\frac{2-\tilde{z}}{2}}}$$

$$V_w = \delta \frac{2 - \tilde{z}\, e^{-\frac{2-\tilde{z}}{2}} - \int_{\tilde{z}}^{2} e^{-\frac{2-z}{2}} dz}{1 - \delta e^{-\frac{2-\tilde{z}}{2}}}$$

These are solved from formulae (2) and (4). The latter one is simplified by standard integration

$$V_w = \delta \frac{2 - \tilde{z}\, e^{-\frac{2-\tilde{z}}{2}} - \int_{\tilde{z}}^{2} e^{-\frac{2-z}{2}} dz}{1 - \delta e^{-\frac{2-\tilde{z}}{2}}} = \delta \frac{2 - \tilde{z}\, e^{-\frac{2-\tilde{z}}{2}} - \left[2e^{-\frac{2-z}{2}} \right]_{\tilde{z}}^{2}}{1 - \delta e^{-\frac{2-\tilde{z}}{2}}}$$

$$= \delta \frac{2 - \tilde{z}\, e^{-\frac{2-\tilde{z}}{2}} - \left(2e^{-\frac{2-2}{2}} - 2e^{-\frac{2-\tilde{z}}{2}} \right)}{1 - \delta e^{-\frac{2-\tilde{z}}{2}}} = \delta \frac{2 - \tilde{z} e^{-\frac{2-\tilde{z}}{2}} - 2 + 2e^{-\frac{2-\tilde{z}}{2}}}{1 - \delta e^{-\frac{2-\tilde{z}}{2}}}$$

$$= \delta \frac{-\tilde{z}\, e^{-\frac{2-\tilde{z}}{2}} + 2e^{-\frac{2-\tilde{z}}{2}}}{1 - \delta e^{-\frac{2-\tilde{z}}{2}}} = \delta \frac{e^{-\frac{2-\tilde{z}}{2}} \left(2 - \tilde{z}\right)}{1 - \delta e^{-\frac{2-\tilde{z}}{2}}}$$

$$V_w = \delta \frac{e^{-\frac{2-\tilde{z}}{2}} \left(2 - \tilde{z}\right)}{1 - \delta e^{-\frac{2-\tilde{z}}{2}}}$$

The welfare is then

$$W_{mfw} = V_m + V_w = \delta \frac{1 - e^{-\frac{2-\tilde{z}}{2}}}{1 - \delta e^{-\frac{2-\tilde{z}}{2}}} + \delta \frac{e^{-\frac{2-\tilde{z}}{2}} \left(2 - \tilde{z}\right)}{1 - \delta e^{-\frac{2-\tilde{z}}{2}}}$$

$$= \delta \frac{1 - e^{-\frac{2-\tilde{z}}{2}} + e^{-\frac{2-\tilde{z}}{2}} \left(2 - \tilde{z}\right)}{1 - \delta e^{-\frac{2-\tilde{z}}{2}}} = \delta \frac{1 + e^{-\frac{2-\tilde{z}}{2}} \left(1 - \tilde{z}\right)}{1 - \delta e^{-\frac{2-\tilde{z}}{2}}}$$

$$W_{mfw} = \delta \frac{1 + e^{-\frac{2-\tilde{z}}{2}} \left(1 - \tilde{z}\right)}{1 - \delta e^{-\frac{2-\tilde{z}}{2}}}.$$

2. Women search for men: Derivation of the welfare function

$$U_m = \delta \frac{1-e^{-\frac{2-\hat{z}}{2}}}{1-\delta e^{-\frac{2-\hat{z}}{2}}}$$

$$U_w = \delta \frac{\frac{1-e^{-\frac{2-\hat{z}}{2}}}{\frac{2-\hat{z}}{2}} \int_{\hat{z}}^{2} z dF(z)}{1-\delta e^{-\frac{2-\hat{z}}{2}}}$$

