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Mathematical Appendix to THE ACHILLES HEEL OF THE DUAL INCOME TAX: THE NORWEGIAN CASE

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The equations with numbering (1), (2), (3), ... refer to equations in the paper, while the equations with numbering (A1), (A2), (A3), ... refer to equations found in this appendix.

0.1 Properties of the utility function:

By assumption we have that $v'(C_2) > 0$, $v''(C_2) < 0$, $v'''(C_2) > 0$, and $C_2'(\widetilde{\gamma}) < 0$. It then follows that

$$cov\left[v'(C_2), \widetilde{\gamma}\right] > 0,$$

and

$$cov\left[v''(C_2), \widetilde{\gamma}\right] < 0.$$

0.2 Developing equation (2) in the paper:

$$\begin{aligned} \Theta &= \frac{E\left[v'\left(C_2\right)\cdot\widetilde{\gamma}\right]}{E\left[v'\left(C_2\right)\right]} \\ &= \frac{E\left[v'\left(C_2\right)\right]\cdot E\left[\widetilde{\gamma}\right] + cov\left[v'\left(C_2\right),\widetilde{\gamma}\right]}{E\left[v'\left(C_2\right)\right]} \\ &\Downarrow \ \text{use that } E\left[\widetilde{\gamma}\right] = \delta, \text{ and let } \lambda \equiv \frac{cov\left[v'\left(C_2\right),\widetilde{\gamma}\right]}{E\left[v'\left(C_2\right)\right]} \\ &= \delta + \lambda. \end{aligned}$$

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1 The Sole Proprietor.

1.1 Developing the optimal investment condition (8) in the paper.

The sole proprietor's optimization problem is given by

$$\max_{K_s, B_s} EU_s = u(C_{1,s}) + E\left[v\left(C_{2,s}\right)\right]$$

where $C_{1,s}$ and $C_{2,s}$ are given by equations (3) and (6):

$$C_{1,s} = Y - K_s - B_s.$$

$$\begin{aligned} C_{2,s} &= [1 - t_w] \cdot [F(K_s) - \widetilde{\gamma} \cdot K_s] \\ &\quad + \{1 + (t_w - t_k) \cdot (r + \mu)\} \cdot K_s + [1 + (1 - t_k) \cdot r] \cdot B_s. \end{aligned}$$

The resulting first order conditions are given by

$$FOC_{B_s} : -u'(C_{1,s}) + E\left[v'(C_{2,s}) \cdot \{1 + (1 - t_k) \cdot r\}\right] = 0 \tag{A1}$$

$$FOC_{K_s} : -u'(C_{1,s}) + E\left[v'(C_{2,s}) \cdot \left\{ \begin{array}{c} [1 - t_w] \cdot [F_{K_s} - \widetilde{\gamma}] \\ + [t_w - t_k] \cdot [r + \mu] + 1 \end{array} \right\}\right] = 0. \tag{A2}$$

We know from (A1) that $u'(C_{1,s}) = \{1 + (1 - t_k) \cdot r\} \cdot E\left[v'(C_{2,s})\right]$. Apply this to equation (A2) and rearrange:

$$\begin{aligned} \{1 + (1 - t_k) \cdot r\} \cdot E\left[v'(C_{2,s})\right] &= E\left[v'(C_{2,s}) \cdot \left\{ \begin{array}{c} [1 - t_w] \cdot [F_{K_s} - \widetilde{\gamma}] \\ + [t_w - t_k] \cdot [r + \mu] + 1 \end{array} \right\}\right] \\ &\Downarrow \cdot \frac{1}{E\left[v'(C_{2,s})\right]} \\ 1 + (1 - t_k) \cdot r &= [1 - t_w] \cdot F_{K_s} - \frac{[1 - t_w] \cdot E\left[v'(C_{2,s}) \cdot \widetilde{\gamma}\right]}{E\left[v'(C_{2,s})\right]} \\ &\quad + [t_w - t_k] \cdot [r + \mu] + 1 \\ &\Downarrow \cdot \frac{1}{1 - t_w} \\ F_{K_s} &= r + \frac{E\left[v'(C_{2,s}) \cdot \widetilde{\gamma}\right]}{E\left[v'(C_{2,s})\right]} - \frac{t_w - t_k}{1 - t_w} \cdot \mu \qquad \text{(A3)} \\ &\Downarrow \text{Apply equation (2)} \\ F_{K_s} &= r + \delta + \lambda_s - \frac{t_w - t_k}{1 - t_w} \cdot \mu \qquad \text{(8)} \end{aligned}$$

1.2 Proof of proposition 1, the effect on investments by increased risk compensation rate under the split model.

From (3) and (6) we find that

$$\frac{\partial C_{1,s}}{\partial \mu} = -\frac{\partial K_s}{\partial \mu} - \frac{\partial B_s}{\partial \mu} \tag{A3b}$$

and

$$\frac{\partial C_{2,s}}{\partial \mu} = \{A - (1-t_w) \cdot \widetilde{\gamma}\} \cdot \frac{\partial K_s}{\partial \mu} + \{1 + (1-t_k) \cdot r\} \cdot \frac{\partial B_s}{\partial \mu} + (t_w - t_k) \cdot K, \qquad \text{(A3c)}$$

where A is positive and defined in equation (A4).

Now, differentiate (A1) with respect to the risk compensation rate μ:

$$u''(C_{1,s}) \cdot \left\{ \frac{\partial K_s}{\partial \mu} + \frac{\partial B_s}{\partial \mu} \right\} + \{1 + (1-t_k) \cdot r\} \cdot E\left[v''(C_{2,s}) \cdot \frac{\partial C_{2,s}}{\partial \mu} \right] = 0$$

$$\Downarrow \text{ Apply (A3c) and rearrange}$$

$$\begin{aligned} & u''(C_{1,s}) \cdot \left\{ \frac{\partial K_s}{\partial \mu} + \frac{\partial B_s}{\partial \mu} \right\} \\ & + \{1 + (1-t_k) \cdot r\} \cdot \frac{\partial K_s}{\partial \mu} \cdot \{A \cdot E[v''(C_{2,s})] - [1-t_w] \cdot E[v''(C_{2,s}) \cdot \widetilde{\gamma}]\} \\ & + \{1 + (1-t_k) \cdot r\}^2 \cdot \frac{\partial B_s}{\partial \mu} \cdot E[v''(C_{2,s})] \\ & + \{1 + (1-t_k) \cdot r\} \cdot (t_w - t_k) \cdot K \cdot E[v''(C_{2,s})] \\ = \; & 0 \end{aligned}$$

$$\Downarrow$$

$$\begin{aligned} & \frac{\partial K_s}{\partial \mu} \cdot \langle u''(C_{1,s}) + \{1 + (1-t_k) \cdot r\} \cdot \{A \cdot E[v''(C_{2,s})] - [1-t_w] \cdot E[v''(C_{2,s}) \cdot \widetilde{\gamma}]\}\rangle \qquad \text{(A3d)} \\ & + \frac{\partial B_s}{\partial \mu} \cdot \left\langle u''(C_{1,s}) + \{1 + (1-t_k) \cdot r\}^2 \cdot E[v''(C_{2,s})] \right\rangle \\ = \; & -\{1 + (1-t_k) \cdot r\} \cdot (t_w - t_k) \cdot K \cdot E[v''(C_{2,s})] \end{aligned}$$

Next, differentiate (A2) with respect to the risk compensation rate μ and simplify by applying the definition (2):

$$u''(C_{1,s}) \cdot \left\{ \frac{\partial K_s}{\partial \mu} + \frac{\partial B_s}{\partial \mu} \right\} + E\left[\begin{array}{l} v''(C_{2,s}) \cdot \frac{\partial C_{2,s}}{\partial \mu} \cdot \{A - (1-t_w) \cdot \widetilde{\gamma}\} \\ + v'(C_{2,s}) \cdot \left\{ (1-t_w) \cdot F_{K_s K_s} \cdot \frac{\partial K_s}{\partial \mu} + (t_w - t_k) \right\} \end{array} \right] = 0$$

$$\Downarrow \text{ Apply (A3c) and rearrange}$$

$$\begin{aligned} & u''(C_{1,s}) \cdot \left\{ \frac{\partial K_s}{\partial \mu} + \frac{\partial B_s}{\partial \mu} \right\} \\ & + \frac{\partial K_s}{\partial \mu} \cdot E\left[v''(C_{2,s}) \cdot \{A - (1-t_w) \cdot \widetilde{\gamma}\}^2 \right] \\ & + \frac{\partial B_s}{\partial \mu} \cdot \{1 + (1-t_k) \cdot r\} \cdot E[v''(C_{2,s}) \cdot \{A - (1-t_w) \cdot \widetilde{\gamma}\}] \\ & + (t_w - t_k) \cdot K \cdot E[v''(C_{2,s}) \cdot \{A - (1-t_w) \cdot \widetilde{\gamma}\}] \\ & + \left\{ (1-t_w) \cdot F_{K_s K_s} \cdot \frac{\partial K_s}{\partial \mu} + (t_w - t_k) \right\} \cdot E[v'(C_{2,s})] \end{aligned}$$

$$\Downarrow$$

$$\begin{aligned} & \frac{\partial K_s}{\partial \mu} \cdot \left\langle \begin{array}{l} u''(C_{1,s}) + E\left[v''(C_{2,s}) \cdot \{A - (1-t_w) \cdot \widetilde{\gamma}\}^2\right] \\ + (1-t_w) \cdot F_{K_s K_s} \cdot E\left[v'(C_{2,s})\right] \end{array} \right\rangle \\ & + \frac{\partial B_s}{\partial \mu} \cdot \left\langle u''(C_{1,s}) + \{1 + (1-t_k) \cdot r\} \cdot E\left[v''(C_{2,s}) \cdot \{A - (1-t_w) \cdot \widetilde{\gamma}\}\right] \right\rangle \\ = & -(t_w - t_k) \cdot \{K \cdot E\left[v''(C_{2,s}) \cdot \{A - (1-t_w) \cdot \widetilde{\gamma}\}\right] + E\left[v'(C_{2,s})\right]\} \end{aligned} \tag{A3e}$$

By Cramers's rule, equations (A3d) and (A3e) yield:

$$\frac{\partial K_s}{\partial \mu} = \frac{1}{D} \cdot \left\{ \begin{array}{l} - \left\langle \begin{array}{l} \left\{u''(C_{1,s}) + \{1 + (1-t_k) \cdot r\}^2 \cdot E\left[v''(C_{2,s})\right]\right\} \\ \cdot (t_w - t_k) \cdot \{K \cdot E\left[v''(C_{2,s}) \cdot \{A - (1-t_w) \cdot \widetilde{\gamma}\}\right] + E\left[v'(C_{2,s})\right]\} \end{array} \right\rangle \\ + \left\langle \begin{array}{l} \{u''(C_{1,s}) + \{1 + (1-t_k) \cdot r\} \cdot E\left[v''(C_{2,s}) \cdot \{A - (1-t_w) \cdot \widetilde{\gamma}\}\right]\} \\ \cdot \{1 + (1-t_k) \cdot r\} \cdot (t_w - t_k) \cdot K \cdot E\left[v''(C_{2,s})\right] \end{array} \right\rangle \end{array} \right\}$$

where D is defined in equation (A6) and $D > 0$. By rearranging the above expression, we get the following expression for $\frac{\partial K_s}{\partial \mu}$:

$$\frac{\partial K_s}{\partial \mu} = \frac{1}{D} \cdot \left\{ \begin{array}{l} \{(1-t_k) \cdot r \cdot (t_w - t_k) \cdot K - E\left[v'(C_{2,s})\right]\} \cdot u''(C_{1,s}) \\ - \{1 + (1-t_k) \cdot r\}^2 \cdot E\left[v'(C_{2,s})\right] \cdot E\left[v''(C_{2,s})\right] \end{array} \right\} \tag{A3f}$$

