

OPERATIONAL FISCAL AND MONETARY POLICY WITH STAGGERED WAGE AND PRICE DYNAMICS*

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I consider the optimal setup of simple rules for monetary and tax policy in a model with distortionary taxes, wage and price stickiness. The rules maximize a measure of households' intertemporal utility. The model is solved through the second-order approximation method of Schmitt-Grohé and Uribe (2004). When both prices and wages are indexed to steady-state inflation, the average tax rate responds little to government liabilities. This arises from the need to minimize the inefficient distortions arising from fluctuations in the price level for fiscal reasons. Optimal monetary policy responds strongly to changes in wages. In an economy with only wage rigidity, fiscal considerations prevail over the need to stabilize wage fluctuations, and the policy mix produces a large variability of inflation. Finally, with indexation of wages and prices to lagged inflation, the dynamic behavior of the economy is closer to the frictionless equilibrium, and the optimal policy mix resembles the one under flexible prices and sticky wages. (JEL: E52, E61, E63)

“Truths are known to us in two ways: some are known directly, and of themselves; some through the medium of other truths. The former are the subject of Intuition, or Consciousness; the latter, of Inference. The truths known by intuition are the original premises from which all others are inferred.”

John Stuart Mill, *A System of Logic*

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1. Introduction

The interaction between monetary and fiscal policy is a topic that bears large relevance in economic affairs. For instance, Reinhart and Savastano (2003) present a survey of the post-World War I experience with hyperinflations. The empirical evidence shows that hyperinflations have been typically associated with large fiscal imbalances. Major fiscal retrenchments have been carried out to end all the hyperinflationary episodes. The positive issues from the historical experience raise normative questions.

With reference to the recent European experience, Artis and Winkler (1997) suggest that a set of formal rules for fiscal discipline – the so-

called Stability Pact – is needed to prevent the free-riding behavior of fiscal authorities from endangering the operational independence of the European Central Bank. Combined with the primary mandate for price stability of the European Central Bank, the institutional setting of the European monetary union sets out a clear division of tasks between monetary and fiscal policy, the latter being in charge of stabilizing national business cycles without interfering with the objective of price stability.

The theoretical literature on quantitative macroeconomics proposes a number of normative studies on the optimal design of the fiscal-monetary policy mix. McCallum (1999) reviews the arguments suggesting that a central bank should keep the fiscal behavior into account while setting the policy rates. Taylor (2000) uses an estimated multicountry model to study whether fiscal automatic stabilizers can compensate for the lack of reaction of monetary policy to real variables.

Various recent papers characterize the degree of responsiveness of monetary and fiscal policy to exogenous shocks that maximizes household's welfare. Kollmann (2007), Marzo (2006) and Schmitt-Grohé and Uribe (2006, 2005) are a few example of studies that concentrate on the closed economy dimension. Gali and Monacelli (2005) deal with the optimal degree of coordination between fiscal and monetary policy in the functioning of a currency union. Albeit focusing on optimal monetary policy, the earlier studies of Kollmann (2002, 2004) based on open-economy models have laid down the ground for a quantitative investigation of welfare-maximizing policy. In particular, these papers use numerical methods to solve optimal policy problems of macroeconomic stabilization around distorted (second-best) long-run equilibria.

A key assumption of the contributions mentioned above is that wages adjust elastically to the conditions in the labor market, as prescribed by the standard New Keynesian model. This is at odds with the empirical evidence on the macroeconomic sources of rigidity. Among others, Hall (2005) suggests that wage rigidity is a central feature needed to explain labor-market fluctuations. Hence, the question arises about the

properties of the optimal fiscal-monetary policy mix when there are frictions in the labor markets.

Erceg, Henderson and Levin (2000) consider the optimal monetary policy problem of a central bank operating in a model economy with nominal wage and price stickiness. Households are assumed to provide heterogenous labor services in monopolistically competitive markets. Nominal wages adjust according to the framework of Calvo (1983), implying that there is a fraction of households that cannot change nominal wages in each period. With staggered wage contracts, aggregate wage changes induce inefficiency in the distribution of labor effort across households. This implies that complete stabilization of wage inflation is required in order to attain the Pareto-optimal (frictionless) allocation.

In this paper, I consider the optimal setup of simple rules for monetary and tax policy in a model with both wage and price stickiness. Like in Kollmann (2007), taxes are a source of distortion. The rules maximize a measure of households' intertemporal utility. Since the long-run equilibria are distorted, the model is solved through the second-order approximation method of Schmitt-Grohé and Uribe (2004). I consider different types of rules. The quantitative results confirm the main findings of the literature.

When both prices and wages are indexed to steady-state inflation, the average tax rate responds little to government liabilities. This arises from the need to minimize the inefficient distortions arising from fluctuations in the price level for fiscal reasons (see also Schmitt-Grohé and Uribe, 2006). On average, the response of monetary policy to wage changes should be significant. In an economy with only wage rigidity, fiscal considerations prevail over the need to stabilize wage fluctuations, and the policy mix produces a large variability of inflation (see Kollmann, 2007). Finally, with indexation of wages and prices to lagged inflation, the dynamic behavior of the economy is closer to the frictionless equilibrium, and the optimal policy mix resembles the one under flexible prices and sticky wages. The reason is that indexation to past inflation induces smaller price dispersion than indexation to long-run inflation. This is so

because firms need not incorporate the expectations of future deviations of inflation from trend in their current markups when they set up prices optimally. Hence, indexation to past inflation allows firms to keep up with changes in the aggregate price level.

The outline of the paper is the following. Section 2 presents the real and nominal sources of rigidity of the model economy. Section 3 discusses the simple rules for monetary and fiscal policy. The conditions for aggregated equilibria are presented in section 4. Section 5 deals with the computation of the welfare metric used to compute the optimal policy coefficients. Section 6 outlines the calibration strategy. The results are presented in section 7. Finally, some concluding remarks can be found in section 8.

2. The model

The model follows the New Keynesian tradition. It includes monopolistically competitive firms that set prices according to the standard framework of Calvo (1983). I introduce wage rigidity in the spirit of Erceg, Henderson and Levin (2000) by assuming existence of idiosyncratic labor services. Both wages and prices are fully indexed to long-run inflation.

2.1 The final-good sector

Firms in the final-good sector are mere retailers. They face a perfectly competitive product market. Their production function consists of a Dixit-Stiglitz technology that aggregates intermediate goods:

$$(1) \quad y_t \leq \left[\int_{i \in \varpi_2} y_{it}^{\frac{\theta-1}{\theta}} dt \right]^{\frac{\theta}{\theta-1}}.$$

The demand for each intermediate good y_{it} follows from the static profit maximization problem:

$$(2) \quad \max_{\{y_{it}\}_{i \in \varpi_2}} P_t \left[\int_{i \in \varpi_2} y_{it}^{\frac{\theta-1}{\theta}} dt \right]^{\frac{\theta}{\theta-1}} - \int_{i \in \varpi_2} P_{it} y_{it} dt$$

and takes the form:

$$(3) \quad y_{it} = \left[\frac{P_{it}}{P_t} \right]^{-\theta} y_t.$$

At a zero-profit equilibrium, the following price index of final goods can be derived:

$$(4) \quad P_t = \left[\int_{i \in \varpi_2} P_{it}^{1-\theta} dt \right]^{\frac{1}{1-\theta}}.$$

2.2 The intermediate-good sector

In the intermediate sector, firm $i \in \varpi_2$ uses capital and labour as production inputs according to a constant returns to scale technology:

$$(5) \quad y_{it} \leq z_t k_{it}^\alpha (\ell_{it})^{1-\alpha}$$

where z_t is an exogenous productivity shock common to all firms:

$$(6) \quad \ln[z_{t+1}] = \rho_z \ln[z_t] + \sigma_z \varepsilon_{t+1}^z$$

and $\varepsilon_t^z \sim N(0, 1)$. Capital services are rented from centralized markets, and are perfectly mobile across firms.

Each firm chooses k_{it} and ℓ_{it} by taking their rental rates as given. The allocation problem for production factors is:

$$(7) \quad \max_{\{\ell_{it+n}, k_{it+n}\}_{n=0}^\infty} \mathbb{E}_0 \sum_{n=0}^\infty \Xi_{t+n|t} P_{t+n} \left[\frac{P_{t+n}}{P_{t+n}} y_{t+n} - w_{t+n} \ell_{t+n} - r_{t+n} k_{t+n} \right]$$

subject to the constraints 3 and 5. The stochastic discount factor $\Xi_{t+n|t}$ collects the prices of the claims that pay each one unit of final good for a given state of nature at $t+n$, normalized by the probability of the state.

