*Finnish Economic Papers – Volume 21 – Number 2 – Autumn 2008*

# IN A HERD? HERDING WITH COSTLY OBSERVATION AND AN UNKNOWN NUMBER OF PREDECESSORS*

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*We consider a sequential decision to adopt/not adopt a technology in a herding environment with costly observation. The novelty compared to the previous models on herding with costly observation, such as Kultti and Miettinen (2006a), is that the agents do not know how many other agents have been in the same situation earlier. It turns out that herding no longer arises deterministically. We show that when observation costs are low there exists a unique symmetric pure strategy equilibrium where all agents observe the actions of two immediate predecessors in order to find out whether they are in a herd or not. (JEL: D82, D83)*

## 1. Introduction

Herding is defined as a situation where an agent, who observes other agents, ignores her own information in favour of the actions of the agents she observes. Banerjee (1992) and Bikhchandani et al. (1992) showed that herding arises eventually in a model with sequential decisions. Herding follows directly from applying Bayes's rule to a sequential problem, with private signals, where agents observe the choices of others. A good presentation and overview of the literature on herding is the textbook by Chamley (2004).

The basic set-up is a sequence of $n$ agents who have to make a decision. Each agent observes the choices of those before her but not their private information. Based on these observations and the agent's own information the agent then makes her choice. The models normally lead with positive probability to herding. Kultti and Miettinen (2006a) consider a situation where information about predecessors' actions is costly. They find that with moderate

* *We wish to thank Juuso Välimäki, Hannu Vartiainen, the audience at the XXX Annual Meeting of the Finnish Society for Economic Research 2008 and an anonymous referee for useful comments. Financial support from Yrjö Jahnsson Foundation and Academy of Finland are gratefully acknowledged.*

costs herding arises deterministically since from the fourth agent onwards all agents observe only their immediate predecessor's action and conform to it regardless of their own information.

This paper aims to render the basic set-up more symmetric. We postulate that the agents do not know their position in the sequence. The question then becomes how many predecessors' actions (hereafter predecessors) to observe. There are $n$ agents who decide whether to adopt a new technology or not. Agents must ex ante decide how many predecessors to observe. We show that with a small enough observation cost $\gamma$ there exists a unique symmetric pure strategy equilibrium where all agents observe two immediate predecessors in order to find out, if a herd has formed. If a herd has formed, the agent follows it regardless of her own information. If no herd has formed, the agent acts according to her own information. Thus, in this setting herding arises probabilistically.

A number of other modelling choices could clearly have been made. Closest to the current paper would be a model with costly observation where players do not know who their immediate predecessors are and just observe a sample of players that moved before. This model would be a variant of Smith and Sørensen (2008).

## 2. *The Model*

There are $n$ agents who make a decision whether to adopt a new technology or not. They act sequentially, but do not know the their position in the sequence. Therefore the problem is ex ante symmetric. Otherwise the model is similar to Kultti and Miettinen (2006a): The new technology/ product can be either good or bad. There are two possible states of the world $\Omega = \{\omega, \varpi\}$,where $\omega$ denotes the good and $\varpi$ the bad state of the world. If the state of the world is good the adopting agent receives a benefit $b$, if the state of the world is bad the agent receives benefit $b^L$ which is normalized to zero. The cost of adoption is $c = \frac{b}{2}$ for all agents and all states. Both states are equally likely, i.e., $\Pr[\omega] = \frac{1}{2} = \Pr[\varpi]$. Agent i receives a signal about the state of the world; $S_i$ denotes a signal that the state of the world is good and $\bar{S}_i$ denotes a signal that the state of the world is bad. The conditional probabilities of the signals are $\Pr[S|\omega] = p = \Pr[\bar{S}|\varpi] > \frac{1}{2}$ and $\Pr[S|\varpi] = (1-p) = \Pr[\bar{S}|\omega] < \frac{1}{2}$.

