

Supplement to

TWO-STAGE QUANTITY-SETTING GAMES AND TACIT COLLUSION

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Case 2: $\partial p_i / \partial x_j < 0$ and $\partial R_i / \partial x_j > 0$.

Case 2 is the case of strategic complements in which goods are substitutes. In Case 2, firm i 's reaction curves are upward-sloping because of strategic complements. Firm i 's profit increases (respectively decreases) with the fall (respectively rise) of firm j 's output on firm i 's reaction curve.

Sub-case 2.1: Only one firm can offer LECP.

Sub-case 2.1 is illustrated in Figure 2.1. Lemma 2 shows that firm i 's profit-maximizing output is larger when it offers LECP than when it does not. Therefore, if only firm 1 adopts LECP, then the equilibrium occurs at the appropriate point on NL . In Case 2, the Stackelberg point S is to the left of N on R_2^N . The further the point on R_2^N gets from the Stackelberg point S , the more firm 1's profit decreases. Firm 1's profit is the highest at N on NL . Therefore, firm 1 has no incentive to offer LECP. Since the game is symmetric, firm 2's incentives are the same. Thus, the equilibrium occurs at N and each firm obtains the Cournot profit.

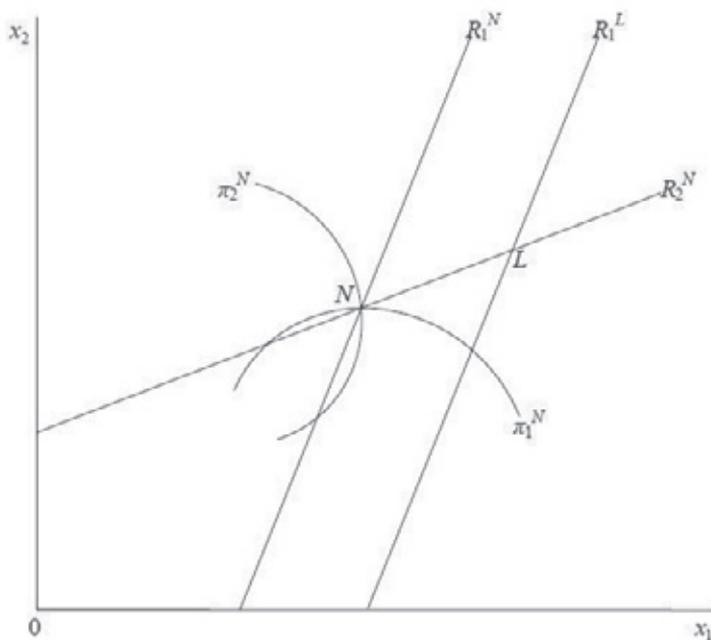


Figure 2.1. Sub-case 2.1

Sub-case 2.2: Only one firm can offer WRCP.

Sub-case 2.2 is illustrated in Figure 2.2. Lemma 4 shows that firm i 's profit-maximizing output is smaller when it offers WRCP than when it does not. Therefore, if only firm 1 offers WRCP, then the equilibrium occurs at the appropriate point on WN . In Case 2, the Stackelberg point S where firm 1 is the leader and firm 2 is the follower is to the left of N on R_2^N . Firm 1's profit is the highest at the Stackelberg point S on R_2^N . Therefore, in the first stage, firm 1 chooses x_1^{*S} corresponding to S and offers WRCP. In the second stage, firm 1's reaction curve is kinked at $x_1 = x_1^{*S}$, and thus the equilibrium occurs at the Stackelberg point S . Since the game is symmetric, firm 2's incentives are the same. Lemma 3 shows if firm i offers WRCP, then in equilibrium $x_i = x_i^*$. Thus, each firm obtains a profit higher than the Cournot profit.

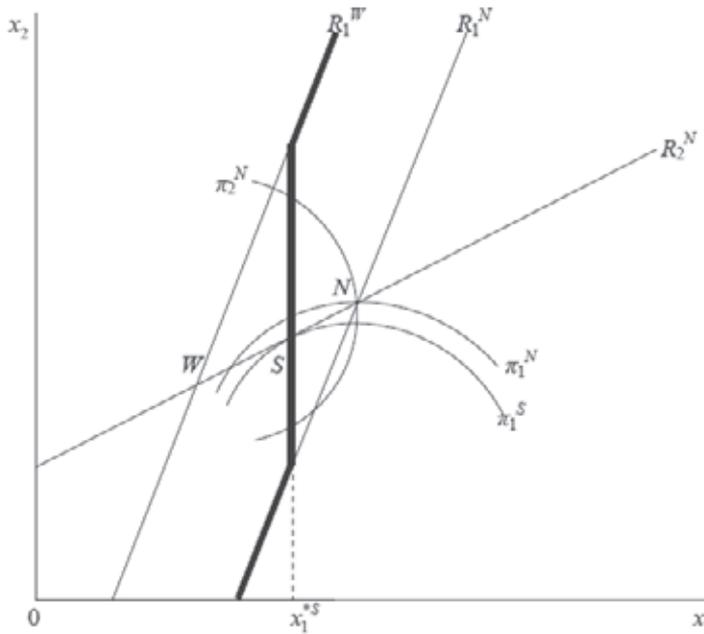


Figure 2.2. Sub-case 2.2.

Sub-case 2.3: One firm can offer LECP and another firm can offer WRCP.