Remark 1 Under the assumptions of complete asset markets, and full ownership of intermediate good firms from households, I can write:

$$(8) \quad \Xi_{t+n|t} = \beta^{t+n} \frac{S_{t+n}}{S_t}$$

where ζ_t is the shadow value of the household budget constraint.

This formulation is based on the assumptions that inputs of the production are perfectly mobile across firms, and that they can be rented from centralized markets. First order conditions are standard:

$$(9) \quad w_t = mc_{\ell t}(1 - \alpha)z_t k_{it}^\alpha (\ell_{it})^{-\alpha}$$

$$(10) \quad r_t = mc_{k t} \alpha z_t \left[\frac{\ell_{it}}{k_{it}} \right]^{1-\alpha}$$

with real marginal costs per unit of output mc_u .

2.3 The price stickiness

Sticky prices arise from staggered price contracts in the tradition of Calvo (1983). Each firm is allowed to change the price of her intermediate good with a fixed probability $1 - \phi_p$. A price that is not negotiable in the current period increases at the steady-state rate of inflation $\bar{\pi}$. Along with the assumption of monopolistically competitive markets, this mechanism implies that firms are willing to satisfy unexpected fluctuations in demand even when they cannot change their prices (see Erceg, Henderson and Levin, 2000). Assuming that no re-optimization has ever taken place, the price-setting decision of firm i in period t involves choosing a contingent plan for \tilde{P}_{it} such that:

$$(11) \quad \max_{\tilde{P}_{it}} \mathbb{E}_t \sum_{n=0}^{\infty} \Xi_{t+n|t} \phi_p^n P_{t+n} \left[\frac{\bar{\pi}^n \tilde{P}_{it}}{P_{t+n}} y_{t+n} - mc_{t+n} y_{t+n} \right].$$

Optimal price decisions for firms that can adjust prices at t follow from the first order condition:

$$(12) \quad \mathbb{E}_t \sum_{n=0}^{\infty} \Xi_{t+n|t} \phi_p^n \left[\frac{1 - \theta}{\theta} \frac{\bar{\pi}^n \tilde{P}_{it}}{P_{t+n}} + mc_{t+n} \right] \left(\frac{\bar{\pi}^n \tilde{P}_{it}}{P_{t+n}} \right)^{-\theta-1} y_{t+n} = 0.$$

This equation indicates that prices are chosen in such a way that the real marginal revenues equal the real marginal costs of the firm in expectations.

2.4 The household sector

The demand side of the economy is populated by a continuum of infinitely lived consumers indexed by $j \in \mathcal{O}_1$. Each agent enjoys utility from current consumption c_{jt} and disutility from hours worked l_{jt} . The history of events $s^t = \{s_0, \dots, s_t\}$ up to date t is attached a time-0 probability mass $\mu(s^t)$. The uncertainty in the choice process is summarized by the conditional-expectation operator $\mathbb{E}_0[\cdot] := \sum_{s^{t+1}} \mu(s_{t+1}|s^t)$. Given this structure, the household j 's allocation problem takes the form:

$$(13) \quad \max_{\{c_{jt}, l_{jt}, k_{jt+1}, A_{b,jt}, W_{jt}\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_{jt}, l_{jt}) \right]$$

s. t. $(1 + \tau_t)C_{jt} + \eta_t A_{b,jt} + I_{jt}$
 $+ \leq (1 - \tau_t)W_{jt}l_{jt} + [(1 - \tau_t)r_t$
 $+ \tau_t \delta] K_{jt} + A_{b,jt-1} + A_{c,jt}$
 $+ TR_t + \Omega_{jt}.$

Lower-case variables are deflated by the price level. The intertemporal discount factor of the consumer is $\beta \in (0, 1)$. Consumer j enjoys utility from consumption c_{jt} , and disutility from hours worked l_{jt} .

The portfolio of financial assets includes one-period riskless nominal bonds $A_{b,jt}$ with price η_t , and state-contingent securities for a total inflow of $A_{c,jt}$. The household also owns the claim to a profit share ω_{jt} of the monopolistically competitive firm i . The gross interest rate on bonds is denoted as R_t . Let Ω_{jit} denote the dividend stream generated by firm i and appropriated by household j . The total dividend payment to household j is

$$(14) \quad \Omega_{jt} := \int_{i \in \mathcal{O}_2} \omega_{ji} \Omega_{jit} dt.$$

For the purpose of analytical simplicity, I assume that the allocation of ownership shares

across agents is constant, and beyond the control of households.

Consumers control the evolution of the individual-specific capital stock k_{jt} through their decisions on investment i_{jt} . Idiosyncratic capital services are rented to the firms of the intermediate good sector at the rate r_t . Capital accumulation follows a linear law of motion:

$$(15) \quad k_{jt+1} = i_{jt} + (1 - \delta)k_{jt}.$$

There is an average distortionary tax rate τ_t that enters the consumer's budget constraint. Following Kim and Kim (2003b), I introduce a depreciation allowance on capital-income taxation, where r_t is the rental rate of capital.

The first order conditions for the allocation problem of households are:

$$(16) \quad u_{c_{jt}} = (1 + \tau_t)\zeta_{jt}$$

$$(17) \quad \eta_t = \beta \mathbb{E}_t \frac{\zeta_{jt+1}}{\zeta_{jt} \pi_{t+1}}$$

$$(18) \quad \mathbb{E}_t \frac{S_{t+1}}{S_t} [(1 - \tau_{t+1})r_{t+1} + \tau_{t+1}\delta] + (1 - \delta)\mathbb{E}_t \frac{S_{t+1}}{S_t} = \frac{1}{\beta}$$

where ζ_{jt} is the Lagrange multiplier on the budget constraint.

2.5 Wage setting

The labor-market structure is based on the following:

1. each household is the monopolistic supplier of a differentiated labor service l_{jt} , $j \in \mathcal{O}_1$;
2. wage setters are 'atomistic' in that the individual labor supply cannot affect the aggregate nominal wage index wage W_t .

Since households have identical preferences, and the types of labor services are idiosyncratic, disutility from labor must enter the utility function in an additively separable way to generate full risk pooling across households. I assume that the felicity function $u : \mathbb{R}_+ \times [0, 1] \rightarrow \mathbb{R}_+$ takes the simple form:

$$(19) \quad u(c_{jt}, \ell_{jt}) := \log(c_{jt}) - \gamma \ell_{jt}.$$

This implies that the state contingent consumption plans are insulated from idiosyncratic wage decisions taken at different dates, thus providing a channel of risk sharing among workers. I should stress that the need for full risk insurance due to wage stickiness motivates the assumption on the existence of a competitive financial market with state contingent bonds.

Heterogenous labor inputs for the production of intermediate goods aggregate to satisfy the economy wide labor demand ℓ_t^d :

$$(20) \quad \ell_t^d \leq \left[\int_{j \in \mathcal{O}_1} \ell_{jt}^{\frac{\vartheta-1}{\vartheta}} dj \right]^{\frac{\vartheta}{\vartheta-1}},$$

where ℓ_t^d indicates labor demand, and where $\vartheta > 1$ is the elasticity of substitution across labor services. The aggregation technology (20) features constant elasticity of substitution like in Dixit (1977). Differentiation in the labor market is due to the decreasing marginal productivity of labor. Firms take the price of labor as given. Hence, the individual labor demand function is:

$$(21) \quad \ell_{jt} = \left[\frac{W_{jt}}{W_t} \right]^{-\vartheta} \ell_t^d$$

where W_{jt} is the real wage paid to household j . The nominal wage index prevailing in the economy takes the standard form:

$$(22) \quad W_t = \left[\int_{j \in \mathcal{O}_1} W_{jt}^{1-\vartheta} dj \right]^{\frac{1}{1-\vartheta}}.$$

Given the aggregate labor-demand equation:

$$(23) \quad \ell_t^d := \int_{i \in \mathcal{O}_2} \ell_{it} di,$$

the total supply of labor can be written as:

$$(24) \quad \ell_t = \int_{j \in \mathcal{O}_1} \ell_{jt} dj = \ell_t^d \int_{j \in \mathcal{O}_1} \left[\frac{W_{jt}}{W_t} \right]^{-\vartheta} dj = \ell_t^d \bar{s}_t$$

with a wage-dispersion term \tilde{s}_t

$$(25) \quad \tilde{s}_t := \int_{j \in \varpi_1} \left[\frac{\tilde{w}_{jt}}{w_t} \right]^{-\vartheta} dj.$$

Schmitt-Grohé and Uribe (2006) show that $\tilde{s}_t \geq 1$. This points to the macroeconomic effect of the source of labor market friction included in the model. That is, the number of hours supplied to the market should not be lower than the number of productive units of labor.

