Finnish Economic Papers – Volume 16 – Number 1 – Spring 2003

MULTILATERAL AND BILATERAL MEETINGS WITH PRODUCTION HETEROGENEITY

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We study different trading processes in the context of a search-based model of endogenous money. We incorporate heterogeneity into the model by allowing multiple meetings of agents and divisible production. We then determine the equilibrium using three different trading mechanisms: auctions, pairwise bargaining and price posting. The welfare under these mechanisms is compared using specific functional forms for utility and cost functions. The analysis is done numerically with the bench mark of a social welfare maximising planner. (JEL: D44, D52, D83)

1. Introduction

The basic set-up in the theory of endogenous money is a search model with pairwise meetings (e.g., Kiyotaki and Wright, 1993). This quite naturally leads to bargaining as the mechanism to determine the terms of trade when money or goods are divisible. Price determination with divisible money is very difficult since one has to determine the distribution of money holdings, too. A simple way to circumvent this problem is to assume that money is indivisible and that the parties negotiate on how much is produced (e.g., Trejos and Wright, 1995).

In this work we adopt this set-up to study different trading mechanisms in the context of a search-based model of money. We do not regard this set up as particularly good to address the problem of price formation. Quite to the contrary we think that divisible money is essential to make these models fruitful at all. Our purpose, however, is to demonstrate that the mechanism for price determination makes a difference, and that it is not at all clear that bargaining is the best mechanism. For this purpose the assumption about indivisible money and divis-

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ible production suits well. To make the point we show that there are at least two other modes of price determination that are technically feasible, namely auction and price posting. Both of these have been used in labour search models, and one clear message from there is that bargaining introduces inefficiencies at the outset while auctions and price posting many times yield efficient outcomes (e.g., Jansen 2000, Peters 1991, Shimer 1999). Interestingly, this is not the case here. The difference from the labour search models is that the object for sale is not uniform but the amount traded depends on the way the terms of trade are determined.

One other thing that is worth mentioning, even though it is not specific to this model, is that when terms of trade are determined in auction there is price distribution even though all agents are identical before meetings take place. A recent paper by Soller-Curtis and Wright (2000) introduces a model where preference shocks among buyers lead to price distribution. This is as such not so surprising but in their model there can be at most two prices even when the preference shocks generate many different kinds of buyers. They call this finding "The law of two prices". This is an interesting result but to get price distribution it is necessary to have some kind of heterogeneity only if the agents are restricted to pairwise meetings and consequently to bargaining. If the aim is to generate price distribution the easiest way is to postulate that trades are consummated in auction.

In our set-up the sellers produce goods of various qualities or quantities depending on the number of trading partners they meet. We use specific functional forms of utility and cost functions, namely linear utility and quadratic costs, and show that there exists an equilibrium with heterogeneity when auctions are considered. We also determine the terms of trade when trades are consummated in pairwise bargaining and when sellers post prices. On the background there is an urn-ball-type meeting technology that makes multiple meetings possible (e.g., Lu and McAfee, 1996). We investigate numerically how the equilibrium prices depend on the share of money holding agents. Finally, we also solve the model with a social planner, and compare the welfare generated by different trading mechanisms.

In section 2 we introduce the model and determine the equilibrium when prices are determined by auction or bilateral meetings. The equilibrium terms of trade using bargaining and price posting are also determined. In section 3 we make welfare comparisons, and in section 4 we present concluding remarks.

2. *The model*

Consider an economy with identical buyers and sellers. Each seller has a convex and increasing cost function $c(q)$ which can be interpreted either as the cost of quality or quantity. Since the good is assumed indivisible the former interpretation may be preferable. Each buyer has a concave and increasing utility function $u(q)$. The functions are zero at zero and satisfy standard assumptions about derivatives so that there exist gains from trade.

Sellers are randomly contacted by buyers. This means that a seller may meet any number of buyers. Sellers can't consume their own goods. The only possible way to trade is to use a medium of exchange called money. The number of agents in the economy is normalised to unity so that M agents, called buyers, hold a unit of indivisible money while $1 – M$ (sellers) don't. Time is discrete, extends to infinity and the agents' common discount factor is $\delta \in (0, 1)$. The order of events within one period is fixed: Buyers and sellers meet randomly, terms of trade are agreed upon, production takes place, sellers receive a unit of money and buyers consume the newly acquired good. After trading takes place the roles of successful buyers and sellers are reversed.