The agents are not able to observe their predecessors unless they pay an observation fee $\gamma$ per predecessor. Let $A_i$ denote agent i's choice of adopting and $\bar{A}_i$ denote her choice of not adopting, and let $k$ denote the number of immediate predecessors the agent observes. For instance, $k = 2$ denotes a situation in which the agent chooses to observe the actions of two immediate predecessors. The agents use Bayes's rule to update their beliefs about the state of the world after making the observation.

The timing is as follows: Each agent gets a signal $S$ or $\bar{S}$, after this the agent decides how many immediate predecessors to observe (given the observation cost $\gamma$). Then the agent chooses whether to adopt the new technology or not.

## 3. *Symmetric equilibria*

We construct a symmetric pure strategy equilibrium of the game. The first candidate for an equilibrium is one where no agent pays to observe the actions of her predecessors.

### *3.1 Observing no predecessors*

It is clear that we could construct an equilibrium where no agent observes any predecessors by setting the cost of observation high enough. We are, however, interested in finding symmetric equilibria where all agents observe some immediate predecessors. These equilibria might exist when observation costs are low or moderate. We define moderate observation costs as costs that are lower than prohibitive so that an agent $i$ breaks the tentative $k = 0$ equilibrium, where the agents don't observe anyone.

If no-one observes anyone and the agent receives signal $S$ her expected utility from adopting is $E[U(A_i)|S_i] = \Pr[\omega|S_i]\,b - c = \frac{b(2p-1)}{2} > 0$. The expected utility from adopting when receiving signal $\bar{S}$ is $E[U(A_i)|\bar{S}_i] = \Pr[\omega|\bar{S}_i]\,b - c = (\frac{1}{2} - p)\,b < 0$. Therefore the agent adopts only if she receives signal $S$.

A deviation to $k = 1$ cannot be profitable for an agent with signal $S$. To see this first note that the agent observing a predecessor expects positive utility from adopting only if her predecessor (who acted on her own signal only) also has adopted. As $\{\Pr[A_{i-1}, S_i]\ E\ [U(A_i) | S_i;\ A_{i-1}]\} = \frac{2p-1}{2}\ b$ for all agents $i > 1$ (the first agent would expect to get $\frac{2p-1}{2}\ b$ as well) and the observation cost $\gamma$ is positive, the deviation is not profitable. With signal $\bar{S}$ the argument is similar.

Can it be optimal to deviate from the tentative $k = 0$ equilibrium by choosing $k = 2$, i.e. observing the actions of two predecessors? Again, remember that in a tentative $k = 0$ equilibrium the actions are completely informative of the signals. Note that the first agent cannot observe any predecessors even though she has paid to observe two, likewise the second agent will only be able to observe the action of the first agent. If the second agent's signal is different from the first agent's action the second agent will be indifferent between adopting and not adopting. To simplify the analysis we assume that an indifferent agent follows her own signal. Then both the first and the second agents learn their position in the sequence by observing two predecessors and play according to their own signal $S$ and thus receive $\frac{2p-1}{2}$. If an agent observes two negative actions $\bar{A}$ she will not adopt and will therefore receive 0. If the agent observes one or two positive actions $A$ she will adopt. We denote an agent's expected utility under the optimal decision by $E^*\ [U(A_i) | S_i]$. Thus when an agent receives signal $S$ the expected utility from a $k = 2$ deviation under the described optimal decision is

$$(1)\quad E^*_{deviation}\,[U(A_i)|S_i] = \frac{2}{n}\left(\frac{2p-1}{2}\right)b + \frac{n-2}{n}\left[p^3(3-2p) - (1+2p)(1-p)^3\right]\frac{b}{2}.$$

If observation cost $\gamma$ is small enough the deviation is profitable as

$$E^*_{deviation}\,[U(A_i)|S_i] - E^*\,[U(A_i)|S_i] = \frac{n-2}{n}\frac{p(1-p)[2p-1]}{2}b > 0.$$ [1]