As $D > 0$, we see that $\frac{\partial K_s}{\partial \mu} > 0$ if $\left\{ \begin{array}{l} \{(1-t_k) \cdot r \cdot (t_w - t_k) \cdot K - E\left[v'(C_{2,s})\right]\} \cdot u''(C_{1,s}) \\ - \{1 + (1-t_k) \cdot r\}^2 \cdot E\left[v'(C_{2,s})\right] \cdot E\left[v''(C_{2,s})\right] \end{array} \right\} > 0$.
As $E\left[v''(C_{2,s})\right] < 0$ and $E\left[v'(C_{2,s})\right] > 0$, then $-\{1 + (1-t_k) \cdot r\}^2 \cdot E\left[v'(C_{2,s})\right] \cdot E\left[v''(C_{2,s})\right] > 0$. Then the sign of $\frac{\partial K_s}{\partial \mu}$ depends on the sign of the first part in the paranthesis. $u''(C_{1,s}) < 0$. We then see that
1) If $(1-t_k) \cdot r \cdot (t_w - t_k) \cdot K < E\left[v'(C_{2,s})\right]$ then $\frac{\partial K_s}{\partial \mu}$ is definitively positive. That means if the expected marginal utility of second period consumption is greater than the after tax return to the tax savings under the split model from investing in real capital.
2) If $(1-t_k) \cdot r \cdot (t_w - t_k) \cdot K > E\left[v'(C_{2,s})\right]$ then $\frac{\partial K_s}{\partial \mu}$ is positive only if $\{(1-t_k) \cdot r \cdot (t_w - t_k) \cdot K - E\left[v'(C_{2,s})\right]\} \cdot u''(C_{1,s}) > \{1 + (1-t_k) \cdot r\}^2 \cdot E\left[v'(C_{2,s})\right] \cdot E\left[v''(C_{2,s})\right]$.

1.3 Conditions for the existence of a local maximum for the sole proprietor:

$$\begin{aligned} 1) &: \quad EU_{BB} < 0 \\ 2) &: \quad EU_{KK} < 0 \\ 3) &: \quad EU_{BB} \cdot EU_{KK} - (EU_{BK})^2 > 0 \end{aligned}$$

From equation (A1) it follows that

$$EU_{BB} = u''(C_{1,s}) + \{1 + (1-t_k) \cdot r\}^2 \cdot E\left[v''(C_{2,s})\right] < 0.$$

From equation (A2) it follows that

$$\begin{aligned} EU_{KK} &= u''(C_{1,s}) + [1-t_w] \cdot F_{K_sK_s} \cdot E[v'(C_{2,s})] + A^2 \cdot E[v''(C_{2,s})] \\ &\quad -2 \cdot A \cdot [1-t_w] \cdot E[v''(C_{2,s}) \cdot \widetilde{\gamma}] + [1-t_w]^2 \cdot E[v''(C_{2,s}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}] \\ &< 0 \end{aligned}$$

where

$$A \equiv [1-t_w] \cdot F_{K_s} + 1 + [t_w - t_k] \cdot [r + \mu] \tag{A4}$$

$$\Downarrow \text{ use condition (A3)}$$

$$A = [1 + [1-t_k] \cdot r] + [1-t_w] \cdot \frac{E[v'(C_{2,s}) \cdot \widetilde{\gamma}]}{E[v'(C_{2,s})]} \tag{A5}$$

Also, from equation (A1) it follows that

$$EU_{BK} = u''(C_{1,s}) + [1 + (1-t_k) \cdot r] \cdot \{A \cdot E[v''(C_{2,s})] - [1-t_w] \cdot E[v''(C_{2,s}) \cdot \widetilde{\gamma}]\}$$

Define

$$\begin{aligned} D &\equiv EU_{BB} \cdot EU_{KK} - (EU_{BK})^2 > 0 \\ &\Downarrow \end{aligned}$$

$$\begin{aligned} D &= \left\{ u''(C_{1,s}) + \{1 + (1-t_k) \cdot r\}^2 \cdot E[v''(C_{2,s})] \right\} \\ &\quad \cdot \left\{ \begin{array}{c} u''(C_{1,s}) + [1-t_w] \cdot F_{K_sK_s} \cdot E[v'(C_{2,s})] + A^2 \cdot E[v''(C_{2,s})] \\ -2 \cdot A \cdot [1-t_w] \cdot E[v''(C_{2,s}) \cdot \widetilde{\gamma}] + [1-t_w]^2 \cdot E[v''(C_{2,s}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}] \end{array} \right\} \\ &\quad - \left\{ u''(C_{1,s}) + [1 + (1-t_k) \cdot r] \cdot \left\{ \begin{array}{c} A \cdot E[v''(C_{2,s})] \\ -[1-t_w] \cdot E[v''(C_{2,s}) \cdot \widetilde{\gamma}] \end{array} \right\} \right\}^2 \\ &> 0 \end{aligned} \tag{A6}$$

1.4 The income effect of the sole proprietor.

From the first order condition (A2) it follows that

$$\begin{aligned} &\Downarrow \\ &\frac{\partial K_s}{\partial Y} \cdot \left\{ \begin{array}{c} u''(C_{1,s}) + [1-t_w] \cdot F_{K_sK_s} \cdot E[v'(C_{2,s})] + A^2 \cdot E[v''(C_{2,s})] \\ -2 \cdot A \cdot [1-t_w] \cdot E[v''(C_{2,s}) \cdot \widetilde{\gamma}] + [1-t_w]^2 \cdot E[v''(C_{2,s}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}] \end{array} \right\} \\ &+ \frac{\partial B_s}{\partial Y} \cdot \left\{ u''(C_{1,s}) + [1 + (1-t_k) \cdot r] \cdot \left\{ \begin{array}{c} A \cdot E[v''(C_{2,s})] \\ -[1-t_w] \cdot E[v''(C_{2,s}) \cdot \widetilde{\gamma}] \end{array} \right\} \right\} \\ &= u''(C_{1,s}) \end{aligned} \tag{A7}$$

And from the first order condition (A1) it follows that

$$\begin{aligned} & u''(C_{1,s}) \cdot \left[\frac{\partial K_s}{\partial Y} + \frac{\partial B_s}{\partial Y} - 1 \right] \\ & + [1 + (1 - t_k) \cdot r] \cdot E \left[v''(C_{2,s}) \cdot \left\{ \begin{array}{c} [1 - t_w] \cdot (F_{K_s} - \widetilde{\gamma}) \cdot \frac{\partial K_s}{\partial Y} \\ + (1 + [t_w - t_k] \cdot [r + \mu]) \cdot \frac{\partial K_s}{\partial Y} \\ + [1 + (1 - t_k) \cdot r] \cdot \frac{\partial B_s}{\partial Y} \end{array} \right\} \right] \\ = \; & 0 \end{aligned}$$

$$\Downarrow \tag{A8}$$

$$\begin{aligned} & \frac{\partial K_s}{\partial Y} \cdot \left\{ u''(C_{1,s}) + [1 + (1 - t_k) \cdot r] \cdot \left\{ \begin{array}{c} A \cdot E[v''(C_{2,s})] \\ - [1 - t_w] \cdot E[v''(C_{2,s}) \cdot \widetilde{\gamma}] \end{array} \right\} \right\} \\ & + \frac{\partial B_s}{\partial Y} \cdot \left\{ u''(C_{1,s}) + [1 + (1 - t_k) \cdot r]^2 \cdot E[v''(C_{2,s})] \right\} \\ = \; & u''(C_{1,s}) \end{aligned}$$

By Cramer's rule, equations (A7) and (A8) yield:

$$\frac{\partial K_s}{\partial Y} = -\frac{u''(C_{1,s})}{D} \cdot \left\{ \begin{array}{c} u''(C_{1,s}) + [1 + (1 - t_k) \cdot r] \\ \cdot \left\{ \begin{array}{c} A \cdot E[v''(C_{2,s})] \\ - [1 - t_w] \cdot E[v''(C_{2,s}) \cdot \widetilde{\gamma}] \end{array} \right\} \\ - \left\{ u''(C_{1,s}) + [1 + (1 - t_k) \cdot r]^2 \cdot E[v''(C_{2,s})] \right\} \end{array} \right\}$$

$\Downarrow$ apply (A5)

$$\frac{\partial K_s}{\partial Y} = -\frac{u''(C_{1,s})}{D} \cdot \left\{ \begin{array}{c} u''(C_{1,s}) + [1 + (1 - t_k) \cdot r] \\ \cdot \left\{ \begin{array}{c} [1 + (1 - t_k) \cdot r] \cdot E[v''(C_{2,s})] \\ + [1 - t_w] \cdot \frac{E[v'(C_{2,s}) \cdot \widetilde{\gamma}]}{E[v'(C_{2,s})]} \cdot E[v''(C_{2,s})] \\ - [1 - t_w] \cdot E[v''(C_{2,s}) \cdot \widetilde{\gamma}] \end{array} \right\} \\ -u''(C_{1,s}) - [1 + (1 - t_k) \cdot r]^2 \cdot E[v''(C_{2,s})] \end{array} \right\}$$

$$\Downarrow \tag{A9}$$

$$\frac{\partial K_s}{\partial Y} = \frac{u''(C_{1,s}) \cdot [1 + (1 - t_k) \cdot r] \cdot [1 - t_w]}{D \cdot E[v'(C_{2,s})]} \cdot \left\{ \begin{array}{c} E[v''(C_{2,s}) \cdot \widetilde{\gamma}] \cdot E[v'(C_{2,s})] \\ \\ -E[v'(C_{2,s}) \cdot \widetilde{\gamma}] \cdot E[v''(C_{2,s})] \end{array} \right\}$$

As $\frac{u''(C_{1,s}) \cdot [1 + (1 - t_k) \cdot r] \cdot [1 - t_w]}{D \cdot E[v'(C_{2,s})]} < 0$, the sign of the income effect is determined by the expressions in the parenthesis. Thus

$$\frac{\partial K_s}{\partial Y} > 0 \text{ if } E[v''(C_{2,s}) \cdot \widetilde{\gamma}] \cdot E[v'(C_{2,s})] - E[v'(C_{2,s}) \cdot \widetilde{\gamma}] \cdot E[v''(C_{2,s})] < 0$$

We know that

$$\begin{aligned} & E[v''(C_{2,s}) \cdot \widetilde{\gamma}] \cdot E[v'(C_{2,s})] \\ = \; & \{cov[v''(C_{2,s}), \widetilde{\gamma}] + E[v''(C_{2,s})] \cdot E[\widetilde{\gamma}]\} \cdot E[v'(C_{2,s})] \end{aligned}$$

and

$$\begin{aligned} & E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\right] \\ = & \left\{cov\left[v'(C_{2,s}),\widetilde{\gamma}\right]+E\left[v'(C_{2,s})\right]\cdot E\left[\widetilde{\gamma}\right]\right\}\cdot E\left[v''(C_{2,s})\right] \end{aligned}$$

This means that

$$\frac{\partial K_s}{\partial Y}>0 \text{ if } cov\left[v''(C_{2,s}),\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right]-cov\left[v'(C_{2,s}),\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\right]<0.$$

We already know that

$$E\left[v'(C_{2,s})\right]>0,\ cov\left[v'(C_{2,s}),\widetilde{\gamma}\right]>0,\ E\left[v''(C_{2,s})\right]<0, \text{ and } cov\left[v''(C_{2,s}),\widetilde{\gamma}\right]<0.$$

Thus

$$\frac{\partial K_s}{\partial Y}>0 \text{ if } cov\left[v''(C_{2,s}),\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right]<cov\left[v'(C_{2,s}),\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\right],$$

which are both negative. This means that in order for the above condition to be met, the absolute value of the left hand side must be larger than the absolute value of the right hand side.