Sub-case 2.3 is illustrated in Figure 2.3. We suppose that firm 1 can offer LECP and firm 2 can offer WRCP. Lemma 2 shows that firm i 's profit-maximizing output is larger when it offers LECP than when it does not. On the other hand, Lemma 4 shows that firm i 's profit-maximizing output is smaller when it offers WRCP than when it does not. Therefore, the equilibrium occurs at the appropriate point on the quadrilateral $NLVM$. Firm 1 prefers a point on NM directly left of LV to the corresponding point on LV . Therefore, firm 1 has no incentive to shift its reaction curve. If firm 1 does not shift its reaction curve, the equilibrium occurs at the appropriate point on NM . From the discussion of Sub-case 2.2 above, firm 2 chooses x_2^{*S} corresponding to the Stackelberg point and offers WRCP. Since firm 2 shifts its reaction curve, the equilibrium occurs at the appropriate point on NM . Lemma 3 shows that in equilibrium firm i does not pay its employees wage premiums. Thus, each firm obtains a profit higher than the Cournot profit.

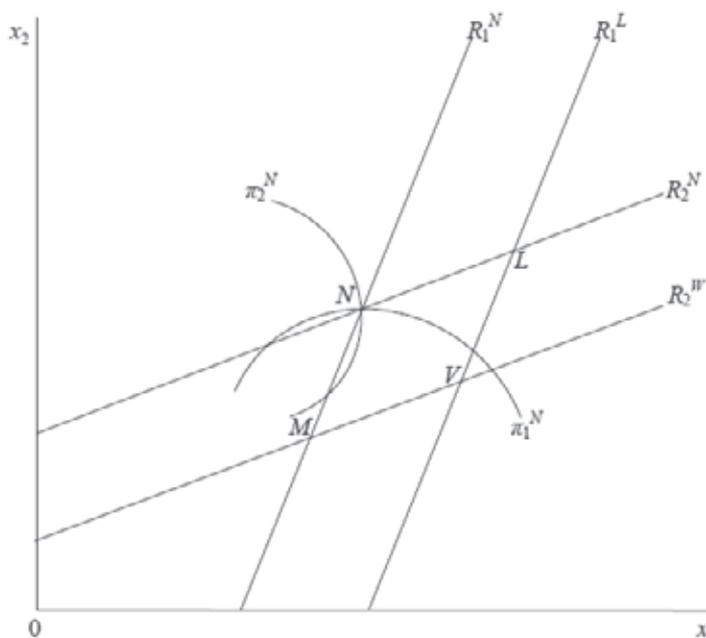


Figure 2.3. Sub-case 2.3

Sub-case 2.4: Each firm can offer LECP.

Sub-case 2.4 is illustrated in Figure 2.4. Lemma 2 shows that firm i 's profit-maximizing output is larger when it offers LECP than when it does not. Therefore, if each firm offers LECP, then the equilibrium occurs at the appropriate point on the quadrilateral $NQTL$. Each firm's profit is the highest at N on $NQTL$. Each firm has no incentive to offer LECP. Thus, the equilibrium occurs at N and each firm obtains the Cournot profit.

Sub-case 2.5: Each firm can offer WRCP.

Sub-case 2.5 is illustrated in Figure 2.5. Lemma 4 shows that firm i 's profit-maximizing output is smaller when it offers WRCP than when it does not. Therefore, if each firm offers WRCP, then the equilibrium occurs at the appropriate point on the quadrilateral $NMUW$ in Figure 2.5. Firm 1 prefers a point on UM directly below WN to the corresponding point on WN , and firm 2 prefers a point on WU directly left of NM to the corresponding point on NM . Therefore, each firm offers WRCP and shifts its reaction curve. Lemma 3 shows that in equilibrium firm i does not pay its employees wage premiums. Thus, each firm obtains a profit higher than the Cournot profit.

Sub-case 2.6: Neither firm offers the policies.

The equilibrium occurs at N in Figures 2.1-2.5, and each firm obtains the Cournot profit.

From the preceding results, we can consider the following matrix:

			Firm 2	
			LECP	WRCP
				Neither policy
Firm1	LECP	2, 2	3, 1	2, 2
	WRCP	3, 3	3, 3	3, 3
	Neither policy	2, 2	3, 3	2, 2
		Profits to: (Firm 1, Firm 2)		