Each household chooses the wage rate for her idiosyncratic labor service. Like for prices, I model nominal wage stickiness *à la* Calvo (1983). Wage contracts have a random duration. In every period, households have a probability $1 - \phi_w$ of resetting their contingent wage plans. Since the probability of changing wages is constant, a fraction $1 - \phi_w$ of households can reset contracts in each period. The expected time between price changes is $1/(1 - \phi_w)$. Following Erceg, Henderson and Levin (2000), I assume that the wages that cannot be changed, increase automatically by the steady state inflation rate $\bar{\pi}$. Suppose a worker has not adjusted her labor contract since period t . Then, her wage at $t + m$ is

$$(26) \quad W_{jt+m} = \bar{\pi}^m W_{jt}.$$

Since each household needs to keep track only of the histories of no re-optimization, the wage setting problem consists in maximizing the following:

$$(27) \quad \mathbb{E}_t \sum_{m=0}^{\infty} (\beta \phi_w)^m \left[u(c_{jt+m}, \ell_{jt+m}) + \varsigma_{t+m} (1 - \tau_{t+m}) \frac{(\bar{\pi}^m \tilde{W}_{jt})^{1-\vartheta}}{P_{t+m} W_{t+m}^{-\vartheta}} \ell_{t+m}^d \right].$$

The first order condition with respect to \tilde{W}_{jt} reads as:

$$(28) \quad \mathbb{E}_t \sum_{m=0}^{\infty} (\beta \phi_w)^m \left[\gamma \left(\frac{\bar{\pi}^m \tilde{W}_{jt}}{W_{t+m}} \right)^{-\vartheta} \ell_{t+m}^d + \varsigma_{t+m} (1 - \tau_{t+m}) \frac{1 - \vartheta}{\vartheta} \frac{\bar{\pi}^m \tilde{W}_{jt}}{P_{t+m}} \left(\frac{\bar{\pi}^m \tilde{W}_{jt}}{W_{t+m}} \right)^{-\vartheta} \ell_{t+m}^d \right] = 0.$$

The intuition behind this equation is that, when the wage rate is optimized, it will be set so that the discounted marginal utility of labour income – i.e. the second expression in brackets – is equal to the discounted marginal disutility from work – i.e. the first expression in brackets. Notice that $\vartheta/(1 - \vartheta)$ is the wage markup over the marginal cost of labour in absence of wage rigidity.

3. The fiscal and monetary policy rules

The government faces a standard flow budget constraint:

$$(29) \quad \int_{j \in \omega_1} D_{jt} dj + P_t \tau_t = R_{t-1} \int_{j \in \omega_1} D_{jt-1} dj + P_t g_t^c.$$

Real total taxation is denoted as τ_t , and g_t^c indicates total government spending. The government issues one-period riskless (non-contingent) nominal bonds denoted by D_t . Real total revenues from taxation are

$$\tau_t := \tau_t \int_{j \in \omega_1} c_{jt} dj + \tau_t \int_{j \in \omega_1} (r_t - \delta) k_{jt} dj + \tau_t \int_{j \in \omega_1} w_{jt} \ell_{jt} dj.$$

Public spending follows the exogenous process

$$(30) \quad \ln[g_{t+1}^c] = \rho_g \ln[g_t^c] + (1 - \rho_g) \ln[\bar{g}^c] + \sigma_g \varepsilon_{t+1}^g,$$

with $\varepsilon_t^g \sim N(0, 1)$. Following Schmitt-Grohé and Uribe (2006), I define the total amount of government liabilities l_t in equilibrium $l_t := R_t d_t$. Hence, the flow government budget constraint takes the form

$$(31) \quad l_t = \frac{R_t l_{t-1}}{\pi_t} + R_t (g_t - \tau_t).$$

The intertemporal budget constraint of the government is written as:

$$(32) \quad R_t \int_{j \in \omega_1} D_{jt} dj \leq \sum_{p=0}^{\infty} \mathbb{E}_{t+p} \left(\frac{1}{R_{t+p}} \right)^p \left[P_{t+p} \tau_{t+p} - P_{t+p} g_{t+p}^c \right].$$

Although not emerging from the notation, the intertemporal budget constraint should hold for every realization of the stochastic shocks (see Bohn, 1995).

Like Kollmann (2007), I assume that the level of the tax instrument τ_t evolves according to the feedback rule

$$(33) \quad \ln \left[\frac{\tau_t}{\bar{\tau}} \right] = \psi_1 \ln \left[\frac{l_{t-1}}{\bar{l}} \right] + \psi_2 \ln \left[\frac{g_t^c}{\bar{g}^c} \right] + \psi_3 \ln [z_t].$$

Finally, the central bank sets the policy rate in the style proposed by Taylor (1993). Since the model includes wage rigidity, I study the case of a simple monetary-policy rule with both price and wage-inflation targets

$$(34) \quad \ln \left[\frac{R_t}{\bar{R}} \right] = \alpha_\pi \ln \left[\frac{\pi_t}{\bar{\pi}} \right] + \alpha_w \ln \left[\frac{w_t}{\bar{w}} \right] + \alpha_y \ln \left[\frac{y_t}{\bar{y}} \right] + \alpha_R \ln \left[\frac{R_{t-1}}{\bar{R}} \right].$$

It should be noted that both the rules 33 and 34 for fiscal and monetary policy are operational despite the fact that the targeting variables on the right-hand side are dated at time t . The reason for this lies in the timing assumptions of the New Keynesian model where, after the realization of the exogenous shocks, output and inflation are realized, and monetary policy is set.

4. Aggregation and equilibrium

Assume that all the households and firms that can change their idiosyncratic wage and price contracts choose, respectively, the same new wages and prices. Aggregate prices and wages evolve according to:

$$(35) \quad (P_t)^{1-\theta} = \phi_p (\bar{\pi} P_{t-1})^{1-\theta} + (1 - \phi_p) (\tilde{P}_t)^{1-\theta}$$

$$(36) \quad (W_t)^{1-\vartheta} = \phi_w (\bar{\pi} W_{t-1})^{1-\vartheta} + (1 - \phi_w) (\tilde{W}_t)^{1-\vartheta}$$

which can be re-written as:

$$(37) \quad 1 = \phi_p \left(\frac{\bar{\pi}}{\pi_t} \right)^{1-\theta} + (1 - \phi_p) (\tilde{p}_t)^{1-\theta}$$

$$(38) \quad w_t^{1-\vartheta} = \phi_w \left[\frac{\bar{\pi} w_{t-1}}{\pi_t} \right]^{1-\vartheta} + (1 - \phi_w) \tilde{w}_t^{1-\vartheta}$$

with $\tilde{p}_t := \frac{\tilde{P}_t}{P_t}$ and $\tilde{w}_t := \frac{\tilde{W}_t}{W_t}$. As a result, the economy wide resource constraint is:

$$(39) \quad y_t = \left[\int_{j \in \omega_1} c_{jt} + \int_{j \in \omega_1} i_{jt} + \int_{j \in \omega_1} g_{jt} \right] s_t$$

where s_t denotes a price-dispersion term (see Yun, 1996; Schmitt-Grohé and Uribe, 2006)

$$(40) \quad s_t := \int_{l \in \omega_2} \left[\frac{P_{lt}}{P_t} \right]^{-\theta} dl.$$

A consequence of the assumptions on aggregation is that households that start out with an endowment of financial assets (bonds) that gives the same initial intertemporal budget constraint will choose the same consumption and asset allocation plans. This is due to the fact that households face the same prices for goods, the same marginal utility of consumption, and enjoy full risk pooling from wage uncertainty. Hence, the budget constraint is identical across households. Also, households will choose asset allocations to ensure that their intertemporal budget constraints are the same both at each point in time, and at each state of the world. This derives from the assumption of strong separability of the disutility from idiosyncratic labor supply, which allows all the households to equalize the marginal utility of income at each state by holding state contingent securities (see Woodford, 2003).