1.5 The effect on the investment portfolio and risk profile of the sole proprietor by changed tax on labor income.

Differentiate equation (A1) with respect to t_w to find that:

$$\begin{aligned} & -u''(C_{1,s})\cdot\left\{-K'(t_w)-B'(t_w)\right\} \\ & +\left\{1+(1-t_k)\cdot r\right\}\cdot E\left[v''(C_{2,s})\cdot\frac{\partial C_{2,s}}{\partial t_w}\right] \\ = & \ 0 \end{aligned}$$

From equation (6) we find that

$$\begin{aligned} \frac{\partial C_{2,s}}{\partial t_w} = & -F(K_s)+(r+\mu+\widetilde{\gamma})\cdot K_s+\left\{1+(1-t_k)\cdot r\right\}\cdot B_s'(t_w) \\ & +\left\{(1-t_w)\cdot(F'(K_s)-\widetilde{\gamma})+1+(t_w-t_k)\cdot(r+\mu)\right\}\cdot K_s'(t_w). \end{aligned} \tag{A10}$$

Then

$$\begin{aligned} E\left[v''(C_{2,s})\cdot\frac{\partial C_{2,s}}{\partial t_w}\right] = & \ E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot\left\{K_s-(1-t_w)\cdot K_s'(t_w)\right\} \\ & +E\left[v''(C_{2,s})\right]\cdot\left\{1+(1-t_k)\cdot r\right\}\cdot B_s'(t_w) \\ & +E\left[v''(C_{2,s})\right]\cdot\left\{A\cdot K_s'(t_w)+(r+\mu)\cdot K_s-F(K_s)\right\} \end{aligned}$$

By using this in the above equation and rearranging, we find that

$$\Downarrow \tag{A11}$$

$$\begin{aligned} & K'(t_w)\cdot\left\{u''(C_{1,s})+\left[1+(1-t_k)\cdot r\right]\cdot\left[\begin{array}{c} A\cdot E\left[v''(C_{2,s})\right] \\ -\left[1-t_w\right]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \end{array}\right]\right\} \\ & +B'(t_w)\cdot\left\{u''(C_{1,s})+\left[1+(1-t_k)\cdot r\right]^2\cdot E\left[v''(C_{2,s})\right]\right\} \\ = & \left[1+(1-t_k)\cdot r\right]\cdot\left\{\begin{array}{c} \left[F(K_s)-(r+\mu)\cdot K_s\right]\cdot E\left[v''(C_{2,s})\right] \\ -K_s\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \end{array}\right\} \end{aligned}$$

For simplicity, denote the different parts of the above equation as

$$K'(t_w) \cdot a_{BK} + B'(t_w) \cdot a_{BB} = b_B$$

Next, condition (A2) is differentiated:

$$\begin{aligned} & u''(C_{1,s}) \cdot \{K_s'(t_w) + B_s'(t_w)\} \\ & + \{-F_{K_s} + [1-t_w] \cdot F_{K_sK_s} \cdot K_s'(t_w) + r + \mu\} \cdot E\left[v'(C_{2,s})\right] \\ & + A \cdot E\left[v''(C_{2,s}) \cdot \frac{\partial C_{2,s}}{\partial t_w}\right] \\ & + E\left[v'(C_{2,s}) \cdot \widetilde{\gamma}\right] \\ & - [1-t_w] \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma} \cdot \frac{\partial C_{2,s}}{\partial t_w}\right] \\ = \; & 0 \end{aligned}$$

Use the expression in equation (A10) and rearrange the above equation to find the following:

$$\begin{aligned} & K_s'(t_w) \cdot \left\{ \begin{array}{c} u''(C_{1,s}) + [1-t_w] \cdot F_{K_sK_s} \cdot E\left[v'(C_{2,s})\right] \\ +A^2 \cdot E\left[v''(C_{2,s})\right] - 2 \cdot [1-t_w] \cdot A \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] \\ +[1-t_w]^2 \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \end{array} \right\} \\ & +B_s'(t_w) \cdot \left\{ u''(C_{1,s}) + [1+(1-t_k) \cdot r] \cdot \left\{ \begin{array}{c} A \cdot E\left[v''(C_{2,s})\right] \\ -[1-t_w] \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] \end{array} \right\} \right\} \\ = & \left\{ \begin{array}{c} A \cdot [F(K_s) - (r+\mu) \cdot K_s] \cdot E\left[v''(C_{2,s})\right] \\ +[F_{K_s} - (r+\mu)] \cdot E\left[v'(C_{2,s})\right] \\ -A \cdot K_s \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] - E\left[v'(C_{2,s}) \cdot \widetilde{\gamma}\right] \\ -[1-t_w] \cdot [F(K_s) - (r+\mu) \cdot K_s] \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] \\ +[1-t_w] \cdot K_s \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \end{array} \right\} \end{aligned} \tag{A12}$$

Denote the parts of the above equation

$$K_s'(t_w) \cdot a_{KK} + B_s'(t_w) \cdot a_{KB} = b_K$$

By Cramer's rule equations (A11) and (A12) yield:

$$K_s'(t_w) = \frac{1}{-D} \cdot (b_B \cdot a_{KB} - b_k \cdot a_{BB}) \tag{A13}$$

where

$$\begin{aligned} & b_B \cdot a_{KB} \\ = & \left\{ u''(C_{1,s}) + \left[\begin{array}{c} [1+(1-t_k) \cdot r] \cdot \left\{ A - [1-t_w] \cdot \frac{E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right]}{E\left[v''(C_{2,s})\right]} \right\} \\ \cdot E\left[v''(C_{2,s})\right] \end{array} \right] \right\} \\ & \cdot [1+(1-t_k) \cdot r] \cdot \left\{ \begin{array}{c} [F(K_s) - (r+\mu) \cdot K_s] \\ -K_s \cdot \frac{E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right]}{E\left[v''(C_{2,s})\right]} \end{array} \right\} \cdot E\left[v''(C_{2,s})\right] \end{aligned}$$

Apply the definition for A:

$$b_B \cdot a_{KB} = \left\{ \begin{array}{c} u''(C_{1,s}) + [1 + [1 - t_k] \cdot r] \cdot E\left[v''(C_{2,s})\right] \\ + \frac{[1+(1-t_k)\cdot r]\cdot[1-t_w]}{E[v'(C_{2,s})]} \cdot \left\{ \begin{array}{c} E\left[v'(C_{2,s}) \cdot \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,s})\right] \\ -E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] \cdot E\left[v'(C_{2,s})\right] \end{array} \right\} \end{array} \right\}$$
$$\cdot [1 + (1 - t_k) \cdot r] \cdot \left\{ \begin{array}{c} [F(K_s) - (r + \mu) \cdot K_s] \\ -K_s \cdot \frac{E[v''(C_{2,s})\cdot\widetilde{\gamma}]}{E[v''(C_{2,s})]} \end{array} \right\} \cdot E\left[v''(C_{2,s})\right]$$

We know from the income effect (A9) that

$$-\frac{\partial K_s}{\partial Y} \cdot \frac{D}{u''(C_{1,s})} = \frac{[1 + (1 - t_k) \cdot r] \cdot [1 - t_w]}{E\left[v'(C_{2,s})\right]} \cdot \left\{ \begin{array}{c} -E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] \cdot E\left[v'(C_{2,s})\right] \\ +E\left[v'(C_{2,s}) \cdot \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,s})\right] \end{array} \right\}$$

$$\Downarrow \tag{A14}$$

$$\begin{aligned} b_B \cdot a_{KB} &= \left\{ \frac{u''(C_{1,s})}{E\left[v''(C_{2,s})\right]} + [1 + (1 - t_k) \cdot r]^2 \right\} \cdot [1 + (1 - t_k) \cdot r] \\ &\quad \cdot \left[F(K_s) - \left(r + \mu + \frac{E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right]}{E\left[v''(C_{2,s})\right]} \right) \cdot K_s \right] \cdot E\left[v''(C_{2,s})\right]^2 \\ &\quad - \frac{D \cdot E\left[v''(C_{2,s})\right]}{u''(C_{1,s})} \cdot \frac{\partial K_s}{\partial Y} \cdot [1 + (1 - t_k) \cdot r] \\ &\quad \cdot \left[F(K_s) - \left(r + \mu + \frac{E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right]}{E\left[v''(C_{2,s})\right]} \right) \cdot K_s \right] \end{aligned}$$

Next,

$$b_k \cdot a_{BB} = \left\{ \begin{array}{c} A \cdot [F(K_s) - (r + \mu) \cdot K_s] \cdot E\left[v''(C_{2,s})\right] \\ + [F_{K_s} - (r + \mu)] \cdot E\left[v'(C_{2,s})\right] \\ -A \cdot K_s \cdot \frac{E[v''(C_{2,s})\cdot\widetilde{\gamma}]}{E[v''(C_{2,s})]} \cdot E\left[v''(C_{2,s})\right] \\ -\frac{E[v'(C_{2,s})\cdot\widetilde{\gamma}]}{E[v'(C_{2,s})]} \cdot E\left[v'(C_{2,s})\right] \\ - [1 - t_w] \cdot [F(K_s) - (r + \mu) \cdot K_s] \\ \cdot \frac{E[v''(C_{2,s})\cdot\widetilde{\gamma}]}{E[v''(C_{2,s})]} \cdot E\left[v''(C_{2,s})\right] \\ \\ + [1 - t_w] \cdot K_s \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \end{array} \right\}$$
$$\cdot \left\{ u''(C_{1,s}) + [1 + (1 - t_k) \cdot r]^2 \cdot E\left[v''(C_{2,s})\right] \right\}$$