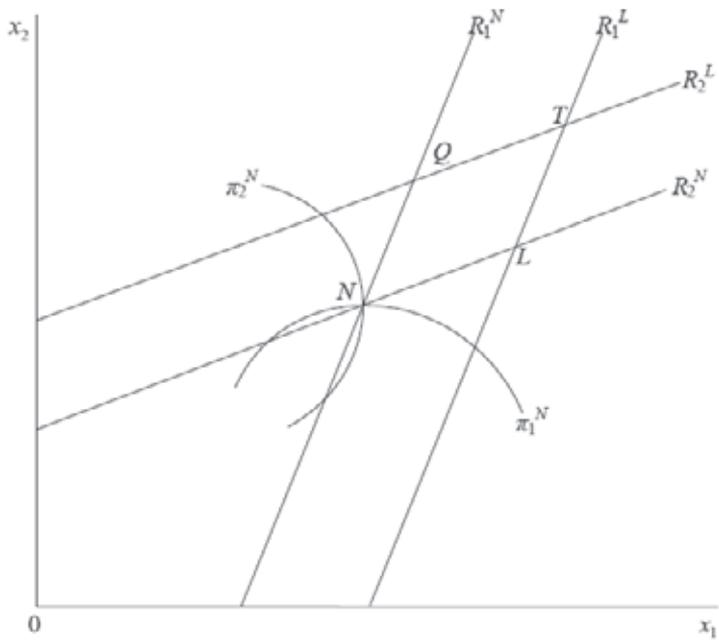


Figure 2.4. Sub-case 2.4

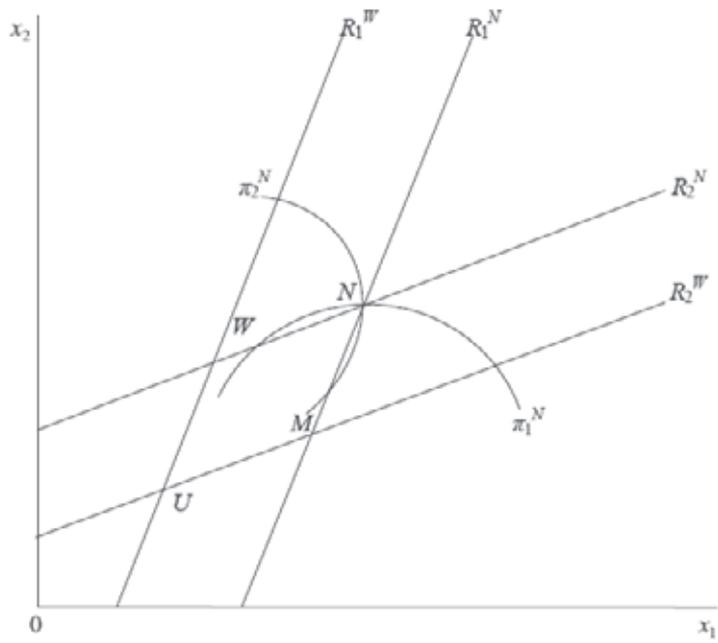


Figure 2.5. Sub-case 2.5

Here, the equilibrium is WRCP for each firm. Each firm's profit higher than the Cournot profit is shown as 3 uniformly. Thus, each firm obtains a profit higher than the Cournot profit. We can now state the following proposition.

Proposition 2. *In Cases 2 (substitute goods and strategic complements), there exists an equilibrium in which each firm offers WRCP. At equilibrium, each firm obtains a higher profit than in the Cournot game with no policies.*

Case 3: $\partial p_i / \partial x_j > 0$ and $\partial R_i / \partial x_j > 0$.

Case 3 is the case of strategic complements in which goods are complements. In Case 3, firm i 's reaction curves are upward-sloping because of strategic complements. Firm i 's profit increases (respectively decreases) with the rise (respectively fall) of firm j 's output on firm i 's reaction curve.

Sub-case 3.1: Only one firm can offer LECP.

This sub-case is illustrated in Figure 3.1. Lemma 2 shows that firm i 's profit-maximizing output is larger when it offers LECP than when it does not. Therefore, if only firm 1 offers LECP, then the equilibrium occurs at the appropriate point on NL . In Case 3, the Stackelberg point S where firm 1 is the leader and firm 2 is the follower is to the right of N on R_2^N . If the Stackelberg point S is on NL , then firm 1's profit is the highest at S on NL . Therefore, in the first stage, firm 1 chooses x_1^{*S} corresponding to S and offers LECP. In the second stage, firm 1's reaction curve is kinked at $x_1 = x_1^{*S}$. Thus, the equilibrium occurs at S .

On the other hand, if the Stackelberg point S is to the right of L on R_2^N , then firm 1's profit is the highest at L on R_2^N . Because the further the point on R_2^N gets from the Stackelberg point S , the more firm 1's profit decreases. Therefore, in the first stage, firm 1 chooses x_1^{*L} corresponding to L and offers LECP. In the second stage, firm 1's reaction curve is kinked at $x_1 = x_1^{*L}$. Thus, the equilibrium occurs at L .

Since the game is symmetric, firm 2's incentives are the same. Lemma 1 shows that in equilibrium firm i does not employ extra employees. Thus, each firm obtains a profit higher than the Cournot profit.