The discussion presented above shows that equilibria for this set of economies are stationary sequences of prices $\{\mathbf{P}_t\}_{t=0}^\infty := \{P_t^*, \tilde{P}_t^*, R_t^*, \eta_t^*, w_t^*, \tilde{w}_t^*, r_t^*, s_t^*, \tilde{s}_t^*\}_{t=0}^\infty$, quantities $\{\mathbf{Q}_t\}_{t=0}^\infty := \{\{Q_t^h\}_{t=0}^\infty, \{Q_t^f\}_{t=0}^\infty, \{Q_t^g\}_{t=0}^\infty, \{Q_t^i\}_{t=0}^\infty := \{c_t^*, \ell_t^*, \ell_{t+1}^{d*}, k_{t+1}^*, i_t^*, a_{b,t}^*\}_{t=0}^\infty, \{Q_t^j\}_{t=0}^\infty := \{y_t^*, k_t^*, \ell_t^*\}_{t=0}^\infty, \{Q_t^g\}_{t=0}^\infty := \{g_t^{c*}, \tau_t, d_t^*\}_{t=0}^\infty$ and stochastic shocks $\{\mathcal{E}_t\}_{t=0}^\infty := \{\epsilon_t^z, \epsilon_t^g\}_{t=0}^\infty$ that aggregate over $\varpi_1 = [0, 1]$ and $\varpi_2 = [0, 1]$, that are bounded in a neighborhood of the steady state, and such that¹

- (i) given prices $\{\mathbf{P}_t\}_{t=0}^\infty$ and shocks $\{\mathcal{E}_t\}_{t=0}^\infty$, $\{Q_t^h\}_{t=0}^\infty$ is a solution to the representative household's problem;
- (ii) given prices $\{\mathbf{P}_t\}_{t=0}^\infty$ and shocks $\{\mathcal{E}_t\}_{t=0}^\infty$, $\{Q_t^f\}_{t=0}^\infty$ is a solution to the representative firms' problem;
- (iii) given quantities $\{\mathbf{Q}_t\}_{t=0}^\infty$ and shocks $\{\mathcal{E}_t\}_{t=0}^\infty$, $\{\mathbf{P}_t\}_{t=0}^\infty$ clears the markets for both goods and factors of production:

$$(41) \quad y_t^* = [c_t^* + i_t^* + g_t^*] s_t^*$$

¹ Formal proofs for deriving the resource constraints is presented in section 1 of a detailed appendix available from the journal's website.

$$(42) \quad s_t^* = (1 - \phi_p) [\tilde{p}_t^*]^{-\theta} + \phi_p \left[\frac{\pi_t^*}{\bar{\pi}} \right]^\theta s_{t-1}^*$$

$$(43) \quad \tilde{s}_t^* = (1 - \phi_w) \left[\frac{\tilde{w}_t^*}{w_t^*} \right]^{-\vartheta} + \phi_w \left[\frac{\bar{\pi} w_{t-1}^*}{\pi_t^* w_t^*} \right]^{-\vartheta} \tilde{s}_{t-1}^*$$

and the markets for bonds:

$$(44) \quad R_t^* = \frac{1}{\eta_t^*}$$

$$(45) \quad a_{b,t}^* = d_t^*$$

$$(46) \quad a_{c,t}^* = 0$$

- (iv) given quantities $\{\mathbf{Q}_t\}_{t=0}^\infty$, prices $\{\mathbf{P}_t\}_{t=0}^\infty$ and shocks $\{\mathcal{E}_t\}_{t=0}^\infty$, $\{Q_t^g\}_{t=0}^\infty$ satisfies the government flow and the intertemporal budget constraint;
- (v) fiscal and monetary policies are set according to the simple rules outlined in section 3.

An additional point is worth stressing. With full indexation to steady-state inflation, there is no price heterogeneity at the deterministic steady state, i.e. $\bar{p}^* = 1$. This implies that there is no price dispersion either, i.e. $s^* = 1$. In this case, Yun (1996) shows that the price dispersion term can be ignored for a log-linear first order approximation. In order to account for the full nonlinear dynamics, I include the price dispersion term as an endogenous predetermined variable². The same type of argument applies to wage dispersion in the case of indexation to steady state inflation.

5. Computational aspects

5.1 Local validity of approximate solutions

Second order perturbation methods are defined only around small neighbourhoods of the approximation points, unless the approximated function is globally analytic (see Anderson,

² See section 2 of the supplementary appendix.

Levin and Swanson, 2004). Since the condition for an analytic form of the policy function are hardly establishable, the problem of validity of the Taylor expansion remains. I impose an *ad hoc* bound that restricts the unconditional mean of each state variable – denoted by x – to be arbitrarily close to its deterministic counterpart \bar{x} :

$$(47) \quad |\mathbb{E}[x_t] - \bar{x}| < \kappa_1$$

Kollmann (2007) imposes this type of constraint only on public debt. The results from numerous experiments suggest that this is not enough to prevent the problem of large deviations from the deterministic steady state from occurring.

5.2 Welfare evaluation

The policy problem consists in the optimization of the coefficients in the fiscal and monetary policy rules. The welfare criterion upon which the maximization relies is utility based. In this paper, we consider a measure of conditional welfare. This is constructed so that it takes into consideration the transitional welfare costs of moving from an initial state to the stochastic steady state of the model. The traditional strand of literature on monetary policy uses an index of unconditional welfare, thus neglecting the role of model dynamics towards the long run. However, as discussed in Kim, Kim, Schaumburg and Sims (2003), this welfare metric leads to incorrect rankings of alternative policy arrangements when transitional effects matter. This point is discussed further below.

Conditional aggregate welfare is the expected stream of lifetime utility of a randomly drawn household for a given initial distribution \tilde{s} of the state variables:

$$(48) \quad \mathcal{W}_0 := \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t u(s_{jt}) \mid \tilde{s} \sim (\underline{s}, \underline{\Omega}) \right]$$

The terms \underline{s} and $\underline{\Omega}$ indicate, respectively, the mean and the covariance matrix of the distribution of the initial state of the economy. Since all the households face the same budget constraint, they choose the same consumption plans in the equilibrium. Hence, the conditional welfare function is:

$$(49) \quad \begin{aligned} \mathcal{W}_0 &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(c_t) - \gamma \int_{j \in \varpi_1} \ell_{jt} dj \right\} \\ &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{ \log(c_t) - \gamma \ell_t \} \end{aligned}$$

The conditional welfare function is evaluated by computing the second order Taylor expansion around the deterministic steady states of each distorted economy. The reason for adopting this approach is that omitting second order terms can yield spurious welfare reversals (see Kim and Kim, 2003a). Woodford (2003) shows that a first order solution produces accurate welfare computations only if the deterministic steady state of a model is at the first best. For instance, a typical assumption is that there exist lump sum taxes that wipe away the real distortions due to the monopolistic power of the firms in the intermediate product markets. The presence of distortionary taxation, as well as imperfect competition in both labor and product markets makes it unfeasible to comply with this condition. The welfare evaluation problem requires me to approximate the solution to the optimality conditions through a second order Taylor expansion. To that end, I use the algorithm and computer codes described in Schmitt-Grohé and Uribe (2004).

In order to compare the outcomes of different policies, I compute the permanent change in consumption, relative to the model's steady state, that yields the expected utility level of the distorted economy. Given steady states of consumption \bar{c}' and hours worked \bar{l}' of the model ι , this translates into the number Φ'_c such that:

$$(50) \quad \sum_{t=0}^{\infty} \beta^t u([1 - \Phi'_c] \bar{c}', \bar{l}') = \mathcal{W}'_0$$

I decompose the conditional welfare cost Δ'_c into two components denoted as Δ'_E and Δ'_V . Given the approximation

$$(51) \quad \begin{aligned} u([1 - \Delta'_c] \bar{c}, \bar{l}) &\approx u(\bar{c}, \bar{l}) \\ &+ (1 - \beta) \sum_{t=0}^{\infty} \beta^t \left(\mathbb{E} [\hat{c}_t - \bar{\ell} \hat{\ell}_t | s_0] \right. \\ &\left. - \frac{1}{2} \text{VAR} [\hat{c}_t | s_0] \right), \end{aligned}$$

I compute the change in mean consumption Δ_E^t that the household faces while giving up the total fraction of certainty equivalent consumption Δ_c^t

$$(52) \quad u\left([1 - \Delta_E^t] \bar{c}, \bar{\ell}\right) = u\left(\bar{c}, \bar{\ell}\right) + (1 - \beta) \sum_{t=0}^{\infty} \beta^t \left(\mathbb{E}[\hat{c}_t | s_0] - \bar{\ell} \mathbb{E}[\hat{\ell}_t | s_0] \right).$$