From (A3) we know that

$$F_{K_s} - r - \mu = \frac{E\left[v'(C_{2,s}) \cdot \widetilde{\gamma}\right]}{E\left[v'(C_{2,s})\right]} - \frac{1 - t_k}{1 - t_w} \mu$$

$$\Downarrow \quad \text{Also use definition for } A:$$

$$b_k \cdot a_{BB} = \left\{ \begin{array}{c} [1+[1-t_k]\cdot r]\cdot[F(K_s)-(r+\mu)\cdot K_s]\cdot E\left[v''(C_{2,s})\right] \\ +[1-t_w]\cdot \frac{E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]}{E[v'(C_{2,s})]}\cdot[F(K_s)-(r+\mu)\cdot K_s]\cdot E\left[v''(C_{2,s})\right] \\ +\left[\frac{E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]}{E[v'(C_{2,s})]}-\frac{1-t_k}{1-t_w}\mu\right]\cdot E\left[v'(C_{2,s})\right] \\ -[1+[1-t_k]\cdot r]\cdot K_s\cdot\frac{E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]}{E[v''(C_{2,s})]}\cdot E\left[v''(C_{2,s})\right] \\ -[1-t_w]\cdot\frac{E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]}{E[v'(C_{2,s})]}\cdot K_s\cdot\frac{E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]}{E[v''(C_{2,s})]}\cdot E\left[v''(C_{2,s})\right] \\ -\frac{E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]}{E[v'(C_{2,s})]}\cdot E\left[v'(C_{2,s})\right] \\ -[1-t_w]\cdot[F(K_s)-(r+\mu)\cdot K_s] \\ \cdot\frac{E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]}{E[v''(C_{2,s})]}\cdot E\left[v''(C_{2,s})\right] \\ +[1-t_w]\cdot K_s\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right] \end{array} \right\}$$

$$\cdot\left\{u''(C_{1,s})+[1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,s})\right]\right\}$$

$$\Downarrow$$

$$b_k \cdot a_{BB}$$

$$= \left\{ \begin{array}{l} [F(K_s)-(r+\mu)\cdot K_s]\cdot\frac{[1-t_w]}{E[v'(C_{2,s})]}\cdot\left[\begin{array}{c} E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\right] \\ -E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right]\end{array}\right] \\ [F(K_s)-(r+\mu)\cdot K_s]\cdot E\left[v''(C_{2,s})\right]\cdot[1+[1-t_k]\cdot r] \\ -\frac{1-t_k}{1-t_w}\mu\cdot E\left[v'(C_{2,s})\right] \\ -E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot K_s\cdot\left\{[1+[1-t_k]\cdot r]+[1-t_w]\cdot\frac{E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]}{E[v'(C_{2,s})]}\right\} \\ +[1-t_w]\cdot K_s\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right] \end{array} \right\}$$

$$\cdot\left\{u''(C_{1,s})+[1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,s})\right]\right\}$$

We know from the income effect (A9) that

$$-\frac{\partial K_s}{\partial Y}\cdot\frac{D}{[1+(1-t_k)\cdot r]\cdot u''(C_{1,s})}$$

$$= \frac{[1-t_w]}{E\left[v'(C_{2,s})\right]}\cdot\left\{\begin{array}{c} -E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right] \\ +E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\right]\end{array}\right\}$$

$$\Downarrow \tag{A15}$$

$$\begin{aligned}
& b_k \cdot a_{BB} \\
= \; & \left\{ u''(C_{1,s}) + \left[1 + (1-t_k)\cdot r\right]^2 \cdot E\left[v''(C_{2,s})\right] \right\} \\
& \cdot \left\{ \begin{array}{l}
-\frac{D}{u''(C_{1,s})\cdot[1+(1-t_k)\cdot r]} \cdot \frac{\partial K_s}{\partial Y} \cdot \left[F(K_s) - (\delta + r + \mu)\cdot K_s\right] \\
+ \left[1 + (1-t_k)\cdot r\right] \cdot \left[F(K_s) - \left(r + \mu + \frac{E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]}{E\left[v''(C_{2,s})\right]} \right) \cdot K_s \right] \\
\quad \cdot E\left[v''(C_{2,s})\right] \\
\\
-\frac{1-t_k}{1-t_w} \cdot \mu \cdot E\left[v'(C_{2,s})\right] \\
+ \left[1 - t_w\right] \cdot K_s \cdot \left\{ \begin{array}{c} E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right] \\ -E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \cdot \frac{E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]}{E\left[v'(C_{2,s})\right]} \end{array} \right\}
\end{array} \right\}
\end{aligned}$$

This yields

$$\begin{aligned}
K'(t_w) \;=\; & \frac{1}{-D} \cdot \left(b_B \cdot a_{KB} - b_k \cdot a_{BB}\right) \\
= \; & -\frac{u''(C_{1,s}) + \left[1 + (1-t_k)\cdot r\right]^2 \cdot E\left[v''(C_{2,s})\right]}{u''(C_{1,s}) \cdot \left[1 + (1-t_k)\cdot r\right]} \\
& \cdot \frac{\partial K_s}{\partial Y} \cdot \left[F(K_s) - (r + \mu)\cdot K_s\right] \\
& + \frac{u''(C_{1,s}) + \left[1 + (1-t_k)\cdot r\right]^2 \cdot E\left[v''(C_{2,s})\right]}{D} \\
& \cdot \left\{ \begin{array}{l}
\quad \left[1 + (1-t_k)\cdot r\right] \cdot E\left[v''(C_{2,s})\right] \\
\quad \cdot \left[F(K_s) - \left(\begin{array}{c} r + \mu \\ + \frac{E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]}{E\left[v''(C_{2,s})\right]} \end{array} \right) \cdot K_s \right] \\
-\frac{1-t_k}{1-t_w} \cdot \mu \cdot E\left[v'(C_{2,s})\right] \\
+ \left[1 - t_w\right] \cdot K_s \cdot \left\{ E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right] - \frac{E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]}{E\left[v'(C_{2,s})\right]} \cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \right\}
\end{array} \right\} \\
& - \frac{1}{D} \cdot \left\{ u''(C_{1,s}) + \left[1 + (1-t_k)\cdot r\right]^2 \cdot E\left[v''(C_{2,s})\right] \right\} \cdot E\left[v''(C_{2,s})\right] \\
& \cdot \left[1 + (1-t_k)\cdot r\right] \cdot \left[F(K_s) - \left(r + \mu + \frac{E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]}{E\left[v''(C_{2,s})\right]} \right) \cdot K_s \right] \\
& + \frac{E\left[v''(C_{2,s})\right]}{u''(C_{1,s})} \cdot \frac{\partial K_s}{\partial Y} \cdot \left[1 + (1-t_k)\cdot r\right] \\
& \cdot \left[F(K_s) - \left(r + \mu + \frac{E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]}{E\left[v''(C_{2,s})\right]} \right) \cdot K_s \right]
\end{aligned}$$

This yields condition (15), used for proposition (2):

$$K'(t_w) = -\left\{ \begin{array}{l} \frac{F(K_s)-(r+\mu)\cdot K_s}{[1+(1-t_k)\cdot r]} \\ +[1+(1-t_k)\cdot r]\cdot \frac{E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]}{u''(C_{1,s})}\cdot K_s \end{array} \right\} \cdot \frac{\partial K_s}{\partial Y}$$
$$+\frac{u''(C_{1,s})+[1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,s})\right]}{[1-t_w]\cdot D\cdot E\left[v'(C_{2,s})\right]}$$
$$\cdot\left\{ \begin{array}{l} [1-t_w]^2\cdot K_s\cdot\left\{ \begin{array}{l} E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right] \\ -E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \end{array}\right\} \\ -[1-t_k]\cdot\mu\cdot E\left[v'(C_{2,s})\right]^2 \end{array} \right\}$$

As $F(K_s)-(r+\mu)\cdot K_s > 0$ and $\frac{E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]}{u''(C_{1,s})} > 0$,then the sign of the first line (which is the total income effect) depends on the sign of $\frac{\partial K_s}{\partial Y}$, which is positive as long as $cov\left[v''(C_{2,s}),\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right] < cov\left[v'(C_{2,s}),\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\right]$, as previously discussed.

Also, $\frac{u''(C_{1,s})+[1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,s})\right]}{[1-t_w]\cdot D\cdot E\left[v'(C_{2,s})\right]} < 0$, such that the substitution effect of the tax change depends on whether the last parenthesis is positive or negative.

The substitution effect is positive if

$$\left\{ \begin{array}{l} E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right] \\ -E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \end{array}\right\} < \frac{[1-t_k]\cdot\mu\cdot E\left[v'(C_{2,s})\right]^2}{[1-t_w]^2\cdot K_s}$$

As the right hand side is positive, we know that the substitution effect at least is positive if the left hand side is negative:

$$\begin{aligned} & E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right]-E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \\ = \; & cov\left[v''(C_{2,s})\cdot\widetilde{\gamma},\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right]-cov\left[v'(C_{2,s}),\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \end{aligned}$$

To summarize:

$$\begin{array}{lcl} & \textit{The substitution effect} & : \\ \text{Positive if } cov\left[v''(C_{2,s})\cdot\widetilde{\gamma},\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right] & < & cov\left[v'(C_{2,s}),\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \end{array}$$

Thus:

$$\begin{array}{rcl} K'(t_w) & > & 0 \text{ if } cov\left[v''(C_{2,s}),\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right] < cov\left[v'(C_{2,s}),\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\right] \\ & & \text{and} \\ 1) & : & cov\left[v''(C_{2,s})\cdot\widetilde{\gamma},\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right] < cov\left[v'(C_{2,s}),\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \\ \\ 2) & : & \text{The substitution effect dominates the total income effect.} \end{array}$$

1.6 The effect on the investment portfolio and risk profile of the sole proprietor by changed tax on capital income.

Differentiating the first order condition (A1) yields

$$\begin{aligned} & -u''(C_{1,s}) \cdot \{-K'(t_k) - B'(t_k)\} - r \cdot E\left[v'(C_{2,s})\right] \\ & + \{1 + (1 - t_k) \cdot r\} \cdot E\left[v''(C_{2,s}) \cdot \frac{\partial C_{2,s}}{\partial t_k}\right] \\ = \; & 0 \end{aligned}$$

From (7) we get that

$$\frac{\partial C_{2,s}}{\partial t_k} = \{A - (1 - t_w) \cdot \widetilde{\gamma}\} \cdot K_s'(t_k) + [1 + (1 - t_k) \cdot r] \cdot B_s'(t_k) - (r + \mu) \cdot K_s - r \cdot B_s, \qquad \text{(A18)}$$

where A is defined in (A4). Applying this in the above expression yields:

$$\begin{aligned} & K_s'(t_k) \cdot \left\{ u''(C_{1,s}) + [1 + (1 - t_k) \cdot r] \cdot \begin{bmatrix} A \cdot E\left[v''(C_{2,s})\right] \\ -[1 - t_w] \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] \end{bmatrix} \right\} \\ & + B_s'(t_k) \cdot \left\{ u''(C_{1,s}) + [1 + (1 - t_k) \cdot r]^2 \cdot E\left[v''(C_{2,s})\right] \right\} \\ = \; & r \cdot E\left[v'(C_{2,s})\right] + [1 + (1 - t_k) \cdot r] \cdot \{(r + \mu) \cdot K_s + r \cdot B_s\} \cdot E\left[v''(C_{2,s})\right] \end{aligned} \qquad \text{(A19)}$$