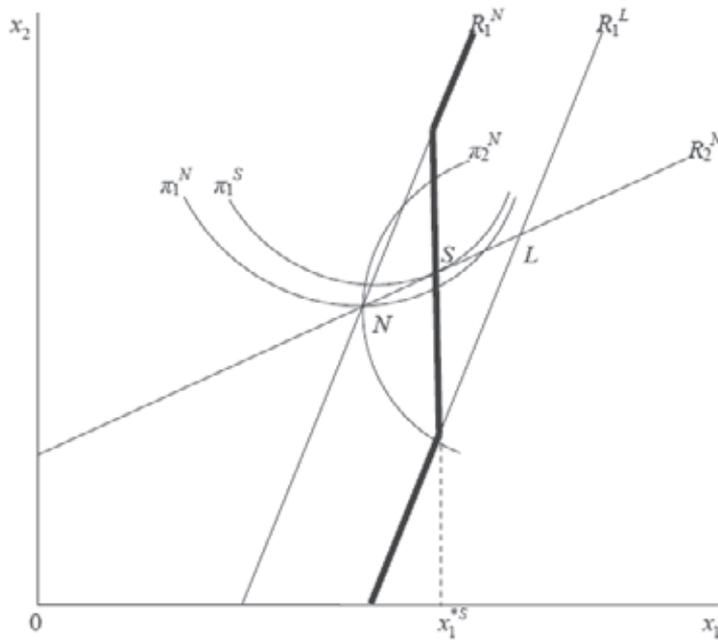


Figure 3.1. Sub-case 3.1

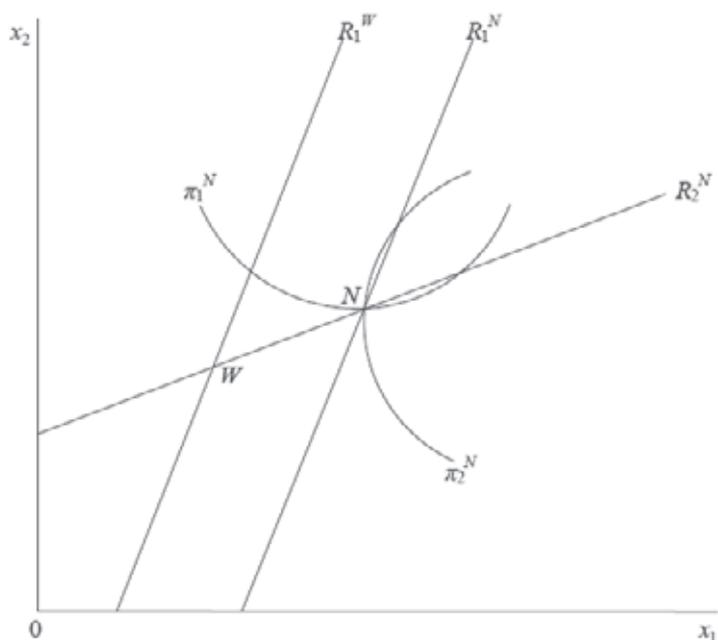


Figure 3.2. Sub-case 3.2

Sub-case 3.2: Only one firm can offer WRCP.

Lemma 4 shows that firm i 's profit-maximizing output is smaller when it offers WRCP than when it does not. Hence, if only firm 1 adopts WRCP, then the equilibrium occurs at the appropriate point on WN in Figure 3.2. In Case 3, the Stackelberg point S is to the right of N on R_2^N . The further the point on R_2^N gets from the Stackelberg point S , the more firm 1's profit decreases. Firm 1's profit is the highest at N on WN . Therefore, firm 1 has no incentive to offer WRCP. Since the game is symmetric, firm 2's incentives are the same. Thus, the equilibrium occurs at N and each firm obtains the Cournot profit.

Sub-case 3.3: One firm can offer LECP and another firm can offer WRCP.

This sub-case is illustrated in Figure 3.3. We suppose that firm 1 offers LECP and firm 2 offers WRCP. Lemma 2 shows that if firm i offers LECP, then its optimal output increases. On the other hand, Lemma 4 means that if firm i offers WRCP, then its optimal output decreases. Therefore, the equilibrium occurs at the appropriate point on the quadrilateral $NLVM$. Firm 2's profit is the highest at L on $NLVM$. Firm 2 prefers a point on NL directly above MV to the corresponding point on MV . Therefore, firm 2 does not shift its reaction curve. If firm 2 does not shift its reaction curve, the equilibrium occurs at the appropriate point on NL . From the discussion of Sub-case 3.1 above, firm 1 chooses x_1^* and offers LECP. Lemma 1 shows that if firm i offers LECP, then in equilibrium $x_i = x_i^*$. Since the game is symmetric, firm 2's incentives are the same. Thus, each firm obtains a profit higher than the Cournot profit.

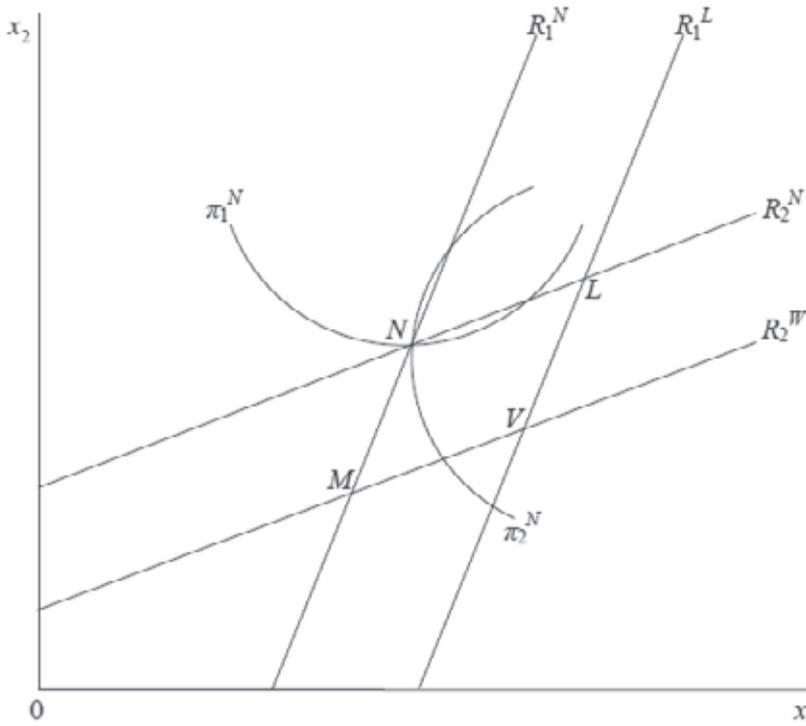


Figure 3.3. Sub-case 3.3

Sub-case 3.4: Each firm can offer LECP.