Since the solution method is non certainty equivalent, I can also calculate the change in conditional variance of consumption Δ_V^t that is consistent with the total welfare cost of policies

$$(53) \quad u\left([1 - \Delta_V^t] \bar{c}, \bar{\ell}\right) = u\left(\bar{c}, \bar{\ell}\right) - (1 - \beta) \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \text{VAR}[\hat{c}_t | s_0],$$

where hats denote log-deviations from the deterministic steady states. It can be shown that the three measures of welfare are linked as

$$(54) \quad (1 - \Delta_c^t) = (1 - \Delta_E^t)(1 - \Delta_V^t).$$

As there are no closed-form solutions for the infinite summations in the expressions for Δ_V^t and Δ_E^t , the discounted sums are computed by Monte Carlo simulations through the analytical expressions summarized in Appendix 5 of Zagaglia (2007).³

³ An anonymous referee suggested to use the ‘pruning’ algorithm for second order approximations presented in Kim, Kim, Schaumburg and Sims (2003). That would then permit to derive analytical expressions for the conditional expectations and variances of equations 51–53 and, eventually, a recursive procedure for the conditional welfare costs. However, practical experience suggests that the ‘pruning’ algorithm makes the problem computationally intractable in terms of time and resource required. The attractiveness of the approximation method proposed by Schmitt-Grohé and Uribe (2004) relies in the distinction, decided by the user, between state and endogenous variables. This reduces the dimensionality of the second order approximation, and makes the computational solution faster. In the ‘pruning’ algorithm, instead, states and non-states are assigned as a function of the solution, which adds a further dimension to the problem.

6. Calibration

I calibrate the model on quarterly data for an average G-7 economy. The long-run inflation rate is set to 4% a year. The intertemporal discount factor β equals 0.9949.⁴ Households devote a steady state share of time to market activity equal to 0.4. I choose a steady-state ratio between investment and output equal to 0.25. The quarterly depreciation rate is consistent with an annual rate of 10%. The capital elasticity of output is 0.4 like in Kim and Kim (2003b).

The price-marginal cost markup factor is set at $\theta/(\theta-1) = 1.25$, as suggested by Bayoumi, Laxton and Pesenti (2004). The wage markup $\vartheta/(\vartheta-1)$ takes the value of 1.5. These figures are in lines with those suggested by Bayoumi, Laxton and Pesenti (2004). Following Pappa (2004), I choose $\phi_p = \phi_w = 2/3$, which implies an average contract duration of $1/(1-2/3) = 3$ quarters.

The steady-state ratio between real public debt and output is 50%. The output share of government consumption is 32%. The average tax rate on income is 0.29, and matches the G-7 average from Kim and Kim (2003b). The calibration of the government spending shock is from Marzo (2006) and Schmitt-Grohé and Uribe (2005).

The average deviations of the state variables from the deterministic steady state – i.e. the parameter on the stationarity constraint (47) – is set to a maximum of 5%.⁵

7. Results

Table 2 reports the optimized coefficients for several variants of the policy rules with both

⁴ As pointed out by an anonymous referee, this value is slightly higher than what is usually assumed. In equation 32 of the supplementary appendix, I show that I calibrate β from the optimality condition of the household’s allocation problem with respect to capital. The reason for this strategy is that I assume that both the average tax rate on capital income and the steady-state ratio between output and capital are known.

⁵ An exhaustive number of numerical experiments suggest that the results do not vary substantially within a range of deviations from 2% to 10%.

Table 1: Calibration of the model

| Description | Parameter | Value |
|--|---------------------|-------------------|
| Subjective discount factor | β | 0.9948 |
| Weight on disutility from work | γ | 1 |
| Time share devoted to market activity | ℓ | 0.4 |
| Elasticity of substitution of labor services | ϑ | 7 |
| Elasticity of substitution of intermediate goods | θ | 5 |
| Rate of capital depreciation | δ | $1.1^{(1/4)} - 1$ |
| Capital elasticity of intermediate output | α | 0.4 |
| Fraction of workers not setting wages optimally | ϕ_w | 2/3 |
| Fraction of firms not setting prices optimally | ϕ_p | 2/3 |
| Steady-state inflation | $\bar{\pi}$ | $1.04^{(1/4)}$ |
| Persistence of productivity shock | ρ_z | 0.98 |
| Variance of productivity shock | σ_z^2 | 0.0003 |
| Steady-state average tax rate | $\bar{\tau}$ | 0.30 |
| Steady state ratio of gov. consumption to output | \bar{g}^c/\bar{y} | 0.15 |
| Persistence of government spending shock | ρ_g | 0.95 |
| Variance of government spending shock | σ_g^2 | 0.000126 |
| Parameter on the stationarity constraint | κ_1 | 0.05 |

Table 2: Optimal policy rules with steady-state indexation

| | Optimized coefficients | | | | |
|-------------------|----------------------------------|-----------|-----------|-----------|-----------|
| | Mix 1 | Mix 2 | Mix 3 | Mix 4 | Mix 5 |
| ψ_1 | 0.93 | 2.93 | 0.55 | 1.01 | 2.74 |
| ψ_2 | 9.61 | 47.91 | 7.93 | - | - |
| ψ_3 | -19.97 | 49.97 | -19.97 | - | - |
| α_w | 24.76 | - | 3.30 | - | 9.24 |
| α_π | 1.14 | 4.50 | - | 3.09 | - |
| α_y | 10.99 | 3.96 | 19.18 | - | - |
| α_R | -2.50 | 4.32 | 1.82 | - | - |
| | Welfare | | | | |
| \mathcal{W}_0^c | -126.8560 | -126.8685 | -126.8621 | -127.6832 | -127.6818 |
| Δ_c^t | -0.441 | -0.435 | -0.438 | -0.052 | -0.052 |
| Δ_E^t | -0.441 | -0.494 | -0.438 | -0.512 | -0.052 |
| Δ_V^t | 0.0002 | 0.058 | 0.0006 | -0.00008 | 0.00009 |
| | Standard deviations (in percent) | | | | |
| y_t | 0.568 | 2.414 | 0.037 | 1.009 | 0.831 |
| π_t | 0.419 | 1.357 | 0.218 | 0.104 | 0.107 |
| R_t | 0.401 | 2.263 | 0.201 | 0.009 | 0.034 |
| c_t | 0.358 | 4.910 | 0.197 | 0.140 | 0.143 |
| ℓ_t | 0.980 | 4.969 | 0.190 | 1.724 | 1.422 |
| b_t | 4.049 | 4.884 | 5.649 | 5.634 | 5.429 |
| τ_t | 1.009 | 3.086 | 0.798 | 0.056 | 0.054 |

Legend: Mix 1: fully-optimized rules. Mix 2: no wage-inflation target. Mix 3: No price-inflation target. Mix 4: Only price-inflation and government liability targets. Mix 5: Only wage-inflation and government liability targets.

Note: All the measures of conditional welfare costs are expressed in percentage points.

prices and wages indexed to steady-state inflation. Since the focus of the paper is on implementable rules for monetary and fiscal policy, I disregard the optimal Ramsey policy. Ramsey plans typically involve complicated functions of the state variables. Unfortunately the literature provides no insight on the properties of implementable rules for Ramsey plans. Moreover, the available quantitative studies on Ramsey fiscal policy use global approximation methods to deal with the nonstationarities induced by the presence of endogenous public debt (e.g. see Cosimano, 2005). That would raise additional problems of comparisons with the main results of the text. The results consider five types of interaction between optimal monetary and fiscal policy. The maximization of welfare over all the parameters in the monetary and fiscal policy rules is denoted as policy 'mix 1'. The other combinations of rules follow from setting to zero some of the policy coefficients. In the 'mix 2', monetary policy is assumed not to respond to wage inflation. In the 'mix 3', instead, the policy rates do not react to price inflation. Finally, in the policy mixes 4 and 5, monetary policy responds to deviations of inflation and real wages from the steady state, respectively, whereas the tax rate reacts to the steady-state deviations of government liabilities.