Write this as

$$K_s'(t_k) \cdot x_{BK} + B_s'(t_k) \cdot x_{BB} = h_B$$

Next, condition (A2) is differentiated:

$$\begin{aligned} & -u''(C_{1,s}) \cdot \{-K_s'(t_k) - B_s'(t_k)\} \\ & + \frac{\partial A}{\partial t_k} \cdot E\left[v'(C_{2,s})\right] + A \cdot E\left[v''(C_{2,s}) \cdot \frac{\partial C_{2,s}}{\partial t_k}\right] \\ & - [1 - t_w] \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma} \cdot \frac{\partial C_{2,s}}{\partial t_k}\right] \\ = \; & 0 \end{aligned}$$

Use the expression for $\frac{\partial C_{2,s}}{\partial t_k}$ from (A18) and rearrange to get:

$$\begin{aligned} & \Downarrow \\ & K_s'(t_k) \cdot \left\{ \begin{array}{c} u''(C_{1,s}) + [1 - t_w] \cdot F_{K_s K_s} \cdot E\left[v'(C_{2,s})\right] \\ + A^2 \cdot E\left[v''(C_{2,s})\right] - 2 \cdot A \cdot [1 - t_w] \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] \\ + [1 - t_w]^2 \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \end{array} \right\} \\ & B_s'(t_k) \cdot \left\{ \begin{array}{c} u''(C_{1,s}) + [1 + (1 - t_k) \cdot r] \\ \cdot \left\{ \begin{array}{c} A \cdot E\left[v''(C_{2,s})\right] \\ -[1 - t_w] \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] \end{array} \right\} \end{array} \right\} \\ = \; & \left\{ \begin{array}{c} (r + \mu) \cdot E\left[v'(C_{2,s})\right] + A \cdot ([r + \mu] \cdot K_s + r \cdot B_s) \cdot E\left[v''(C_{2,s})\right] \\ -[1 - t_w] \cdot ([r + \mu] \cdot K_s + r \cdot B_s) \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] \end{array} \right\} \end{aligned} \qquad \text{(A20)}$$

Write this as

$$K_s'(t_k)\cdot x_{KK} + B_s'(t_k)\cdot x_{KB} = h_K$$

By applying Cramer's rule and using the definition of A, equations (A19) and (A20) yield:

$$K_s'(t_k) = \frac{h_B\cdot x_{KB} - h_K\cdot x_{BB}}{-D}$$

where

$$\begin{aligned} h_B\cdot x_{KB} \quad &= \left\{ \begin{array}{c} r\cdot E\left[v'(C_{2,s})\right] \\ +\left[1+(1-t_k)\cdot r\right]\cdot\left\{[r+\mu]\cdot K_s + r\cdot B_s\right\}\cdot E\left[v''(C_{2,s})\right] \end{array} \right\} \\ &\cdot\left\{ \begin{array}{c} u''(C_{1,s}) + \left[1+(1-t_k)\cdot r\right]^2\cdot E\left[v''(C_{2,s})\right] \\ +\frac{[1+(1-t_k)\cdot r]\cdot[1-t_w]}{E[v'(C_{2,s})]}\cdot\left\{ \begin{array}{c} E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\right] \\ -E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right] \end{array} \right\} \end{array} \right\} \end{aligned}$$

We know from the income effect (A9) that

$$\begin{aligned} -\frac{\partial K_s}{\partial Y}\cdot\frac{D}{u''(C_{1,s})} \quad &= \quad \frac{\left[1+(1-t_k)\cdot r\right]\cdot\left[1-t_w\right]}{E\left[v'(C_{2,s})\right]} \\ &\cdot\left\{ \begin{array}{c} -E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right] \\ +E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\right] \end{array} \right\} \end{aligned}$$

$$\Downarrow$$

$$\begin{aligned} h_B\cdot x_{KB} \quad &= \left\{ \begin{array}{c} r\cdot E\left[v'(C_{2,s})\right] \\ +\left[1+(1-t_k)\cdot r\right]\cdot\left\{[r+\mu]\cdot K_s + r\cdot B_s\right\}\cdot E\left[v''(C_{2,s})\right] \end{array} \right\} \\ &\cdot\left\{ \begin{array}{c} u''(C_{1,s}) + \left[1+(1-t_k)\cdot r\right]^2\cdot E\left[v''(C_{2,s})\right] \\ -\frac{\partial K_s}{\partial Y}\cdot\frac{D}{u''(C_{1,s})} \end{array} \right\} \end{aligned}$$

Now, we have that

$$\begin{aligned} h_K\cdot x_{BB} \quad &= \left\{ \begin{array}{c} (r+\mu)\cdot E\left[v'(C_{2,s})\right] \\ +\left\{ \begin{array}{c} \left[1+(1-t_k)\cdot r\right] \\ +\left[1-t_w\right]\cdot\frac{E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]}{E[v'(C_{2,s})]} \end{array} \right\}\cdot\left([r+\mu]\cdot K_s + r\cdot B_s\right)\cdot E\left[v''(C_{2,s})\right] \\ -\left[1-t_w\right]\cdot\left([r+\mu]\cdot K_s + r\cdot B_s\right)\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \end{array} \right\} \\ &\cdot\left\{u''(C_{1,s}) + \left[1+(1-t_k)\cdot r\right]^2\cdot E\left[v''(C_{2,s})\right]\right\} \\ &= \left\{u''(C_{1,s}) + \left[1+(1-t_k)\cdot r\right]^2\cdot E\left[v''(C_{2,s})\right]\right\} \\ &\cdot\left\{ \begin{array}{l} (r+\mu)\cdot E\left[v'(C_{2,s})\right] \\ +\left[1+(1-t_k)\cdot r\right]\cdot\left([r+\mu]\cdot K_s + r\cdot B_s\right)\cdot E\left[v''(C_{2,s})\right] \\ +\left[1-t_w\right]\cdot\frac{[r+\mu]\cdot K_s + r\cdot B_s}{E[v'(C_{2,s})]}\cdot\left\{ \begin{array}{c} E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\right] \\ -E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right] \end{array} \right\} \end{array} \right\} \end{aligned}$$

$$\Downarrow$$

$$\begin{aligned} h_K \cdot x_{BB} &= \left\{ u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,s})\right] \right\} \\ &\cdot \left\{ \begin{array}{c} (r+\mu)\cdot E\left[v'(C_{2,s})\right] \\ +[1+(1-t_k)\cdot r]\cdot([r+\mu]\cdot K_s + r\cdot B_s)\cdot E\left[v''(C_{2,s})\right] \end{array} \right\} \\ &- \left\{ u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,s})\right] \right\} \\ &\cdot \frac{\partial K_s}{\partial Y} \cdot \frac{D\cdot\{[r+\mu]\cdot K_s + r\cdot B_s\}}{[1+(1-t_k)\cdot r]\cdot u''(C_{1,s})} \end{aligned}$$

Thus

$$\begin{aligned} K_s'(t_k) &= \frac{h_B \cdot x_{KB} - h_K \cdot x_{BB}}{-D} \\ &= -\frac{r\cdot E\left[v'(C_{2,s})\right] + [1+(1-t_k)\cdot r]\cdot \left\{ \begin{array}{c} [r+\mu]\cdot K_s \\ +r\cdot B_s \end{array} \right\} \cdot E\left[v''(C_{2,s})\right]}{D} \\ &\cdot \left\{ u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,s})\right] \right\} \\ &+ \left\{ \begin{array}{c} r\cdot E\left[v'(C_{2,s})\right] \\ +[1+(1-t_k)\cdot r]\cdot\{[r+\mu]\cdot K_s + r\cdot B_s\}\cdot E\left[v''(C_{2,s})\right] \end{array} \right\} \\ &\cdot \frac{\partial K_s}{\partial Y} \cdot \frac{D}{u''(C_{1,s})} \\ &+ \frac{u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,s})\right]}{D} \\ &\cdot \left\{ \begin{array}{c} (r+\mu)\cdot E\left[v'(C_{2,s})\right] \\ +[1+(1-t_k)\cdot r]\cdot([r+\mu]\cdot K_s + r\cdot B_s)\cdot E\left[v''(C_{2,s})\right] \end{array} \right\} \\ &- \left\{ u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,s})\right] \right\} \\ &\cdot \frac{\partial K_s}{\partial Y} \cdot \frac{[r+\mu]\cdot K_s + r\cdot B_s}{[1+(1-t_k)\cdot r]\cdot u''(C_{1,s})} \end{aligned}$$

$$\Downarrow$$

$$\begin{aligned} K_s'(t_k) &= \frac{u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,s})\right]}{D} \cdot \left\{\mu \cdot E\left[v'(C_{2,s})\right]\right\} \\ &+ \frac{\partial K_s}{\partial Y} \cdot \frac{1}{[1+(1-t_k)\cdot r]\cdot u''(C_{1,s})} \\ &\cdot \left\{ \begin{array}{l} -[r+\mu]\cdot K_s \cdot u''(C_{1,s}) - r\cdot B_s \cdot u''(C_{1,s}) \\ -[r+\mu]\cdot K_s \cdot [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,s})\right] \\ -r\cdot B_s \cdot [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,s})\right] \\ +[1+(1-t_k)\cdot r]\cdot r\cdot E\left[v'(C_{2,s})\right] \\ +[1+(1-t_k)\cdot r]^2 \cdot [r+\mu]\cdot K_s \cdot E\left[v''(C_{2,s})\right] \\ +[1+(1-t_k)\cdot r]^2 \cdot r\cdot B_s \cdot E\left[v''(C_{2,s})\right] \end{array} \right\} \end{aligned}$$

$$\Downarrow \tag{A21}$$
$$\begin{aligned} K_s'(t_k) &= \frac{\partial K_s}{\partial Y} \cdot \left\{ r \cdot \frac{E\left[v'(C_{2,s})\right]}{u''(C_{1,s})} - \left[\frac{(r+\mu) \cdot K_s + r \cdot B_s}{1+(1-t_k) \cdot r} \right] \right\} \\ &\quad + \left\{ u''(C_{1,s}) + \left[1+(1-t_k) \cdot r\right]^2 \cdot E\left[v''(C_{2,s})\right] \right\} \cdot \frac{\mu \cdot E\left[v'(C_{2,s})\right]}{D} \\ &< 0 \end{aligned}$$

The first part of (A21) is the full income effect. As the expression in the parenthesis is negative, the full income effect is negative if $\frac{\partial K_s}{\partial Y} > 0$. From (A9) we know that this is true if $E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] \cdot E\left[v'(C_{2,s})\right] - E\left[v'(C_{2,s}) \cdot \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,s})\right] < 0$. The second part of (A21) is the substitution effect, which is negative. Thus,

$$\frac{\partial K_s}{\partial t_k} < 0 \text{ if } E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] \cdot E\left[v'(C_{2,s})\right] - E\left[v'(C_{2,s}) \cdot \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,s})\right] < 0. \tag{A22}$$