Therefore, given that $n > 2$ a deviation from the $k = 0$ equilibrium to $k = 2$ is profitable if player $i$ receives a good signal $S$ and the observation cost $\gamma$ is small enough. Given signal $\bar{S}$ we have under the optimal decision $E\ [U(A_i) | \bar{S}_i] = 0 + 0 + \frac{n-2}{n}\ \frac{p(1-p)[2p-1]}{2}b$. As $\frac{n-2}{n}\ \frac{p(1-p)[2p-1]}{2}\ b > 0$ it is again profitable for the agent to observe her two preceding agents if observation cost $\gamma$ is small enough. We define costs as moderate when a deviation from the $k = 0$ strategy to $k = 2$ becomes profitable, i.e., when

$$(2)\quad \gamma < \frac{1}{2}\frac{n-2}{n}\frac{p(1-p)[2p-1]}{2}b.$$

### 3.2 *Observing exactly one predecessor*

*Claim 1* It can not be an equilibrium for all agents to observe exactly one predecessor.

*Proof* To see this note that the nth agent observing the $(n - 1)$th agent can observe one of the following: $SA$, $S\bar{A}$, $\bar{S}A$, $\bar{S}\bar{A}$. With information $SA$ the agent will adopt for sure and with information $\bar{S}\bar{A}$ the agent will not adopt. In order for observing and adopting (also in case of conflicting signals) to be an equilibrium the predecessor's action would have to be a stronger signal on the state of the world than the agent's own signal. Assuming signal $S_n$, observing $\bar{A}_{n-1}$ would then lead agent n to not adopt (and vice versa). This is because there would be more information in $\bar{A}_{n-1}$ than in $S_n$. But if observing the previous agent's decision and imitating her action were an equilibrium strategy it would lead to agent 1's signal determining the action of all n agents. Thus, observing one's immediate predecessor and adopting to her behavior cannot be optimal as the signals $S_i$ are assumed to be independent and equally informative of the state and the observation cost $\gamma$ is positive.

If the previous agent's action is a weaker signal of the state of the world than the agents own signal then the agent can not gain enough information by observing its immediate predecessor

[1] *Note that we have omitted the observation costs from the numbered equations in this section. We will, however, explicitely use them later when we compare the expected utility from the k = 2 equilibrium to the expected utilities from possible deviations.*

to reverse her own action. Given that $\gamma$, the cost of observing a predecessor, is positive the agent will never observe only one predecessor. Observing only one predecessor is therefore not an equilibrium of the game. ■

### *3.3 Observing two predecessors*

In this section we construct a symmetric equilibrium where all agents observe the actions of two immediate predecessors. We then define low observation costs that allow for this equilibrium as low.

*Claim 2* When observation costs are low it is an equilibrium for all players to observe the actions of two predecessors.

Before considering a sequence with n agents we illustrate the situation in a sequence where there are only four agents.

#### *3.3.1 A sequence with four agents*

Assume that an agent receives signal $S$ and observes two predecessors, i.e., $k = 2$. A player's probability of being the first or second in the sequence is $\frac{1}{2}$. She would then observe either one or two empty actions and learn her position in the sequence and act according to her own signal only. The expected utility of adopting would be $(\frac{2p-1}{2})b$ (minus the observation cost $2\gamma$). The actions of the first and second agents are thus completely informative of their signals. The agent knows that if she is third in the sequence her possible observations are $A_1$, $A_2$ or $\bar{A}_1$, $A_2$ or $A_1$, $\bar{A}_2$ or $\bar{A}_1$, $\bar{A}_2$. In case she observes $\bar{A}_1$, $\bar{A}_2$ her expected utility from adopting would be negative and she would not adopt, therefore her expected utility under the optimal decision would be zero. The expected utility under the optimal decision assuming that the agent is third, observes two predecessors and receives signal $S_i$ is

$$
\begin{aligned}
(3) \quad & E^*\left[U_3(A_3)|S_3\right] \\
&= \Pr\left[A_1, A_2|S_3\right] E\left[U\left(A_3\right)|S_3; A_1, A_2\right] \\
&+ 2\Pr\left[\overline{A}_1 A_2|S_3\right] E\left[U\left(A_3\right)|S_3; \overline{A}_1, A_2\right] \\
&= \left[p^3(3-2p) - (1+2p)(1-p)^3\right] \frac{b}{2}.
\end{aligned}
$$