With the fully-optimal rule for monetary policy, four results emerge. First, optimal monetary policy responds to inflation according to that the standard version of the Taylor principle, i.e. $\alpha_\pi > 1$. Second, the reaction of the average tax rate to past liabilities is small in relation to the one of monetary policy to inflation. This result is also obtained by Kollmann (2007) and Schmitt-Grohé and Uribe (2006) in a model with only price rigidity.⁶ Third, the welfare-maximizing rule for monetary policy is very

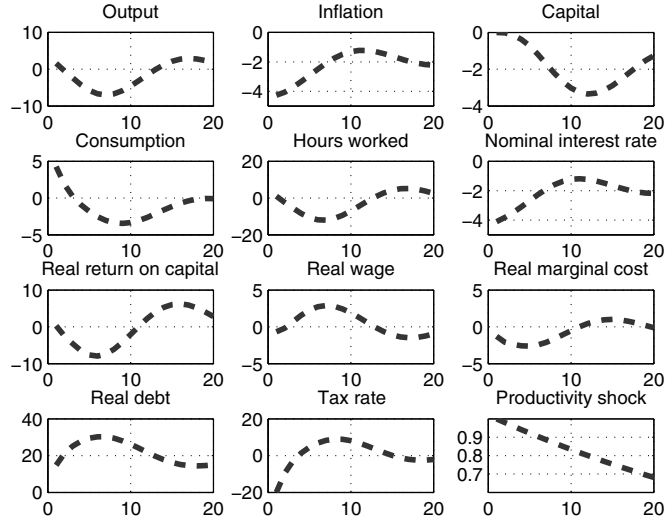
responsive to output fluctuations.⁷ As noticed by Marzo (2006), a high level of distortion requires a strong policy reaction towards fluctuations in nominal income. However, the model embeds an aggregate-supply curve. A policy prescription that is too aggressive in terms of fighting inflation can determine either a recession, or excessive volatility in output (see Roberts, 1995). This requires either a relaxation of the anti-inflationary stance, or a stronger responsiveness to output. Finally, the optimal response of unrestricted monetary policy to wage changes is large, and stronger than the reaction to price inflation. The intuition is that, since all job varieties are ex ante identical in the Calvo setting, any wage dispersion is inefficient. Hence, policies that respond to wage fluctuations maximize welfare.

Figures 1(a) and 1(b) report the impulse responses of the model with the unconstrained policy mix. The pattern is similar to the one documented by Schmitt-Grohé and Uribe (2005) in a setting with a larger number of real rigidities. After a positive productivity shock, the capital stock drops for a prolonged period of time (see figure 1(a)). This implies that the response of investment is very sluggish. As a result, the rental rate of capital falls mildly on impact towards an inverse hump-shaped path. The small initial reaction of investment is driven by the large response of consumption on impact, which is determined by an increase in disposable income. Two factors play a key role. First, the average tax rate displays a large drop. Second, the increase in the real value of outstanding public overcompensates the initial drop of the nominal interest rate, thus sustaining the response of consumption. The drop of the tax rate allows households to keep the supply of labour services at a low level. It is interesting to notice that workers use their market power to allow for a small negative response of the wage rate, following the limited increase in the demand for labour. Finally, the response of consumption features the rise and fall shape that arises typically from internal habit formation

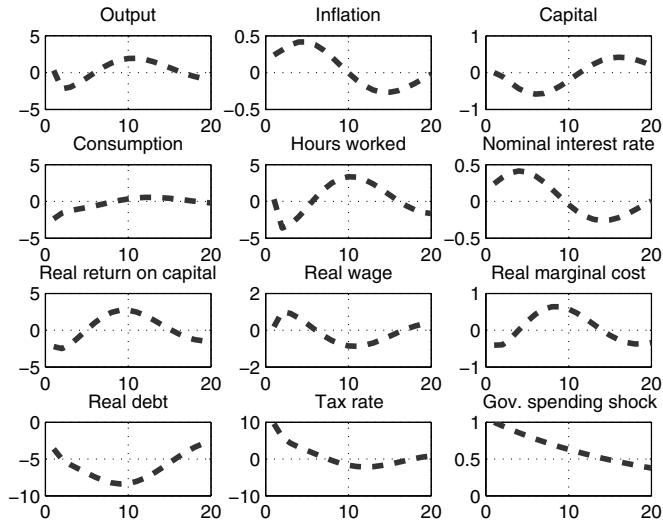
⁶ The reader should notice that the size of the optimal reaction to inflation does not allow a clear-cut classification between 'active' and 'passive' policy in the style of Leeper (1991). The reason is that taxes are distortionary in the model discussed here. Unlike in Leeper (1991)'s framework with lump-sum taxation, tax changes affect the real allocations directly at each point in time. This goes beyond the expectation channel stressed by Leeper (1991) that arises from misalignments between the expected value of future primary surpluses and the discounted stream of debt servicing costs.

⁷ Imposing a constraint on the size of α_y suggests that welfare maximization picks up a level of responsiveness to output that is often close to the largest level attainable.

Figure 1. Impulse responses (%) under the fully-optimal policy mix with wage and price rigidity, indexation to long-run inflation.



(a) Productivity shock



(b) Government spending shock

(see Christiano, Eichenbaum and Evans, 2005). Figure 1(b) shows that a government spending shock entails an inflationary waste of resources, in that it raises both output and inflation on impact. Differently from the case of a productivity

shock, the positive response of the tax rate depresses disposable income, and induces the workers to use their market power. As a result, the real wage rate rises. However, since also real debt holdings fall by a large extent, this is not enough to prevent consumption spending from falling. These considerations suggest that

the mildly positive response of output on impact is determined by a sluggish increase in investment.

With a monetary policy rule that does not respond to the deviation of real wages from their steady state – policy mix 2 –, the prescribed reaction of monetary policy to price inflation is higher than that of the fully-optimal rule (see table 2). At the same time, the responsiveness of tax policy to changes in government liabilities becomes sizeable.

Table 2 suggests that not reacting to price inflation – policy mix 3 – leads to welfare losses smaller than those generated by not responding to real wages. This finding can be motivated in several ways. The lack of reaction of monetary policy to inflation also leads to a small optimal feedback of the average tax rate from government liabilities. This implies that the inflation rate is pushed around by cyclical responses of the policy mix only to a limited extent. Thus, a source of welfare reducing changes in markups is partly eliminated (see Schmitt-Grohé and Uribe, 2006). This constrained policy configuration appears less damaging in terms of welfare also because it features the largest reaction of monetary policy to output. The main lesson is that shutting down the responsiveness to inflation can destabilize the economy by enhancing booms and busts in inflation as a response to exogenous fluctuations. Hence, the extent of output stabilization implied by the rule needs to be supplemented through a large coefficient on the output gap.

The fully-optimal rules 33 and 34 incorporate a rich array of variables that can make the optimal policy mix difficult to implement. As a benchmark for comparison, I also consider two configurations of feedback rules whereby the average tax rate responds only to government liabilities, as suggested in the literature on the fiscal theory of price level determination (see Leeper, 1991). Monetary policy is instead assumed to react to either price inflation – policy mix 4 – or real wage fluctuations – policy mix 5. Once again, the results indicate that targeting wages alone achieves a slightly higher level of welfare at the cost of supporting a negligible worsening in the variability of consumption.

The third panel of table 2 reports some de-

scriptive statistics from the model solution for each rule. Since the responsiveness of monetary policy to inflation is not large, rule 1 does not achieve the lowest variance of inflation across the range of policy rules. This finding can be reconciled with the quantitative results of Ercog, Henderson and Levin (2000). Their evidence indicates that inflation stabilization cannot deliver the long run allocation when both wage and price stickiness are present. Following the intuition outlined earlier, table 2 also shows that constraining monetary policy to not responding to real wages, as in rule 2, produces the largest variability in both inflation and output. Instead, when monetary policy does not react to movements in prices, the variance of both inflation and output is low. Rule 3 also displays the largest fluctuations in public debt. Like for the other rules, this corresponds to a somewhat negligible variability in the tax rates.

7.1 An economy with only wage rigidity

Kollmann (2007) shows that, in a framework where both prices and wages are fully flexible, and the government issues only nominal debt, fiscal rules that prescribe unexpected variations in the price level to support debt changes are welfare maximizing. This takes the form of a strong reaction of the average tax rate to government liabilities. The intuition is that the markup fluctuations induced by price changes do not generate inefficient dispersion in equilibrium (see Schmitt-Grohé and Uribe, 2006). Hence, the policy planner can comply with the intertemporal budget constraint by changing the real value of outstanding debt through unexpected inflation.