2 The widely held corporation

2.1 Developing equation (12):

The individual's maximization problem when organizing as a widely held organization is given by

$$\max_{K_l, B_l} EU_l = u(C_{1,l}) + E\left[v\left(C_{2,l}\right)\right],$$

where $C_{1,l}$ and $C_{2,l}$ are given by equations (10):

$$C_{1,l} = Y - \beta \cdot K_l - B_l$$

and (11):

$$C_{2,l} = \beta \cdot [1-t_k] \cdot [F(K_l) - \widetilde{\gamma} \cdot K_l] + \beta \cdot K_l + [1+(1-t_k) \cdot r] \cdot B_l.$$

The resulting first order conditions are:

$$FOC_{B_l} : -u'(C_{1,l}) + [1+(1-t_k) \cdot r] \cdot E\left[v'(C_{2,l})\right] = 0 \tag{A23}$$

$$FOC_{K_l} : -\beta \cdot u'(C_{1,l}) + E\left[v'(C_{2,l}) \cdot \{\beta \cdot [1-t_k] \cdot [F_{K_l} - \widetilde{\gamma}] + \beta\}\right] = 0 \tag{A24}$$

Combining the first order conditions yields the optimal investment condition (13):

$$\begin{aligned} F_{K_l} &= r + \frac{E\left[v'(C_2) \cdot \widetilde{\gamma}\right]}{E\left[v'(C_2)\right]} \\ &\Downarrow \\ F_{K_l} &= r + \delta + \lambda_l \end{aligned}$$

Define

$$\begin{aligned} G &\equiv [1-t_k] \cdot F_{K_l} + 1 \\ &\Downarrow \text{ by (13)} \\ G &= [1+(1-t_k) \cdot r] + \frac{E\left[v'(C_2) \cdot \widetilde{\gamma}\right]}{E\left[v'(C_2)\right]} \end{aligned} \tag{A25}$$

2.2 The conditions for the existence of a maximum of the widely held corporation.

By differentiating (A23) and (A24) with regard to K_l and B_l we get:

$$EU_{BB} = u''(C_{1,l}) + [1 + (1 - t_k) \cdot r]^2 \cdot E\left[v''(C_{2,l})\right] < 0,$$

$$EU_{KK} = \beta \cdot \left\{ \begin{array}{c} [1 - t_k] \cdot F_{K_l K_l} \cdot E\left[v'(C_{2,l})\right] + \beta \cdot u''(C_{1,l}) \\ +\beta \cdot G^2 \cdot E\left[v''(C_{2,l})\right] - 2 \cdot \beta \cdot G \cdot [1 - t_k] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \\ +\beta \cdot [1 - t_k]^2 \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \end{array} \right\} < 0$$

$$EU_{BK} = \beta \cdot u''(C_{1,l}) + \beta \cdot [1 + (1 - t_k) \cdot r] \cdot \left\{ \begin{array}{c} G \cdot E\left[v''(C_{2,l})\right] \\ - [1 - t_k] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \end{array} \right\}.$$

Thus

$$EU_{BB} \cdot EU_{KK} - (EU_{BK})^2 \equiv M > 0$$

in order for a maximum to exist.

$$\begin{aligned} M \;=\; & \left\{u''(C_{1,l}) + [1 + (1 - t_k) \cdot r]^2 \cdot E\left[v''(C_{2,l})\right]\right\} \cdot \beta \\ & \cdot \left\{ \begin{array}{c} [1 - t_k] \cdot F_{K_l K_l} \cdot E\left[v'(C_{2,l})\right] + \beta \cdot u''(C_{1,l}) \\ +\beta \cdot G^2 \cdot E\left[v''(C_{2,l})\right] - 2 \cdot \beta \cdot G \cdot [1 - t_k] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \\ +\beta \cdot [1 - t_k]^2 \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \end{array} \right\} \\ & -\beta^2 \cdot \left\{ u''(C_{1,l}) + [1 + (1 - t_k) \cdot r] \cdot \left\{ \begin{array}{c} G \cdot E\left[v''(C_{2,l})\right] \\ - [1 - t_k] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \end{array} \right\} \right\}^2 \end{aligned} \tag{A26}$$

2.3 The income effect in the widely held corporation.

Differentiate (A23) with respect to initial income Y :

$$\begin{aligned} & -u''(C_{1,l}) \cdot \{1 - \beta \cdot K_l'(Y) - B_l'(Y)\} \\ & + [1 + (1 - t_k) \cdot r] \cdot \left\{ \begin{array}{c} \beta \cdot G \cdot E\left[v''(C_{2,l})\right] \cdot K_l'(Y) \\ -\beta \cdot [1 - t_k] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \cdot K_l'(Y) \\ + [1 + (1 - t_k) \cdot r] \cdot E\left[v''(C_{2,l})\right] \cdot B_l'(Y) \end{array} \right\} \\ = \; & 0 \end{aligned}$$

$$\Downarrow$$

$$\begin{aligned} & K_l'(Y) \cdot \beta \cdot \left\{ u''(C_{1,l}) + [1 + (1 - t_k) \cdot r] \cdot \left\{ \begin{array}{c} G \cdot E\left[v''(C_{2,l})\right] \\ - [1 - t_k] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \end{array} \right\} \right\} \\ & + B_l'(Y) \cdot \left\{u''(C_{1,l}) + [1 + (1 - t_k) \cdot r]^2 \cdot E\left[v''(C_{2,l})\right]\right\} \\ = \; & u''(C_{1,l}) \end{aligned}$$

Define this as

$$K_l'(Y)\cdot o_{BK}+B_l'(Y)\cdot o_{BB}=q_B$$

Next, equation (A24) is differentiated:

$$\begin{aligned}
& -\beta\cdot u''(C_{1,l})\cdot\{1-\beta\cdot K_l'(Y)-B_l'(Y)\} \\
& +\beta\cdot[1-t_k]\cdot F_{K_lK_l}\cdot E\left[v'(C_{2,l})\right]\cdot K_l'(Y) \\
& +\beta\cdot G\cdot\left\{\begin{array}{c}\beta\cdot G\cdot E\left[v''(C_{2,l})\right]\cdot K_l'(Y)- \\ \beta\cdot[1-t_k]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot K_l'(Y) \\ +\left[1+(1-t_k)\cdot r\right]\cdot E\left[v''(C_{2,l})\right]\cdot B_l'(Y)\end{array}\right\} \\
& -\beta\cdot[1-t_k]\cdot\left\{\begin{array}{c}\beta\cdot G\cdot E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot K_l'(Y) \\ -\beta\cdot[1-t_k]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\cdot K_l'(Y) \\ +\left[1+(1-t_k)\cdot r\right]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot B_l'(Y)\end{array}\right\} \\
= {} & 0
\end{aligned}$$

$$\Downarrow$$

$$\begin{aligned}
& K_l'(Y)\cdot\beta\cdot\left\{\begin{array}{c}\beta\cdot u''(C_{1,l})+\beta\cdot[1-t_k]\cdot F_{K_lK_l}\cdot E\left[v'(C_{2,l})\right] \\ +\beta\cdot G^2\cdot E\left[v''(C_{2,l})\right] \\ -2\cdot\beta\cdot G\cdot[1-t_k]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right] \\ +\beta\cdot[1-t_k]^2\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\end{array}\right\} \\
& +B_l'(Y)\cdot\beta\cdot\left\{u''(C_{1,l})+\left[1+(1-t_k)\cdot r\right]\cdot\left\{\begin{array}{c}G\cdot E\left[v''(C_{2,l})\right] \\ -[1-t_k]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\end{array}\right\}\right\} \\
= {} & \beta\cdot u''(C_{1,l})
\end{aligned}$$

Define this as

$$K_l'(Y)\cdot o_{KK}+B_l'(Y)\cdot o_{KB}=q_K$$

By Cramer's rule,

$$K_l'(Y)=\frac{q_B\cdot o_{KB}-q_K\cdot o_{BB}}{o_{BK}\cdot o_{KB}-o_{KK}\cdot o_{BB}}=\frac{q_K\cdot o_{BB}-q_B\cdot o_{KB}}{M}$$

$$\begin{aligned}
& K_l'(Y) \\
= {} & \frac{\beta\cdot u''(C_{1,l})}{M}\cdot\left\{\begin{array}{l}\left\{u''(C_{1,l})+\left[1+(1-t_k)\cdot r\right]^2\cdot E\left[v''(C_{2,l})\right]\right\} \\ -u''(C_{1,l}) \\ -\left[1+(1-t_k)\cdot r\right]\cdot\left\{\begin{array}{c}G\cdot E\left[v''(C_{2,l})\right] \\ -[1-t_k]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\end{array}\right\}\end{array}\right\}
\end{aligned}$$

$$
\begin{aligned}
& \Downarrow \quad \text{by the definition of } G \\
K_l'(Y) &= \frac{\beta \cdot u''(C_{1,l})}{M} \\
& \cdot \left\{ \begin{array}{l} [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,l})\right] \\ -[1+(1-t_k)\cdot r] \\ \cdot \left\{ \begin{array}{c} [1+(1-t_k)\cdot r]\cdot E\left[v''(C_{2,l})\right] \\ +(1-t_k)\cdot \frac{E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]}{E\left[v'(C_{2,l})\right]} \cdot E\left[v''(C_{2,l})\right] \\ -[1-t_k]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right] \end{array} \right\} \end{array} \right\}
\end{aligned}
$$

$$
\begin{aligned}
& \Downarrow \\
\frac{\partial K_l}{\partial Y} &= -\beta \cdot \frac{u''(C_{1,l})\cdot[1+(1-t_k)\cdot r]\cdot[1-t_k]}{M\cdot E\left[v'(C_{2,l})\right]} \\
& \cdot \left[\begin{array}{c} E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,l})\right] \\ -E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,l})\right] \end{array} \right]
\end{aligned} \tag{A27}
$$

where M is positive and defined in (A26). The first part of (A27) is then positive. Thus the income effect is positive as long as the second part of (A27) is positive, that is if

$$E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,l})\right] - E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,l})\right] > 0.$$

$$
\begin{aligned}
& \Downarrow \\
\frac{\partial K_l}{\partial Y} &> 0 \quad \text{if } \ cov\left[v'(C_{2,l}),\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,l})\right] > cov\left[v''(C_{2,l}),\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,l})\right].
\end{aligned} \tag{A28}
$$

2.4 The effect on real capital investments from increased tax on capital income.

Differentiate (A23) with respect to t_k :

$$
\begin{aligned}
& K_l'(t_k)\cdot\beta\cdot\left\{u''(C_{1,l}) + [1+(1-t_k)\cdot r]\cdot \left\{ \begin{array}{c} +G\cdot E\left[v''(C_{2,l})\right] \\ -[1-t_k]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right] \end{array} \right\}\right\} \\
& +B_l'(t_k)\cdot\left\{u''(C_{1,l}) + [1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,l})\right]\right\} \\
= \ & r\cdot E\left[v'(C_{2,l})\right] \\
& +[1+(1-t_k)\cdot r]\cdot E\left[v''(C_{2,l})\right]\cdot[\beta\cdot F(K_l)+r\cdot B_l] \\
& -\beta\cdot K_l\cdot[1+(1-t_k)\cdot r]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]
\end{aligned}
$$