Lemma 2 shows that firm i 's profit-maximizing output is higher when it offers LECP than when it does not. Therefore, if each firm offers LECP, then the equilibrium occurs at the appropriate point on the quadrilateral $NQTL$ in Figure 3.4. Firm 1 prefers a point on QT directly above NL to the corresponding point on NL , and firm 2 prefers a point on LT directly right of NQ to the corresponding point on NQ . Therefore, each firm offers LECP and shifts its reaction curve. Lemma 1 shows that if firm i offers LECP, then in equilibrium $x_i = x_i^*$. Thus, each firm obtains a profit higher than the Cournot profit.

Sub-case 3.5: Each firm can offer WRCP.

This sub-case is illustrated in Figure 3.5. Lemma 4 shows that firm i 's profit-maximizing output is smaller when it offers WRCP than when it does not. Hence, if each firm offers WRCP, then the equilibrium occurs at the appropriate point on the quadrilateral $NMUW$. Each firm's profit is the highest at N on $NMUW$. In Sub-case 3.5, if each firm shifts its reaction curve, then its own profit decreases. Each firm has no incentive to shift its reaction curve. Thus, the equilibrium occurs at N and each firm obtains the Cournot profit.

Sub-case 3.6: Neither firm offers the policies.

The equilibrium occurs at N in Figures 3.1-3.5, and each firm obtains the Cournot profit. From the preceding results, we can consider the following matrix:

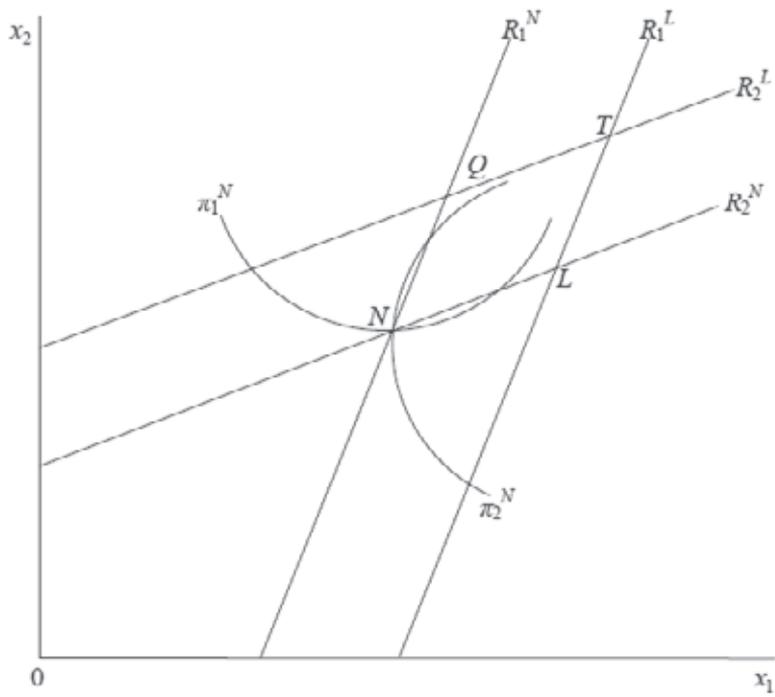


Figure 3.4. Sub-case 3.4

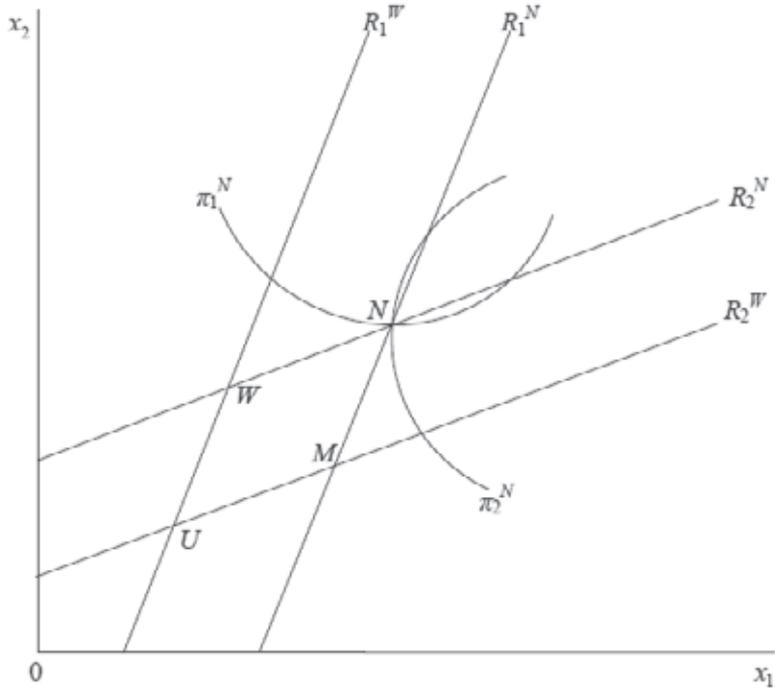


Figure 3.5. Sub-case 3.5

		Firm 2		
		LECP	WRCP	Neither policy
Firm 1	LECP	3, 3	3, 3	3, 1
	WRCP	3, 3	2, 2	2, 2
	Neither policy	1, 3	2, 2	2, 2

Profits to: (Firm 1, Firm 2)

Here, the equilibrium is LECP for each firm. Thus, each firm obtains a profit higher than the Cournot profit. The following proposition states that tacit collusion is facilitated in Case 3.