In order to investigate the role of alternative sources of rigidity, table 3 reports the optimized feedback coefficients in an economy with flexible prices, and sticky wages indexed to steady-state inflation. With the unconstrained set of policy rules, the finding of Kollmann (2007) is confirmed. The interaction between tax distortions and wage rigidity is consistent with a low response of monetary policy to real wages. This implies that, with the unconstrained rule, wage stickiness becomes a minor objective for the

Table 3: Optimal policy rules with only wage rigidity

| | Mix 1 | Mix 2 | Mix 3 | Mix 4 | Mix 5 |
|----------------------------------|-----------|-----------|-----------|-----------|-----------|
| ψ_1 | -13.68 | -0.01 | -0.30 | -0.2 | -0.33 |
| ψ_2 | 26.08 | -1.95 | -0.24 | - | - |
| ψ_3 | -14.10 | -0.19 | 2.28 | - | - |
| α_w | 0.80 | - | -0.06 | - | 0.05 |
| α_π | 2.88 | -1.11 | - | -0.82 | - |
| α_y | -0.65 | -0.08 | -0.97 | - | - |
| α_R | -1.90 | 0.66 | 0.66 | - | - |
| Welfare | | | | | |
| W_0^c | -126.3725 | -127.7938 | -127.7011 | -127.7989 | -127.7999 |
| Δ_c^c | -0.669 | 0.0008 | -0.043 | 0.003 | 0.004 |
| Δ_E^c | -0.722 | -0.003 | -0.042 | 0.003 | 0.002 |
| Δ_V^c | 0.052 | 0.001 | 0.0001 | 0.0001 | 0.001 |
| Standard deviations (in percent) | | | | | |
| y_t | 0.362 | 0.564 | 0.987 | 0.754 | 0.428 |
| π_t | 3.905 | 2.756 | 2.552 | 2.206 | 2.139 |
| R_t | 0.395 | 0.711 | 1.252 | 0.994 | 0.002 |
| c_t | 0.279 | 0.536 | 0.181 | 0.184 | 0.616 |
| ℓ_t | 0.608 | 0.756 | 1.677 | 1.283 | 0.258 |
| b_t | 3.383 | 2.443 | 7.908 | 7.031 | 1.524 |
| τ_t | 0.545 | 0.469 | 0.099 | 0.236 | 0.503 |

Legend: Mix 1: fully-optimized rules. Mix 2: no wage-inflation target. Mix 3: No price-inflation target. Mix 4: Only price-inflation and government liability targets. Mix 5: Only wage-inflation and government liability targets.

Note: All the measures of conditional welfare costs are expressed in percentage points.

optimal policy mix. Table 3 also reports the descriptive statistics for the model with flexible prices and rigid wages. Coherently with the arguments discussed earlier, the variability of inflation is ten times higher than in the setting with both wage and price stickiness.

7.2 A comparison with full indexation to lagged inflation

A framework that has gained considerable support in both the empirical and the theoretical literature on nominal rigidities assumes that wage and price contracts are indexed to lagged inflation (see Christiano, Eichenbaum and Evans, 2005). Like in the previous part of the paper, I consider only the case of full indexation. The allocation of price variations follows the Calvo scheme, with unchanged prices that

are updated on the rule $P_{it} = P_{it-1}\pi_{t-1}$. At the same time, unchanged real wages evolve according to $w_{jt} = w_{jt-1}\pi_{t-1}/\pi_t$. In particular, n periods after the last optimization, prices and nominal wages are, respectively, $P_{it+n} = \tilde{P}_{it}\prod_{p=1}^n \pi_{t+p-1}$ and $W_{jt+n} = \tilde{W}_{jt}\prod_{p=1}^n \pi_{t+p-1}$. The first order conditions from the optimization problem of firms and households are, respectively,

$$\begin{aligned}
 (55) \quad \mathbb{E}_t \sum_{n=0}^{\infty} \Xi_{t+n|t} \phi_p^n & \left[\frac{1-\theta}{\theta} \frac{\tilde{P}_{it}}{P_t} \prod_{p=1}^n \left(\frac{\pi_{t+p-1}}{\pi_{t+p}} \right) \right. \\
 & \left. + mc_{t+n} \right] \times \left(\frac{\tilde{P}_{it}}{P_t} \right)^{-\theta-1} \\
 & \left(\prod_{p=1}^n \frac{\pi_{t+p-1}}{\pi_{t+p}} \right)^{-\theta-1} y_{t+n} = 0
 \end{aligned}$$

Table 4: Optimal policy rules with indexation to past inflation

| | Mix 1 | Mix 2 | Mix 3 | Mix 4 | Mix 5 |
|--------------|--------|--------|-------|-------|-------|
| ψ_1 | 9.99 | -2.01 | 0.18 | 0.01 | -2.00 |
| ψ_2 | 4.47 | 1.35 | 0.37 | - | - |
| ψ_3 | 2.98 | 20.02 | -0.07 | - | - |
| α_w | 18.87 | - | 2.38 | - | 3.26 |
| α_π | 17.18 | 18.67 | - | 6.35 | - |
| α_y | 2.12 | -12.55 | 2.19 | - | - |
| α_R | -19.52 | 0.2 | 0.68 | - | - |

| Welfare | | | | | |
|--------------|-----------|-----------|-----------|-----------|-----------|
| W_0^l | -120.9957 | -126.8761 | -127.2854 | -127.3969 | -127.3988 |
| Δ_c^l | -0.280 | -0.007 | -0.007 | 0.002 | 0.003 |
| Δ_E^l | -9.790 | -0.107 | -0.008 | 0.002 | 0.003 |
| Δ_V^l | 8.661 | 0.099 | 0.001 | 0.00009 | 0.0001 |

| Standard deviations (in percent) | | | | | |
|----------------------------------|-------|-------|-------|-------|-------|
| y_t | 3.780 | 3.267 | 0.362 | 1.301 | 2.752 |
| π_t | 2.103 | 0.803 | 0.385 | 0.348 | 0.216 |
| R_t | 2.838 | 1.863 | 0.386 | 0.093 | 0.219 |
| c_t | 2.610 | 4.418 | 0.527 | 0.150 | 0.171 |
| ℓ_t | 7.413 | 7.967 | 0.129 | 0.129 | 4.676 |
| b_t | 3.693 | 6.701 | 1.923 | 5.893 | 5.444 |
| τ_t | 5.897 | 6.732 | 0.423 | 0.058 | 0.011 |

Legend: Mix 1: fully-optimized rules. Mix 2: no wage-inflation target. Mix 3: No price-inflation target. Mix 4: Only price-inflation and government liability targets. Mix 5: Only wage-inflation and government liability targets.

Note: All the measures of conditional welfare costs are expressed in percentage points.

$$\begin{aligned}
 (56) \quad \mathbb{E}_t \sum_{m=0}^{\infty} (\beta\phi_w)^m & \left[\gamma \left(\frac{\prod_{p=1}^m (\pi_{t+p-1}) \tilde{W}_{jt}}{W_{t+m}} \right)^{-\vartheta} \right. \\
 & + \varsigma_{t+m} (1 - \tau_{t+m}) \frac{1 - \vartheta}{\vartheta} \frac{\prod_{p=1}^m (\pi_{t+p-1}) \tilde{W}_{jt}}{P_{t+m}} \\
 & \left. \left(\frac{\prod_{p=1}^m (\pi_{t+p-1}) \tilde{W}_{jt}}{W_{t+m}} \right)^{-\vartheta} \right] \ell_{t+m}^d = 0
 \end{aligned}$$

The intuition behind these two first order conditions is the same as in the case of indexation to steady-state inflation. The only difference lies in the fact that the firms' expected marginal costs and revenues are now discounted by the change in the price level.

Table 4 reports the optimized policy coefficients. There are two results of interest. The first one is that the maximized welfare levels are higher than those under indexation to steady

state inflation for all the rules. The second result is that the optimal responsiveness of the tax rule to government liabilities is also larger than those of table 2. The reason lies in the fact that a model where prices are indexed to past inflation is closer to a flexible price economy than a model with indexation to steady-state inflation (see Schmitt-Grohé and Uribe, 2005). Indexation to lagged inflation induces smaller price dispersion than indexation to long-run inflation. This is so because firms need not incorporate the expectations of future deviations of inflation from trend in their current markups when they set up prices optimally. Overall, indexation to past inflation allows firms to keep up with changes in the aggregate price level. Under indexation to long-run inflation instead, it is essential for firms to incorporate expected inflation deviations from trend in their profit-maximizing prices.

8. Conclusions

This paper studies welfare-maximizing simple rules for monetary and tax policy. I amend the standard New Keynesian model by including wage rigidity and distortionary taxes on consumption, capital and labour income. According to the nature of the optimal policy problem, I solve the model by the second order approximation method to the policy function proposed by Schmitt-Grohé and Uribe (2004). I consider several variants of the model with full steady-state indexation of both prices and wages, indexation to past inflation and wages, as well as a case with only wage rigidity.

With indexation of wages and prices to steady-state inflation, the optimal tax rate displays a small response to government liabilities. This minimizes the effects of the distortions from price-level fluctuations. At the same time, monetary policy reacts largely to wage inflation. When prices are flexible and wages are sticky, a large variability of inflation is optimal. Finally, with nominal wages and prices indexed to lagged wages and inflation, respectively, the frictionless equilibrium is almost replicated. This implies that the optimal policy mix is close to the one without nominal rigidities.