The agent would, in a similar way, reason that if she were fourth and observed $\bar{A}_2, \bar{A}_3$ the expected utility from adopting would be negative and she would not adopt whereas $A_2$, $A_3$ would lead her to adopt for sure. The only possible difference in behavior between the fourth and third agent stems from the fact that $E[U_4\ (A_4) \mid S_4;\ A_2, \bar{A}_3]$ is zero for the fourth agent whereas $E[U_3(A_3) \mid S_3;\ A_1, \bar{A}_2]$ is positive for the third agent. (This is because having seen $A_2, \bar{A}_3$ the fourth agent knows that if she is fourth the signals must have been $\bar{S}_1$, $S_2$, $\bar{S}_3$ in equilibrium.) Note, however, that the agent adopts (and follows her own signal) as the expected utility from adopting is positive (she doesn't know whether she is third or fourth) The expected utility for an agent with signal $S$ who knows that she is fourth and observes two predecessors is, thus, under the optimal decision

$$
\begin{aligned}
(4) \quad & E^*\left[U_4(A_4)|S_4\right] \\
&= \Pr\left[A_2, A_3|S_4\right] E\left[U_4\left(A_4\right)|S_4; A_2, A_3\right] \\
&+ \Pr\left[\overline{A}_2, A_3|S_4\right] E\left[U_4(A_4)|S_4; \overline{A}_2, A_3\right].
\end{aligned}
$$

Now in equilibrium the signals leading to $A_2$, $A_3$ are either $S_1$, $S_2$, $S_3$ or $\bar{S}_1$, $S_2$, $S_3$, or $S_1$, $S_2$, $\bar{S}_3$. The last two are equally likely and give the same expected utility. The only set of signals leading to $\bar{A}_2$, $A_3$ under optimal decisions are $S_1$, $\bar{S}_2$, $S_3$. This is due to the fact that in equilibrium the third agent has seen the actions of two predecessors.

We now have

$$
\begin{aligned}
(5) \quad & E^*\left[U_4(A_4)|S_4\right] \\
&= \Pr\left[S_1, S_2, S_3|S_4\right] E\left[U_4\left(A_4\right)|S_4; S_1, S_2, S_3\right] \\
&+ \Pr\left[\overline{S}_1, S_2, S_3|S_4\right] E\left[U_4\left(A_4\right)|S_4; \overline{S}_1, S_2, S_3\right] \\
&+ \Pr\left[S_1, S_2, \overline{S}_3|S_4\right] E\left[U_4\left(A_4\right)|S_4; S_1, S_2, \overline{S}_3\right] \\
&+ \Pr\left[S_1, \overline{S}_2, S_3|S_4\right] E\left[U_4\left(A_4\right)|S_4; S_1, \overline{S}_2, S_3\right] \\
&= \Pr\left[S_1, S_2, S_3|S_4\right] E\left[U_4\left(A_4\right)|S_4; S_1, S_2, S_3\right] \\
&+ 3\Pr\left[\overline{S}_1, S_2, S_3|S_4\right] E\left[U_4\left(A_4\right)|S_4; \overline{S}_1, S_2, S_3\right]
\end{aligned}
$$

as the signals of each player are of equal strength. After some manipulations we get

$$
(6) \quad E^*\left[U_4(A_4)|S_4\right] = \frac{b}{2}(2p-1)\left(1+p-p^2\right).
$$