Proposition 3. *In Case 3 (complementary goods and strategic complements), there exists an equilibrium in which each firm offers LECP. At equilibrium, each firm obtains a higher profit than in the Cournot game with no policies.*

Case 4: $\partial p_i / \partial x_j > 0$ and $\partial R_i / \partial x_j < 0$.

Case 4 is the case of strategic substitutes in which goods are complements. In Case 4, firm i 's reaction curves are downward-sloping because of strategic substitutes. Firm i 's profit increases (respectively decreases) with the rise (respectively fall) of firm j 's output on firm i 's reaction curve.

Sub-case 4.1: Only one firm can offer LECP.

This sub-case is illustrated in Figure 4.1. Lemma 2 shows that firm i 's profit-maximizing output is higher when it offers LECP than when it does not. Hence, if only firm 1 offers LECP, then the equilibrium occurs at the appropriate point on NL . In Case 4, the Stackelberg point S where firm 1 is the leader and firm 2 is the follower is to the left of N on R_2^N . The further the point on R_2^N gets from the Stackelberg point S , the more firm 1's profit decreases. Firm 1's profit is the highest at N on NL . Therefore, firm 1 has no incentive to offer LECP. Since the game is symmetric, firm 2's incentives are the same. Thus, the equilibrium occurs at N and each firm obtains the Cournot profit.

Sub-case 4.2: Only one firm can offer WRCP.

Lemma 4 shows that firm i 's profit-maximizing output is smaller when it offers WRCP than when it does not. Therefore, if only firm 1 offers WRCP, then the equilibrium occurs at the appropriate point on WN in Figure 4.2. In Case 4, the Stackelberg point S where firm 1 is the leader and firm 2 is the follower is to the left of N on R_2^N . Firm 1's profit is the highest at the Stackelberg point S on R_2^N . Therefore, in the first stage, firm 1 chooses output x_1^{*S} corresponding to S and offers WRCP. In the second stage, firm 1's reaction curve is kinked at $x_1 = x_1^{*S}$. Thus, the equilibrium occurs at the Stackelberg point S . Since the game is symmetric, firm 2's incentives are the same. Lemma 1 indicates that in equilibrium firm i does not employ extra employees. Thus, one firm which offers WRCP obtains a profit higher than the Cournot profit and another firm obtains a profit lower than the Cournot profit.