Several extensions can be envisaged. Current work in progress considers the relation between different instruments for distortionary taxation – namely taxes on consumption, capital and labor income – and optimal monetary policy. In particular, the issue of interest is whether the type of indexation scheme for wages and prices affects the interaction between policy instruments. Schmitt-Grohé and Uribe (2005) suggest that the optimal degree of monetary policy inertia is contingent upon the type of indexation. Since tax shocks have been proposed as a compelling ingredient for matching models to U.S. data (e.g. see Klein and Jonsson, 2005), a consistent characterization of the fiscal structure should be an important ingredient in understanding monetary policy. Finally, as noted by one referee, the model is based on a cashless economy. Adding money would be an interesting avenue of research for it would introduce a reason for policymakers to smooth the policy rate.

References

- Anderson, G., A. T. Levin and E. Swanson (2004). Higher-Order Perturbation Solutions to Dynamic, Discrete-Time Rational Expectations Model. Unpublished manuscript, Federal Reserve Board.
- Artis, M., and B. Winkler (1997). The Stability Pact: Safeguarding the Credibility of the European Central Bank. CEPR Discussion Paper No. 1688.
- Bayoumi, T., D. Laxton and P. Pesenti (2004). Benefits and Spillovers of Greater Competition in Europe. NBER Working Paper No. 10416.
- Blanchard, O. J., and J. Gali (2007). “Real Wage Rigidities and the New Keynesian Model.” Forthcoming in *Journal of Money, Credit and Banking*.
- Bohn, H. (1995). “The Sustainability of Budget Deficits in a Stochastic Economy.” *Journal of Money, Credit and Banking* 27, 257–271.
- Calvo, G. (1983). “Staggered Prices in a Utility Maximizing Framework.” *Journal of Monetary Economics* 12(3), 383–398.
- Christiano, L., M. Eichenbaum and C. Evans (2005). “Nominal Rigidities and the Dynamic Effect of a Shock to Monetary Policy.” *Journal of Political Economy* 113, 1–45.
- Cosimano, T. F., and M. Gapen (2005). “Solving Ramsey Problems with Nonlinear Projection Methods.” *Studies in Nonlinear Dynamics and Econometrics* 9.
- Dixit, A., and J. Stiglitz (1977). “Monopolistic Competition and Optimal Product Diversity.” *American Economic Review* 67, 297–308.
- Erceg, C. J., D. W. Henderson and A. T. Levin (2000). “Optimal Monetary Policy with Staggered Wage and Price Contracts.” *Journal of Monetary Economics* 46(2), 281–313.
- Gali, J., and M. Gertler (1999). “Inflation Dynamics: A Structural Econometric Analysis.” *Journal of Monetary Economics* 44(1), 195–222.
- Gali, J., and T. Monacelli (2005). Optimal Monetary and Fiscal Policy in a Currency Union. NBER Working Paper No. 11815.
- Goffe, W., M. Ferriers and P. Rogers (1994). “Global Optimization of Statistical Functions with Simulated Annealing.” *Journal of Econometrics* 60(1), 65–100.
- Hall, R. (2005). “Employment Fluctuations with Equilibrium Wage Stickiness.” *American Economic Review* 95(1), 50–64.
- Kim, J. (2003). “Functional Equivalence Between Intertemporal and Multisectoral Investment Adjustment Costs.” *Journal of Economic Dynamics and Control* 27(4), 533–549.
- Kim, J., and S. H. Kim (2003a). “Spurious Welfare Reversals in International Business Cycle Models.” *Journal of International Economics* 60(2), 471–500.
- Kim, J., and S. H. Kim (2003b). Welfare Effects of Tax Policy in Open Economies: Stabilization and Cooperation. Unpublished manuscript, Tufts University.
- Kim, J., S. H. Kim, E. Schaumburg and C. Sims (2003). Calculating and Using Second Order Accurate Solutions of Discrete Time Dynamic Equilibrium Models. Unpublished manuscript, Princeton University.
- King, R., and S. T. Rebelo (1999). “Resuscitating Real Business Cycles.” In *Handbook of Macroeconomics* Vol. 1, 927–1007. Eds. J. B. Taylor and M. Woodford. Amsterdam: Elsevier.
- Kirkpatrick, S., C. D. Gelatt Jr and M. P. Vecchi (1983). “Optimization by Simulated Annealing.” *Science* 220,

- 671–680.
- Klein, P., and M. Jonsson (2005).** “Tax Distortions in Sweden and in the United States.” *European Economic Review* 47(4), 711–729.
- Kollmann, R. (2002).** “Monetary Policy Rules in the Open Economy: Effects on Welfare and Business Cycles.” *Journal of Monetary Economics* 27, 989–1015.
- Kollmann, R. (2004).** “Welfare Effects of a Monetary Union: the Role of Trade Openness.” *Journal of the European Economic Association* 2, 289–301.
- Kollmann, R. (2007).** “Welfare Maximizing Fiscal and Monetary Policy Rules.” Forthcoming in *Macroeconomic Dynamics*.
- Leeper, E. (2001).** “Equilibria under ‘Active’ and ‘Passive’ Monetary and Fiscal Policies.” *Journal of Monetary Economics* 27, 129–147.
- Marzo, M. (2006).** “Optimal Monetary Policy with Price and Wage Rigidities.” *Economic Notes* 35(1), 63–95.
- Marzo, M., I. Strid, and P. Zagaglia (2006).** Optimal Opportunistic Monetary Policy in a New Keynesian Model. Unpublished manuscript, Stockholm University.
- McCallum, B. T. (1999).** “Issues in the Design of Monetary Policy Rules.” In *Handbook of Macroeconomics* Vol. 2, 741–1228. Eds. J. B. Taylor and M. Woodford. Amsterdam: Elsevier.
- Mendoza, E., A. Razin, and L. Tesar (1994).** “Effective Tax Rates in Macroeconomics: Cross Country Estimates of Tax Rates on Factor Income and Consumption.” *Journal of Monetary Economics* 34, 297–323.
- Pappa, E. (2004).** “Do the ECB and the Fed Really Need to Cooperate? Optimal Monetary Policy in a Two-Country World.” *Journal of Monetary Economics* 51, 753–779.
- Railavo, J. (2004).** Monetary Consequences of Alternative Fiscal Policy Rules. Bank of Finland Discussion Paper No. 20.
- Reinhart, C., and M. A. Savastano (2003).** “The Realities of Modern Hyperinflation.” *Finance and Development* 1688, 20–23.
- Roberts, J. M. (1995).** “New Keynesian Economics and the Phillips Curve.” *Journal of Money, Credit and Banking* 27(4), 975–984.
- Rotemberg, J. (1982).** “Monopolistic Price Adjustment and Aggregate Output.” *Review of Economic Studies* 49, 517–531.
- Sbordone, A. (2002).** “Prices and Unit Labor Costs: A New Test of Price Stickiness.” *Journal of Monetary Economics* 49, 265–292.
- Schmitt-Grohé, S., and M. Uribe (2004).** “Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function.” *Journal of Economic Dynamics and Control* 28(3), 755–775.
- Schmitt-Grohé, S., and M. Uribe (2005).** Optimal Operational Monetary Policy in the Christiano-Eichenbaum-Evans Model of the U.S. Business Cycle. NBER Working Paper No. 11417.
- Schmitt-Grohé, S., and M. Uribe (2006).** “Optimal Simple and Implementable Monetary and Fiscal Rules.” Forthcoming in *Journal of Monetary Economics*.
- Taylor, J. B. (1993).** “Discretion versus Policy Rules in Practice.” *Carnegie-Rochester Conference Series* 39.
- Taylor, J. B. (2000).** “The Policy Rule Mix: A Macroeconomic Policy Evaluation.” In *Money, Capital Mobility and Trade: Essays in Honor of Robert Mundell*, 505–518. Eds. G. Calvo, R. Dornbusch and M. Obstfeld Boston: MIT Press.
- Woodford, M. (2001).** “Fiscal Requirements for Price Stability.” *Journal of Money, Credit and Banking* 33, 669–728.
- Woodford, M. (2002).** “Inflation Stabilization and Welfare.” *Contributions to Macroeconomics* 2(1).
- Woodford, M. (2003).** *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.
- Yun, T. (1996).** “Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles.” *Journal of Monetary Economics* 37, 345–370.
- Zagaglia, P. (2007).** Distortionary Tax Instruments and Implementable Monetary Policy. Unpublished manuscript, Stockholm University.