Define this as

$$K_l'(t_k)\cdot\Psi_{BK} + B_l'(t_k)\cdot\Psi_{BB} = j_B$$

Next, condition (A24) is differentiated:

$$
\begin{aligned}
& -\beta \cdot u''(C_{1,l}) \cdot \{-\beta \cdot K_l'(t_k) - B_l'(t_k)\} \\
& -\beta \cdot F_{K_l} \cdot E\left[v'(C_{2,l})\right] \\
& +\beta \cdot [1-t_k] \cdot F_{K_l K_l} \cdot E\left[v'(C_{2,l})\right] \cdot K_l'(t_k) \\
& +\beta \cdot G \cdot \left\{ \begin{array}{c} -\beta \cdot F(K_l) \cdot E\left[v''(C_{2,l})\right] + \beta \cdot K_l \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \\ +\beta \cdot G \cdot E\left[v''(C_{2,l})\right] \cdot K_l'(t_k) \\ -\beta \cdot [1-t_k] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \cdot K_l'(t_k) - r \cdot B_l \cdot E\left[v''(C_{2,l})\right] \\ + [1+(1-t_k)\cdot r] \cdot E\left[v''(C_{2,l})\right] \cdot B_l'(t_k) \end{array} \right\} \\
& +\beta \cdot E\left[v'(C_{2,l}) \cdot \widetilde{\gamma}\right] \\
& -\beta \cdot [1-t_k] \left\{ \begin{array}{c} -\beta \cdot F(K_l) \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \\ +\beta \cdot K_l \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \\ +\beta \cdot G \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \cdot K_l'(t_k) \\ -\beta \cdot [1-t_k] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \cdot K_l'(t_k) \\ -r \cdot B_l \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \\ + [1+(1-t_k)\cdot r] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \cdot B_l'(t_k) \end{array} \right\} \\
= \quad & 0
\end{aligned}
$$

$$\Downarrow$$

$$
\begin{aligned}
& K_l'(t_k) \cdot \beta \cdot \left\{ \begin{array}{c} \beta \cdot u''(C_{1,l}) + \beta \cdot G^2 \cdot E\left[v''(C_{2,l})\right] \\ -2 \cdot \beta \cdot G \cdot [1-t_k] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \\ +\beta \cdot [1-t_k]^2 \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] + [1-t_k] \cdot F_{K_l K_l} \cdot E\left[v'(C_{2,l})\right] \end{array} \right\} \\
& +B_l'(t_k) \cdot \beta \cdot \left\{ u''(C_{1,l}) + [1+(1-t_k)\cdot r] \cdot \left\{ \begin{array}{c} G \cdot E\left[v''(C_{2,l})\right] \\ -[1-t_k] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \end{array} \right\} \right\} \\
= \quad & \beta \cdot \left\{ \begin{array}{c} F_{K_l} \cdot E\left[v'(C_{2,l})\right] - E\left[v'(C_{2,l}) \cdot \widetilde{\gamma}\right] - \beta \cdot G \cdot K_l \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \\ + \{\beta \cdot F(K_l) + r \cdot B_l\} \cdot \left\{ \begin{array}{c} G \cdot E\left[v''(C_{2,l})\right] \\ -[1-t_k] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \end{array} \right\} \\ +\beta \cdot [1-t_k] \cdot K_l \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \end{array} \right\}
\end{aligned}
$$

Define this as

$$K_l'(t_k) \cdot \Psi_{KK} + B_l'(t_k) \cdot \Psi_{KB} = j_K$$

By Cramer's rule:

$$K_l'(t_k) = \frac{j_B \cdot \Psi_{KB} - j_K \cdot \Psi_{BB}}{\Psi_{BK} \cdot \Psi_{KB} - \Psi_{KK} \cdot \Psi_{BB}} = \frac{j_K \cdot \Psi_{BB} - j_B \cdot \Psi_{KB}}{M}$$

$$K_l'(t_k) = \frac{\beta}{M} \cdot \left\{ \begin{array}{c} r \cdot E\left[v'(C_{2,l})\right] \\ -\beta \cdot [1 + [1 - t_k] \cdot r] \cdot K_l \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \\ + \{\beta \cdot F(K_l) + r \cdot B_l\} \cdot E\left[v''(C_{2,l})\right] \cdot [1 + (1 - t_k) \cdot r] \\ + \frac{1 - t_k}{E[v'(C_{2,l})]} \cdot \{\beta \cdot F(K_l) + r \cdot B_l\} \\ \cdot \left\{ \begin{array}{c} E\left[v'(C_{2,l}) \cdot \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,l})\right] \\ -E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \cdot E\left[v'(C_{2,l})\right] \end{array} \right\} \\ +\beta \cdot \frac{[1 - t_k] \cdot K_l}{E[v'(C_{2,l})]} \cdot \left\{ \begin{array}{c} E\left[v''(C_{2,l}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \cdot E\left[v'(C_{2,l})\right] \\ -E\left[v'(C_{2,l}) \cdot \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \end{array} \right\} \end{array} \right\}$$

$$\cdot \left\{ u''(C_{1,l}) + [1 + (1 - t_k) \cdot r]^2 \cdot E\left[v''(C_{2,l})\right] \right\}$$

$$- \left\{ \begin{array}{l} r \cdot E\left[v'(C_{2,l})\right] \\ + [1 + (1 - t_k) \cdot r] \cdot E\left[v''(C_{2,l})\right] \cdot \{\beta \cdot F(K_l) + r \cdot B_l\} \\ -\beta \cdot K_l \cdot [1 + (1 - t_k) \cdot r] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \end{array} \right\}$$

$$\cdot \frac{\beta}{M} \cdot \left\{ \begin{array}{c} u''(C_{1,l}) + [1 + (1 - t_k) \cdot r]^2 \cdot E\left[v''(C_{2,l})\right] \\ + \frac{[1 + (1 - t_k) \cdot r] \cdot [1 - t_k]}{E[v'(C_{2,l})]} \cdot \left\{ \begin{array}{c} E\left[v'(C_{2,l}) \cdot \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,l})\right] \\ -E\left[v'(C_{2,l})\right] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \end{array} \right\} \end{array} \right\}$$

$$\Downarrow$$

$$K_l'(t_k)$$

$$= \quad \frac{\beta}{M} \cdot \left\{ \begin{array}{c} -\beta \cdot [1 + [1 - t_k] \cdot r] \cdot K_l \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \\ + \{\beta \cdot F(K_l) + r \cdot B_l\} \cdot E\left[v''(C_{2,l})\right] \cdot [1 + (1 - t_k) \cdot r] \\ +\beta \cdot \frac{[1 - t_k] \cdot K_l}{E[v'(C_{2,l})]} \cdot \left\{ \begin{array}{c} E\left[v''(C_{2,l}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \cdot E\left[v'(C_{2,l})\right] \\ -E\left[v'(C_{2,l}) \cdot \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \end{array} \right\} \\ - \left\{ \begin{array}{c} [1 + (1 - t_k) \cdot r] \cdot E\left[v''(C_{2,l})\right] \cdot \{\beta \cdot F(K_l) + r \cdot B_l\} \\ -\beta \cdot K_l \cdot [1 + (1 - t_k) \cdot r] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \end{array} \right\} \end{array} \right\}$$

$$\cdot \left\{ u''(C_{1,l}) + [1 + (1 - t_k) \cdot r]^2 \cdot E\left[v''(C_{2,l})\right] \right\}$$

$$+ \left\{ \begin{array}{l} \{\beta \cdot F(K_l) + r \cdot B_l\} \cdot \frac{u''(C_{1,l})}{1 + (1 - t_k) \cdot r} \\ -r \cdot E\left[v'(C_{2,l})\right] \\ +\beta \cdot K_l \cdot [1 + (1 - t_k) \cdot r] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \end{array} \right\}$$

$$\cdot \frac{\beta}{M} \cdot \frac{[1 + (1 - t_k) \cdot r] \cdot [1 - t_k]}{E\left[v'(C_{2,l})\right]} \cdot \left\{ \begin{array}{c} E\left[v'(C_{2,l}) \cdot \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,l})\right] \\ -E\left[v'(C_{2,l})\right] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \end{array} \right\}$$

Apply the definition of the income effect in equation (A27):

$$
\begin{aligned}
K_l'(t_k) &= -\left\{ \begin{array}{l} \frac{\beta \cdot F(K_l)+r\cdot B_l}{1+(1-t_k)\cdot r} - r \cdot \frac{E\left[v'(C_{2,l})\right]}{u''(C_{1,l})} \\ +\beta \cdot K_l \cdot [1+(1-t_k)\cdot r] \cdot \frac{E\left[v''(C_{2,l})\cdot \widetilde{\gamma}\right]}{u''(C_{1,l})} \end{array} \right\} \cdot \frac{\partial K_l}{\partial Y} \\
&\quad + \frac{\beta^2 \cdot [1-t_k] \cdot K_l}{M \cdot E\left[v'(C_{2,l})\right]} \cdot \left\{ u''(C_{1,l}) + [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,l})\right] \right\} \\
&\quad \cdot \left\{ \begin{array}{l} E\left[v''(C_{2,l}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \cdot E\left[v'(C_{2,l})\right] \\ -E\left[v'(C_{2,l}) \cdot \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \end{array} \right\}
\end{aligned}
$$

As the expression in the first parenthesis is positive, the total income effect is negative if $\frac{\partial K_l}{\partial Y} > 0$, which we from (A28) know holds if $cov\left[v'(C_{2,l}), \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,l})\right] > cov\left[v''(C_{2,l}), \widetilde{\gamma}\right] \cdot E\left[v'(C_{2,l})\right]$.
We also know that $\frac{\beta^2 \cdot [1-t_k] \cdot K_l}{M \cdot E[v'(C_{2,l})]} > 0$, and that $\left\{ u''(C_{1,l}) + [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,l})\right] \right\} < 0$.
Thus the substitution effect is positive if
$E\left[v''(C_{2,l}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \cdot E\left[v'(C_{2,l})\right] - E\left[v'(C_{2,l}) \cdot \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] < 0$. This can be rewritten as
$cov\left[v''(C_{2,l}) \cdot \widetilde{\gamma}, \widetilde{\gamma}\right] \cdot E\left[v'(C_{2,l})\right] < cov\left[v'(C_{2,l}), \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right]$.

$$
\begin{aligned}
K_l'(t_k) &> 0 \text{ if } cov\left[v''(C_{2,l}), \widetilde{\gamma}\right] \cdot E\left[v'(C_{2,l})\right] < cov\left[v'(C_{2,l}), \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,l})\right] \\
& \quad \text{and} \\
1) &: cov\left[v''(C_{2,l}) \cdot \widetilde{\gamma}, \widetilde{\gamma}\right] \cdot E\left[v'(C_{2,l})\right] < cov\left[v'(C_{2,l}), \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \\
2) &: \text{The substitution effect dominates the total income effect.}
\end{aligned}
$$