Thus an agent with signal $S_i$ expects to get the following utility after she observes two predecessors:

$$(7)\quad E^*[U_i(A_i)|S_i] = \left(\frac{1}{4}\right) E^*[U(A_1)|S_1] + \left(\frac{1}{4}\right) E^*[U(A_2)|S_2] + \left(\frac{1}{4}\right) E^*[U_3(A_3)|S_3] + \left(\frac{1}{4}\right) E^*[U_4(A_4)|S_4] = \frac{1}{4}(2-p)(2p-1)(p+1)\,b.$$

It is possible that she would like to deviate by observing no predecessors and follow her own signal $S$. Her expected utility would then be $(\frac{2p-1}{2})\,b$. This deviation would not be profitable if the cost of observing two predecessors is smaller than or equal to the expected gain from the observations, i.e., if $\frac{1}{4}(2–p)(2p–1)(p+1)\,b - (\frac{2p-1}{2})\,b \geq 2\gamma$. This inequality simplifies to

$$(8)\quad \frac{b}{4}p(2p-1)(1-p) \geq \gamma.$$

It is also conceivable that the agent wants to break $k = 2$ equilibrium by observing only one predecessor. The agent with signal $S$ would then again expect to receive $(\frac{2p-1}{2})\ b-\gamma$ if she were first or second. If she were third she would expect to get $(\frac{2p-1}{2})\ b-\gamma$ as well. The reason is that in an equilibrium where all agents observe two predecessors the first and second agents would learn their positions in the sequence and would follow their own signals. The fourth agent could benefit from the deviation. Her expected utility from observing only one predecessor, given that all other agents observe two predecessors, is $\frac{b}{2}(2p–1)(1+p–p^2)-\gamma$. This is higher than her expected utility from the nondeviation. This can easily be explained as agent three has already based her decision on three signals. As observation is costly an agent who knows that she is fourth would thus observe only the third agent, and make the same choice (see e.g. Kultti and Miettinen (2006 a)). As the agent does not know her position in the sequence her expected utility from deviating and observing only one predecessor is

$$(9)\quad E_{deviation}[U(A_i)|S_i] = \frac{b}{8}(2p-1)(4+p-p^2).$$

The deviation is not profitable when $\frac{b}{4}(2-p)(2p-1)(p+1)–2\gamma - \frac{b}{8}(2p-1)(4+p-p^2)+\gamma \geq 0$.This simplifies to

$$(10)\quad \frac{b}{8}p(2p-1)(1-p) \geq \gamma.$$

It is immediately clear that there is no profitable deviation from the $k = 2$ equilibrium to $k = 3$ or $k = 4$. The game has a pure strategy equilibrium for the positive values of $\gamma$ when the most restrictive of (8) and (10) holds, i.e. when:

$$(11)\quad \frac{b}{8}p(2p-1)(1-p) \geq \gamma.$$

We define low observation cost $\gamma$ as such that (11) holds. The four player minigame then has an equilibrium where each agent observes the actions of two immediate predecessors. Note that due to the symmetric structure of the model the maximum cost of $\gamma$ allowing for a $k = 2$ equilibrium is the same regardless of whether the agent has received signal $S$ or $\bar{S}$.

### *3.3.2 A sequence with n agents*

To derive the expected utility from observing the actions of predecessors becomes inconvenient when $n$ is larger than four. We therefore use a different approach to construct equilibrium.

We focus on the equilibrium where all agents observe two preceding agents ($k = 2$). We show that with $\gamma$ small enough (defined later) there exists no profitable deviation to $k = 1$ or, in fact, any $k \neq 2$. To find out under what parameter values it might be optimal for an agent $i$ to deviate from this equilibrium and observe the actions of only one predecessor we compare costs and expected gains from observing two predecessors to those from observing only one predecessor.