These are solved from (8) and (10), and the integration below is standard

$$U_w = \delta \frac{\frac{1-e^{-\frac{2-\hat{z}}{2}}}{\frac{2-\hat{z}}{2}} \int_{\hat{z}}^{2} z dF(z)}{1-\delta e^{-\frac{2-\hat{z}}{2}}} = \delta \left(\frac{1-e^{-\frac{2-\hat{z}}{2}}}{\frac{2-\hat{z}}{2}} \int_{\hat{z}}^{2} z dF(z) \right) \frac{1}{1-\delta e^{-\frac{2-\hat{z}}{2}}}$$

$$= \delta \left(\frac{2\left(1-e^{-\frac{2-\hat{z}}{2}}\right)}{2-\hat{z}} \int_{\hat{z}}^{2} z d\frac{z}{2} \right) \frac{1}{1-\delta e^{-\frac{2-\hat{z}}{2}}} = \delta \left(\frac{2\left(1-e^{-\frac{2-\hat{z}}{2}}\right)}{2-\hat{z}} \left[\frac{1}{4} z^2 \right]_{\hat{z}}^{2} \right) \frac{1}{1-\delta e^{-\frac{2-\hat{z}}{2}}}$$

$$= \delta \left(\frac{2\left(1-e^{-\frac{2-\hat{z}}{2}}\right)}{2-\hat{z}} \left(\frac{1}{4} 2^2 - \frac{1}{4} \hat{z}^2 \right) \right) \frac{1}{1-\delta e^{-\frac{2-\hat{z}}{2}}} = \delta \left(\frac{2\left(1-e^{-\frac{2-\hat{z}}{2}}\right)}{2-\hat{z}} \left(1 - \frac{\hat{z}^2}{4} \right) \right) \frac{1}{1-\delta e^{-\frac{2-\hat{z}}{2}}}$$

$$= \delta \left(\frac{2\left(1-e^{-\frac{2-\hat{z}}{2}}\right)}{2-\hat{z}} \left(\frac{4-\hat{z}^2}{4} \right) \right) \frac{1}{1-\delta e^{-\frac{2-\hat{z}}{2}}} = \delta \left(\frac{2\left(1-e^{-\frac{2-\hat{z}}{2}}\right)\left(4-\hat{z}^2\right)}{4(2-\hat{z})} \right) \frac{1}{1-\delta e^{-\frac{2-\hat{z}}{2}}}$$

$$= \delta \left(\frac{\left(1-e^{-\frac{2-\hat{z}}{2}}\right)\left(4-\hat{z}^2\right)}{2(2-\hat{z})} \right) \frac{1}{1-\delta e^{-\frac{2-\hat{z}}{2}}} = \delta \left(\frac{\left(1-e^{-\frac{2-\hat{z}}{2}}\right)(2+\hat{z})}{2} \right) \frac{1}{1-\delta e^{-\frac{2-\hat{z}}{2}}}$$

$$= \delta \frac{\left(1-e^{-\frac{2-\hat{z}}{2}}\right)(2+\hat{z})}{2\left(1-\delta e^{-\frac{2-\hat{z}}{2}}\right)}$$

$$U_w = \delta \frac{\left(1 - e^{-\frac{2-\hat{z}}{2}}\right)(2+\hat{z})}{2\left(1 - \delta e^{-\frac{2-\hat{z}}{2}}\right)}$$

$$W_{wfm} = U_m + U_w = \delta \frac{1 - e^{-\frac{2-\hat{z}}{2}}}{1 - \delta e^{-\frac{2-\hat{z}}{2}}} + \delta \frac{\left(1 - e^{-\frac{2-\hat{z}}{2}}\right)(2+\hat{z})}{2\left(1 - \delta e^{-\frac{2-\hat{z}}{2}}\right)}$$

$$= \delta \frac{2\left(1 - e^{-\frac{2-\hat{z}}{2}}\right) + \left(1 - e^{-\frac{2-\hat{z}}{2}}\right)(2+\hat{z})}{2\left(1 - \delta e^{-\frac{2-\hat{z}}{2}}\right)} = \delta \frac{2 - 2e^{-\frac{2-\hat{z}}{2}} + 2 + \hat{z} - 2e^{-\frac{2-\hat{z}}{2}} - \hat{z} e^{-\frac{2-\hat{z}}{2}}}{2\left(1 - \delta e^{-\frac{2-\hat{z}}{2}}\right)}$$

$$= \delta \frac{4 - 4e^{-\frac{2-\hat{z}}{2}} + \hat{z} - \hat{z} e^{-\frac{2-\hat{z}}{2}}}{2\left(1 - \delta e^{-\frac{2-\hat{z}}{2}}\right)} = \delta \frac{\left(1 - e^{-\frac{2-\hat{z}}{2}}\right)(4+\hat{z})}{2\left(1 - \delta e^{-\frac{2-\hat{z}}{2}}\right)}$$

$$W_{wfm} = \delta \frac{\left(1 - e^{-\frac{2-\hat{z}}{2}}\right)(4+\hat{z})}{2\left(1 - \delta e^{-\frac{2-\hat{z}}{2}}\right)}.$$