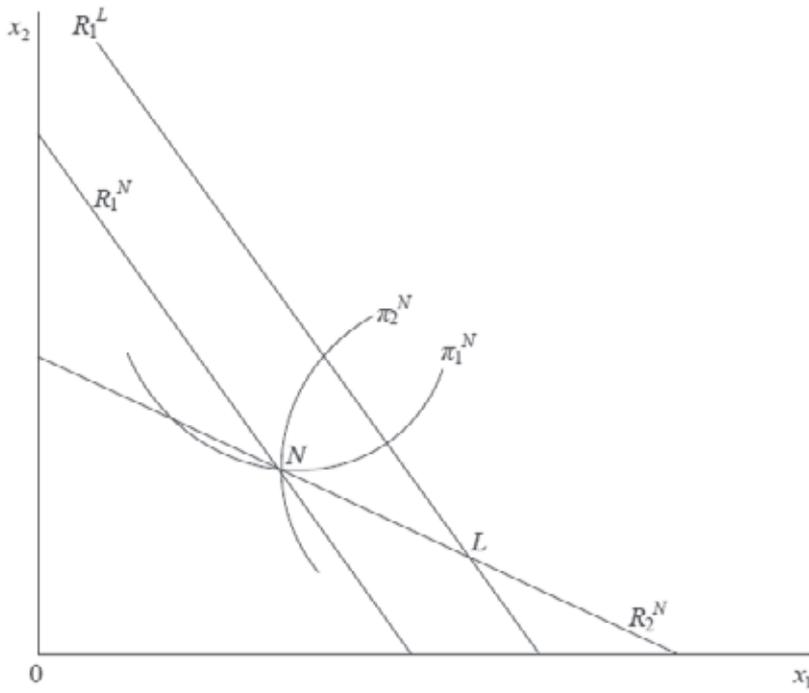


Figure 4.1. Sub-case 4.1

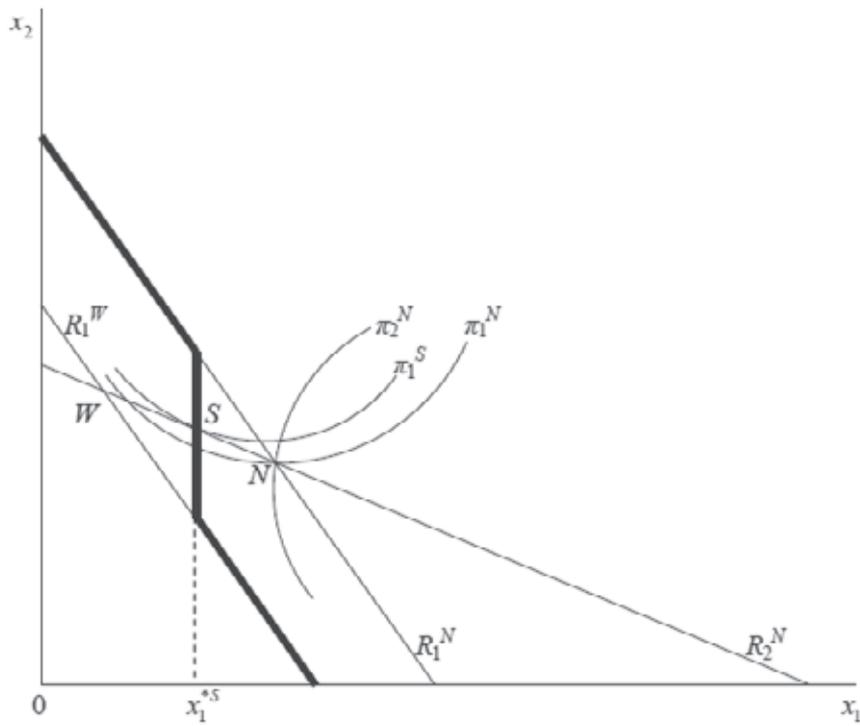


Figure 4.2. Sub-case 4.2

Sub-case 4.3: One firm can offer LECP and another firm can offer WRCP.

This sub-case is illustrated in Figure 4.3. We suppose that firm 1 offers LECP and firm 2 offers WRCP. Lemma 2 shows that if firm i offers LECP, then its profit-maximizing output increases. On the other hand, Lemma 4 means that if firm i offers WRCP, then its profit-maximizing output decreases. Therefore, the equilibrium occurs at the appropriate point on the pentagon $NLVYM$. Firm 1's profit is the highest at N on $NLVYM$. Therefore, firm 1 has no incentive to shift its reaction curve. If firm 1 does not shift its reaction curve, the equilibrium occurs at the appropriate point on NM . From the discussion of Sub-case 4.2 above, firm 2 chooses x_2^* and offers WRCP. Since firm 2 shifts its reaction curve, the equilibrium occurs at the appropriate point on NM . Lemma 3 shows that in equilibrium firm i does not pay its employees wage premiums. Thus, one firm which offers WRCP obtains a profit higher than the Cournot profit and another firm which offers LECP obtains a profit lower than the Cournot profit.

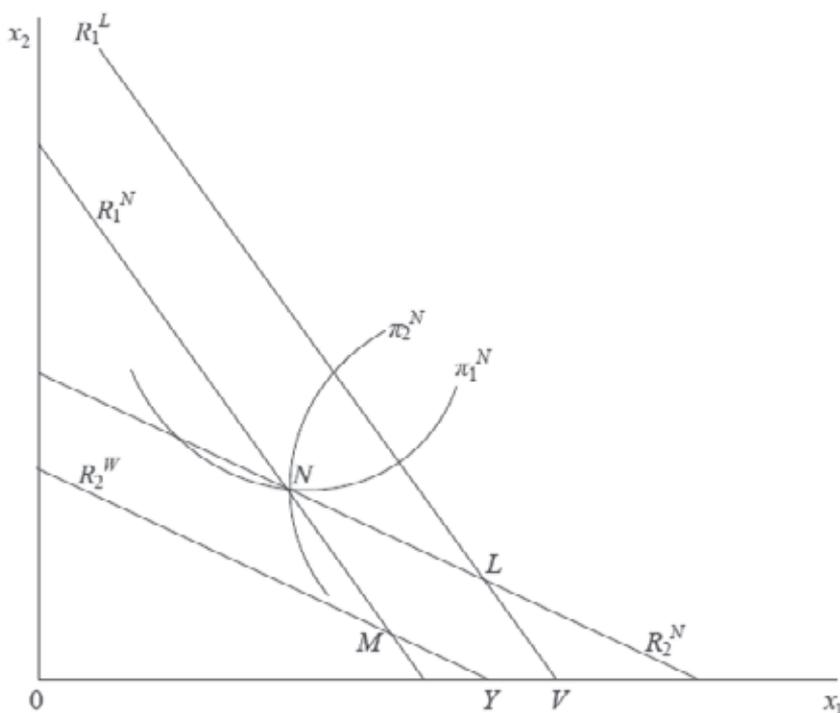


Figure 4.3. Sub-case 4.3

Sub-case 4.4: Each firm can offer LECP.

This sub-case is illustrated in Figure 4.4. Lemma 2 shows that firm i 's profit-maximizing output is higher when it offers LECP than when it does not. Therefore, if each firm offers LECP, then the equilibrium occurs at the appropriate point on the quadrilateral $NQTL$. Firm 1's profit is the highest at Q on $NQTL$, while firm 2's profit is the highest at L on $NQTL$. If each firm offers LECP and shifts its reaction curve, then its own profit increases. However, if only one firm offers LECP and shifts its reaction curve, then its own profit decreases and its rival firm's profit increases. Thus, the equilibrium occurs at N and each firm obtains the Cournot profit.