An agent $i$ observing two predecessors can see one of the following:

$$(12)\quad (A_{i-1},A_{i-2}),(A_{i-1},\overline{A}_{i-2}),(\overline{A}_{i-1},A_{i-2}),(\overline{A}_{i-1},\overline{A}_{i-2}),(A_{i-1},\emptyset),(\overline{A}_{i-1},\emptyset),(\emptyset,\emptyset).$$

If the agent makes any of the last three observations she will know her position in the sequence and act according to her own signal. In the first and the fourth case the agent knows that a herd has formed (or that she should start one) and follows the action of her two predecessors regardless of her own signal. In cases two and three no herd has formed and the agent follows her own signal.

The idea is in the spirit of the equilibrium with adequately informed agents (see Kultti and Miettinen (2006b) for the application of this concept in a different setting). In the current setting an agent will observe two predecessors as opposed to one only in case she then can learn something new and behavior changing. Observing one predecessor and following her action would be optimal if agent $i$ knew that she is in a herd. Observing two immediate predecessors can be optimal only if it reveals enough new information for agent $i$ to change her behavior (from what it was when she observed only one agent). This can be the case only when the actions of the two predecessors are opposite. These actions are different only when a herd has not formed before agent $i-1$ acts.

The only way that a herd does not form is if sequential agents get opposite signals. The actions of the agents are then completely informative of their signals. The two different histories of signals leading to a herd not having formed before agent $i$ are obviously:

$$(13)\quad S_1,\overline{S_2},S_3,\overline{S}_4...S_{i-2},\overline{S}_{i-1}\quad\text{and}\quad \overline{S}_1,S_2,\overline{S}_3,S_4...\overline{S}_{i-2},S_{i-1}.$$

As stated above, observing the actions of two immediate predecessors (as opposed to one) can be beneficial to the agent only if it leads to a reversal of her action. Now assume that agent $i$ has received signal $S_i$ Then the only way in which she could benefit from observing two predecessors instead of one would be if she observed $\bar{A}_{i-1}, A_{i-2}$. She would learn that she is not in a herd and that including her own signal there are either equally many good and bad signals (if i is even) or one more good signal (if i is odd). Agent $i$ would therefore change her behavior from $\bar{A}_i$ to $A_i$.

Thus, with $i$ even, the agents expected utility from adopting/buying would then be zero as all signals are of equal strength. With $i$ odd there would be one more positive than negative signals in the sequence, the expected utility from adopting/buying would then be positive. This means that under the optimal decision the agent expected utility is higher if she observes two predecessors as opposed to one if her position is third, fifth, seventh or any odd integer above that.

In a sequence with $n>2$ players agent $i$ would thus deviate from the tentative $k=2$ equilibrium to $k=1$ if the cost of one more observation $\gamma$ is larger than the expected gain in utility at the optimal decision. With signal $S_i$ agent i would thus deviate only if

$$(14)\quad \gamma > \frac{1}{n}\sum_{i=3}^{n}\Pr\left[A_{i-2},\overline{A}_{i-1}|S_i\right]E\left[U\left(A_i\right)|S_i;A_{i-2},\overline{A}_{i-1}\right].$$

As $E\ [U\ (A_i)\,|\,S_i;\ A_{i-2},\ \bar{A}_{i-1}] = 0$ when $i$ is even, we get the following expression

$$(15)\quad \frac{1}{n}\sum_{i=3}^{n}\Pr\left[A_{i-2},\overline{A}_{i-1}|S_i\right]E\left[U\left(A_i\right)|S_i;A_{i-2},\overline{A}_{i-1}\right]=\frac{1}{n}\sum_{d=1}^{\frac{n}{2}-1}\frac{1}{2}\left(p\left(1-p\right)\right)^d\left(2p-1\right)b$$

where $d = i-2$, (with the obvious changes if $n$ is odd).