3 When to incorporate?

$$
\widehat{EU}_l - \widehat{EU}_s = u(\widehat{C}_{1,l}) + E\left[v(\widehat{C}_{2,l})\right] - u(\widehat{C}_{1,s}) - E\left[v(\widehat{C}_{2,s})\right],
$$

where

$$
\begin{aligned}
\widehat{C}_{1,l} &= Y - \beta \cdot \widehat{K}_l - \widehat{B}_l \\
\widehat{C}_{2,l} &= \beta \cdot [1-t_k] \cdot \left[F(\widehat{K}_l) - \widetilde{\gamma} \cdot \widehat{K}_l\right] + \beta \cdot \widehat{K}_l + [1+(1-t_k)\cdot r] \cdot \widehat{B}_l \\
\widehat{C}_{1,s} &= Y - \widehat{K}_s - \widehat{B}_s \\
\widehat{C}_{2,s} &= [1-t_w] \cdot \left[F(\widehat{K}_s) - \widetilde{\gamma} \cdot \widehat{K}_s\right] + \{1+[t_w - t_k]\cdot[r+\mu]\} \cdot \widehat{K}_s \\
&\quad + [1+(1-t_k)\cdot r] \cdot \widehat{B}_s.
\end{aligned}
$$

3.1 Effect of increased tax on labor income, proposition 5:

$$\begin{aligned}\frac{\partial\left(\widehat{EU}_l-\widehat{EU}_s\right)}{\partial t_w} &= -E\left[\left\{-F(\widehat{K}_s)+\widetilde{\gamma}\cdot\widehat{K}_s+[r+\mu]\cdot\widehat{K}_s\right\}\cdot v(\widehat{C}_{2,s})\right]\\ &= \left\{F(\widehat{K}_s)-[r+\mu]\cdot\widehat{K}_s\right\}\cdot E\left[v(\widehat{C}_{2,s})\right]-\widehat{K}_s\cdot E\left[v(\widehat{C}_{2,s})\cdot\widetilde{\gamma}\right]\\ &= \left\{F(\widehat{K}_s)-[r+\mu]\cdot\widehat{K}_s-\widehat{K}_s\cdot\frac{E\left[v(\widehat{C}_{2,s})\cdot\widetilde{\gamma}\right]}{E\left[v(\widehat{C}_{2,s})\right]}\right\}\cdot E\left[v(\widehat{C}_{2,s})\right]\\ &\Downarrow\ \text{From (2) we know that } \frac{E\left[v(\widehat{C}_{2,s})\cdot\widetilde{\gamma}\right]}{E\left[v(\widehat{C}_{2,s})\right]}=\delta+\widehat{\lambda}\end{aligned}$$

$$\frac{\partial\left(\widehat{EU}_l-\widehat{EU}_s\right)}{\partial t_w}=\left\{F(\widehat{K}_s)-\left[r+\mu+\delta+\widehat{\lambda}_s\right]\cdot\widehat{K}_s\right\}\cdot E\left[v(\widehat{C}_{2,s})\right]$$

We know that $E\left[v(\widehat{C}_{2,s})\right]>0$. By assumption $F(\widehat{K}_s)-[r+\mu+\delta]\cdot\widehat{K}_s>0$. Thus $\frac{\partial\left(\widehat{EU}_l-\widehat{EU}_s\right)}{\partial t_w}>0$ as long as $\frac{F(\widehat{K}_s)}{\widehat{K}_s}-r-\mu-\delta>\widehat{\lambda}_s$.

3.2 Effect of increased tax on capital income, proposition 6:

$$\begin{aligned}&\frac{\partial\left(\widehat{EU}_l-\widehat{EU}_s\right)}{\partial t_k}\\ =\ &\frac{\partial E\left[v(\widehat{C}_{2,l})\right]}{\partial t_k}-\frac{\partial E\left[v(\widehat{C}_{2,s})\right]}{\partial t_k}\\ =\ &E\left[\left\{-\beta\cdot\left(F(\widehat{K}_l)-\widetilde{\gamma}\cdot\widehat{K}_l\right)-r\cdot\widehat{B}_l\right\}\cdot v'(\widehat{C}_{2,l})\right]\\ &-E\left[\left\{-(r+\mu)\cdot\widehat{K}_s-r\cdot\widehat{B}_s\right\}\cdot v'(\widehat{C}_{2,s})\right]\\ =\ &-\left\{\beta\cdot F(\widehat{K}_l)+r\cdot\widehat{B}_l\right\}\cdot E\left[v'(\widehat{C}_{2,l})\right]+\beta\cdot\widehat{K}_l\cdot E\left[v'(\widehat{C}_{2,l})\cdot\widetilde{\gamma}\right]\\ &+\left\{(r+\mu)\cdot\widehat{K}_s+r\cdot\widehat{B}_s\right\}\cdot E\left[v'(\widehat{C}_{2,s})\right]\\ \Downarrow\ &\text{from (13): } E\left[v'(\widehat{C}_{2,l})\cdot\widetilde{\gamma}\right]=(\delta+\widehat{\lambda}_l)\cdot E\left[v'(\widehat{C}_{2,l})\right]\\ =\ &-\left\{\beta\cdot F(\widehat{K}_l)+r\cdot\widehat{B}_l\right\}\cdot E\left[v'(\widehat{C}_{2,l})\right]+\beta\cdot\widehat{K}_l\cdot(\delta+\widehat{\lambda}_l)\cdot E\left[v'(\widehat{C}_{2,l})\right]\\ &+\left\{(r+\mu)\cdot\widehat{K}_s+r\cdot\widehat{B}_s\right\}\cdot E\left[v'(\widehat{C}_{2,s})\right]\\ \Downarrow\ &\end{aligned}$$

$$\begin{aligned}\frac{\partial\left(\widehat{EU}_l-\widehat{EU}_s\right)}{\partial t_k} &= -\left\{\beta\cdot\left[F(\widehat{K}_l)-\left(\delta+\widehat{\lambda}_l\right)\cdot\widehat{K}_l\right]+r\cdot\widehat{B}_l\right\}\cdot E\left[v'(\widehat{C}_{2,l})\right]\\ &\quad+\left\{(r+\mu)\cdot\widehat{K}_s+r\cdot\widehat{B}_s\right\}\cdot E\left[v'(\widehat{C}_{2,s})\right]\end{aligned}$$

We don't know whether $E\left[v'(\widehat{C}_{2,s})\right]$ or $E\left[v'(\widehat{C}_{2,l})\right]$ is larger, as we don't know the $\widehat{C}_{2,s}$ and $\widehat{C}_{2,l}$. We do know that $\widehat{B}_l>\widehat{B}_s$. The reason for this is twofold. Firstly, as shown in section 4, proposition 1, the split model induces the self-employed to over-invest in firm specific real capital, such that $\widehat{K}_l < \widehat{K}_s$. Secondly, the active owner of the widely held corporation only invests a share β of total capital $\widehat{K}_l$. Thus the individual has more capital to invest in the financial market when organizing as a widely held corporation than as a self-employed, $\widehat{B}_l > \widehat{B}_s$.

$\frac{\partial(\widehat{EU}_l - \widehat{EU}_s)}{\partial t_k} < 0$ as long as $\left\{\beta \cdot \left[F(\widehat{K}_l) - \left(\delta + \widehat{\lambda}_l\right) \cdot \widehat{K}_l\right] + r \cdot \widehat{B}_l\right\} \cdot E\left[v'(\widehat{C}_{2,l})\right] > \left\{(r+\mu) \cdot \widehat{K}_s + r \cdot \widehat{B}_s\right\} \cdot E\left[v'(\widehat{C}_{2,s})\right]$, which is likely to hold, as the first part of the left hand side represents the individual's full second period income if he organizes as a widely held corporation, while the first part of the right hand side represents only part of the individual's income (the imputed return to capital in the firm and the return to financial investments) if he organizes as a self-employed individual.

3.3 Effect of increased risk compensation rate under the split model, proposition 7:

$$\frac{\partial\left(\widehat{EU}_l - \widehat{EU}_s\right)}{\partial\mu} = -\left[t_w - t_k\right] \cdot \widehat{K}_s \cdot E\left[v'(\widehat{C}_{2,s})\right] < 0$$

3.4 Effect of increase in the amount of shares allowed held by the active owner in the widely held corporation, proposition 8:

$$\begin{aligned}
\frac{\partial\left(\widehat{EU}_l - \widehat{EU}_s\right)}{\partial\beta} &= -\widehat{K}_l \cdot u'(\widehat{C}_{1,l}) + E\left[\left\{[1-t_k] \cdot \left[F(\widehat{K}_l) - \widetilde{\gamma} \cdot \widehat{K}_l\right] + \widehat{K}_l\right\} \cdot v'(\widehat{C}_{2,l})\right] \\
&= \widehat{K}_l \cdot E\left[v'(\widehat{C}_{2,l})\right] \cdot \left\{1 - \frac{u'(\widehat{C}_{1,l})}{E\left[v'(\widehat{C}_{2,l})\right]}\right\} \\
&\quad + \left\{[1-t_k] \cdot F(\widehat{K}_l)\right\} \cdot E\left[v'(\widehat{C}_{2,l})\right] \\
&\quad - [1-t_k] \cdot \widehat{K}_l \cdot E\left[v'(\widehat{C}_{2,l}) \cdot \widetilde{\gamma}\right] \\
&\Downarrow \; \textit{from } (A23)\text{: } \frac{u'(C_{1,l})}{E\left[v'(C_{2,l})\right]} = \left[1 + (1-t_k) \cdot r\right] \\
&\Downarrow \; \textit{and} \text{ from (13): } E\left[v'(\widehat{C}_{2,l}) \cdot \widetilde{\gamma}\right] = (\delta + \widehat{\lambda}_l) \cdot E\left[v'(\widehat{C}_{2,l})\right] \\
&= -\widehat{K}_l \cdot E\left[v'(\widehat{C}_{2,l})\right] \cdot [1-t_k] \cdot r \\
&\quad + \left\{[1-t_k] \cdot F(\widehat{K}_l)\right\} \cdot E\left[v'(\widehat{C}_{2,l})\right] \\
&\quad - [1-t_k] \cdot \widehat{K}_l \cdot (\delta + \widehat{\lambda}_l) \cdot E\left[v'(\widehat{C}_{2,l})\right] \\
&\Downarrow
\end{aligned}$$

$$\frac{\partial\left(\widehat{EU}_l - \widehat{EU}_s\right)}{\partial\beta} = [1-t_k] \cdot \left\{F(\widehat{K}_l) - \left(r + \delta + \widehat{\lambda}_l\right) \cdot \widehat{K}_l\right\} \cdot E\left[v'(\widehat{C}_{2,l})\right]$$

We know that $[1-t_k]\cdot E\left[v'(\widehat{C}_{2,l})\right] > 0$. Then $\frac{\partial\left(\widehat{EU}_l-\widehat{EU}_s\right)}{\partial\beta} > 0$ if $F(\widehat{K}_l) - \left(r+\delta+\widehat{\lambda}_l\right)\cdot\widehat{K}_l > 0$. By Applying (13) this condition reduces to $\frac{F(\widehat{K}_l)}{\widehat{K}_l} > F_{\widehat{K_l}}$, which means that the average return to capital in optimum is higher than the marginal return to capital. And this holds, as $F_{\widehat{K_l}}$ and $F_{\widehat{K_l}\widehat{K_l}} < 0$.