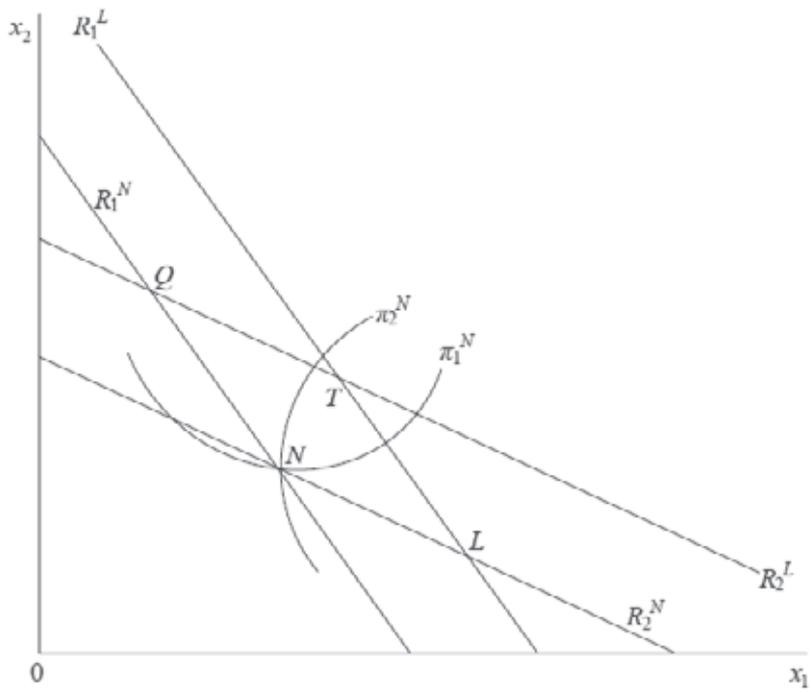


Figure 4.4. Sub-case 4.4

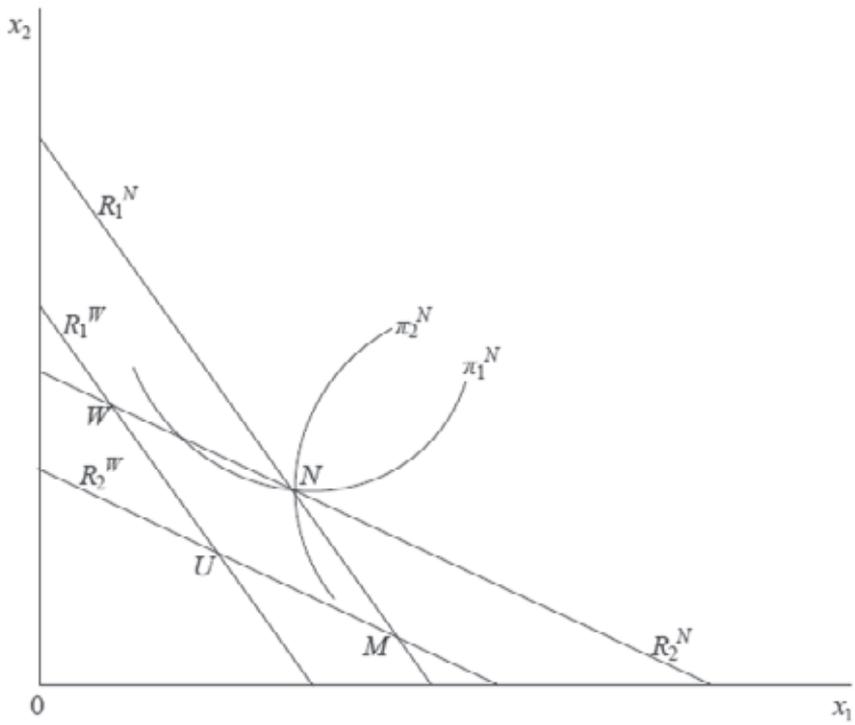


Figure 4.5. Sub-case 4.5

Sub-case 4.5: Each firm can offer WRCP.

Lemma 4 shows that firm i 's profit-maximizing output is smaller when it offers WRCP than when it does not. Therefore, if each firm offers WRCP, then the equilibrium occurs at the appropriate point on the quadrilateral $NMUW$ in Figure 4.5. Here, firm 1 prefers a point on WN directly above UM to the corresponding point on UM , while firm 2 prefers a point on NM directly right of WU to the corresponding point on WU . Each firm chooses x_i^{*N} corresponding to N and offers WRCP. Thus, the equilibrium occurs at N and each firm obtains the Cournot profit.

Sub-case 4.6: Neither firm offers the policies.

The equilibrium occurs at N in Figures 4.1-4.5, and each firm obtains the Cournot profit.

We can now consider the following matrix:

		Firm 2		
		LECP	WRCP	Neither policy
Firm 1	LECP	2, 2	1, 3	2, 2
	WRCP	3, 1	2, 2	3, 1
	Neither policy	2, 2	1, 3	2, 2
		Profits to: (Firm 1, Firm 2)		

Here, the equilibrium occurs at WRCP for each firm. Thus, each firm obtains the Cournot profit. The following proposition states that tacit collusion is not facilitated in Case 4.

Proposition 4. *In Case 4 (complementary goods and strategic substitutes), there exists an equilibrium which coincides with the Cournot solution with no policies.*