The agent does not deviate to $k = 1$ if observation costs are low i.e. if the following inequality holds

$$(16)\quad \gamma \le \frac{1}{n}\sum_{d=1}^{\frac{n}{2}-1}\frac{1}{2}\left(p\left(1-p\right)\right)^d\left(2p-1\right)b.$$

It is obvious that if (16) holds, a positive deviation to $k = 0$ does not exist. It is still possible that the agent might want to deviate from the $k = 2$ equilibrium by observing the actions of more than two predecessors. Assume that an agent receives signal $S$, and observes the actions of two immediate predecessors. If both predecessors have declined from buying, i.e. the observations are both $\bar{A}$, then the agent knows that either both have received signal $\bar{S}$ or that a herd has started earlier. Therefore the expected utility from adopting is at most zero and buying costly information on additional actions could not be profitable. If the actions of the two immediate predecessors are different from each other, the agent knows that the only way for this to be possible is if the signals from the first agent onwards have been alternating. In this case the agent would follow his own signal as the expected utility from this is at least zero. Buying information on additional actions would again be costly without affecting the agent's optimal decision. Therefore no profitable deviation from the $k=2$ equilibrium exists when observation costs are low as defined in (16).

## 4. *Uniqueness*

*Claim 3* When observation costs are low observing the actions of two predecessors is the unique pure strategy equilibrium.

In order to answer the question of whether there are equilibria for some $k > 2$, we first need to find out, if it is possible that sequences of two similar sequential actions can happen without this leading to a herd forming. If this is not possible for any $k$ then it is clear that $k > 2$ cannot be an equilibrium as a profitable deviation to $k = 2$ would then exist. It is immediately obvious that sequences of two similar actions followed by a different action (hereafter a broken sequence)) cannot happen in equilibrium for $k = 3$.

For $k \geq 4$ the following sequences become possible:

(17) $A_1\overline{A}_2\overline{A}_3, A_4...$

and

(18) $\overline{A}_1 A_2, A_3, \overline{A}_4...$

These sequences are possible as the fourth agent would observe that she is fourth and would follow her own signal even after having seen $A_1$, $\bar{A}_2$, $\bar{A}_3$ Assuming the signal of the fourth agent is $S$ the fifth agent would observe $A$, $\bar{A}$, $\bar{A}$, $A$ and figure out that the agent in front of her must have known that including her own signal there were equally many good signals as bad signals. Therefore the fifth agent would again act on her own signal.

In fact, the following observations can be made for $k \geq 4$:

Assume all agents $j \neq i$ observe the actions of $k$ predecessors i.e. play the $k$ equilibrium. Assume further that agent $i$'s signal is $S$

1. If a herd has formed before agent $i$ then it is clear that agent i cannot do better than to follow the herd.
2. If $i \leq k$ and the actions of $i$'s predecessors alternate up to the two closest predecessors s.t. $\bar{A}_{i-2}$, $\bar{A}_{i-1}$ then agent $i$'s expected utility from adopting is negative if $i$ is odd or zero if $i$ is even. This is clear as all actions before agent $i$ would have revealed the signals of the agents. Then the agent cannot do better than to follow the actions of her two immediate predecessors. As she, however, already has tried to observe $k$ predecessors she will follow her own signal if she learns that the expected utility from doing so is at least zero. In other words: if the agent learns that there are equally many good and bad signals she will act according to her own signal. Therefore broken sequences can form for $i \leq k$. This means that an agent $j$ observing agent $i$ breaking a sequence will know that the sequence including agent $i$ must have included equally many good and bad signals.
3. Given 2, subsequent broken sequences can form if $i > k$ and there are one or more broken sequences within the actions of agents $\{i-k, ..., i-1\}$. These subsequent broken sequences must also start at an even $i$ and cannot encompass more than two actions. The broken sequences must be altering so that if the first encompasses actions $AA$ then the second encompasses actions $\bar{A}\bar{A}$. Also note that, as in 2, it follows that the agent who has broken a sequence must have had a signal opposite from that in the sequence and that including the last broken sequence there must be equally many good and bad signals.
4. If $i > k$ and there have not been broken sequences $A$, $A$ or $\bar{A}$, $\bar{A}$ in the $k$ first instances

of the sequence and agent $i$'s two closest predecessors actions are $\bar{A}_{i-2}$, $\bar{A}_{i-1}$ then the utility from adopting is negative if $i$ is odd or zero if i is even. This is due to the fact that all actions have been revealing as no herd has formed. As agent $i$ has no way of knowing if she is odd or even, her expected utility from adopting is negative. Therefore she will follow the action of his two immediate predecessors and herd.

5. If $i > k$ and there has been one or more broken sequences, but there are none for agents $\{i-k, ..., i-1\}$ and the actions of agent $i$'s two closest predecessors are $\bar{A}_{i-2}$, $\bar{A}_{i-1}$, then agent $i$ cannot expect to do better than by following the actions of her two immediate predecessors. This is clear, since up to and including the point where the last sequence of two similar actions was broken, there must have been equally many good and bad signals. Therefore if agent $i$ doesn't observe any broken sequences all actions from the last broken sequence to agent $i$ must also have been informative. Therefore, there must be at least as many negative signals as positive signals in the sequence and agent $i$ can not expect to do better than by following her two closest predecessors.

This means that with positive observation cost $\gamma$ the agents have a profitable deviation from $k > 2$ to $k = 2$. We have in sections 3.1 and 3.2 shown that when observation costs are low, neither $k = 0$ nor $k = 1$ can be equilibria. Thus we have shown that $k = 2$ is the unique equilibrium when observation costs are low.

## 5. Conclusion

We have extended the model of herding with costly observation in Kultti and Miettinen (2006a) by relaxing the assumption that the agents know how many other agents have been in the same situation when they decide on how many predecessors to observe. This extension changes the result as herding now does not arise deterministically. We find that when observation costs are low there exists a unique symmetric pure strategy equilibrium where each agent observes the actions of only her two closest predecessors in order to find out if there is a herd or not. A further topic could be to extend the model by characterizing the equilibria when we fix the observation cost $\gamma$ and let $n$ grow towards infinity. When observation costs are low, we then expect to find mixed equilibria with support consisting of observing zero, one and two immediate predecessors.

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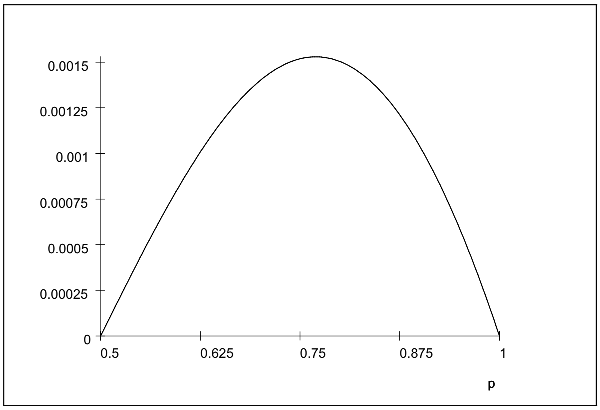
## *Appendix 1*

An example illustrating the maximal cost of observation allowing for a $k = 2$ equilibrium when $n = 40$ and $b = 1$.

$$\gamma^* = \frac{1}{n} \sum_{d=1}^{\frac{n}{2}-1} \frac{1}{2} \left(p\left(1-p\right)\right)^d \left(2p-1\right) b$$




With $p = 0.65$, $n = 40$ and $b = 1$ the max value of $\gamma$ allowing $k = 2$ to be an equilibrium is

$$\frac{1}{40} \sum_{d=1}^{\frac{40}{2}-1} \frac{1}{2} \left(0.65\left(1-0.65\right)\right)^d \left(2p-1\right) = 1.1044 \times 10^{-3}.$$