The probability that a fixed seller meets any particular buyer is $1/(1 – M)$, and since there are M buyers the number of buyers a seller meets is distributed according to *Bin* $\left(M, \frac{M}{1-M}\right)$. This binomial distribution is approximated with a Poisson distribution with parameter $m = \frac{M}{1-M}$.

One should note that even though the model is inspired by the models of money with a double coincidence of wants problem (e.g., Kiyotaki and Wright, 1993) we abstract from that here since our focus is on price determination. This is a simplification that does not affect the results, and the same practice is used in Trejos and Wright (1995) where price determination is also the main point. We also want to emphasise that regardless of the trading mechanism there always exists an equilibrium where money is not accepted at all; if everyone expects no-one else to accept money the optimal response is not to accept money. Here we study only symmetric equilibria in pure strategies. There may exist mixed strategy equilibria when the agents post prices but with auction such do not exist since mixing between accepting and not accepting money would mean that accepting money yields zero utility as does not accepting it. As we want to compare different trading mechanisms it seems reasonable to exclude mixed strategy equilibria.

2.1. Multilateral meetings and auction markets

If a particular seller is not contacted by a buyer, this seller doesn't produce and receives his reservation utility. If exactly one buyer arrives to the seller he makes a take it or leave it offer to the seller who produces and receives once again his reservation utility. If it should happen that two or more buyers meet one particular seller then this seller receives all the surplus meaning that he gets everything except the buyers' reservation utility. This is because with multiple identical buyers meeting one seller, the buyers engage in a competition or auction that ensures that even the buyer who finally gets the good receives no more than his reservation utility.

This way to model auctions results in different qualities being produced depending on whether the seller meets one buyer or more than one buyer. Thus, in equilibrium there are two types of goods, q_l and q_h, produced in the economy. To shorten the formulae we adopt the following notation

(1) $$u_l = u(q_l), u_h = u(q_h)$$
$$c_l = c(q_l), c_h = c(q_h)$$

where $u_{l,h}$ and $c_{l,h}$ denote the utilities and costs of production associated with the respective types of good, and $c_l < c_h$ and $u_l < u_h$.

In the context of our model these different possibilities imply that if a single buyer meets a particular seller the seller is driven to his reservation level and this seller produces a high quality good associated with high production costs c_h. If a multiple buyer meeting takes place, the seller gets all the surplus. Thus the seller produces a low quality good with production costs c_l. Naturally, on the buyers' side things go the other way. If there are no other buyers at a particular meeting the single buyer receives everything except the seller's reservation utility as the seller must produce a high quality good. With other buyers present the one who gets to trade receives his reservation utility, that is the good of low quality. The sellers' and buyers' life time utilities are determined by equations (2) and (3).

(2) $$V_s = e^{-m}\delta V_s + me^{-m}(-c_h + \delta V_b) + \left(1 - e^{-m} - me^{-m}\right)(-c_l + \delta V_b)$$

(3) $$V_b = e^{-m}(u_h + \delta V_s) + \left(1 - e^{-m}\right)(u_l + \delta V_s)$$

where V_s is the life time utility of the seller and V_b that of the buyer. Remarks about utilities of agents in different kind of meetings imply that in equilibrium the following conditions have to be met.

(4) $$u_l + \delta V_s = \delta V_b$$

(5) $$-c_h + \delta V_b = \delta V_s$$

Expressions (4) and (5) imply that in equilibrium $c_h = u_l$. Using (4) and (5) life time utilities from (2) and (3) can be solved.

(6) $$V_s = \frac{\delta e^{-m}(1 - e^{-m} - me^{-m})}{(1-\delta)(1-\delta me^{-m})}u_h - \frac{(1-\delta+\delta e^{-m})(1 - e^{-m} - me^{-m})}{(1-\delta)(1-\delta me^{-m})}c_l$$

(7) $$V_b = \frac{e^{-m}(1 - \delta e^{-m} - \delta me^{-m})}{(1-\delta)(1-\delta me^{-m})}u_h - \delta\frac{e^{-m}(1 - e^{-m} - me^{-m})}{(1-\delta)(1-\delta me^{-m})}c_l$$

The equilibrium in the model is determined, using life time utilities (6), (7) and constraints (4), (5). Finally, in equilibrium a seller should be willing to accept money, a condition which is always satisfied when positive amounts are produced, and a seller should produce rather than do nothing which condition is formally $V_s \geq 0$ and simplifies to

(8) $$\delta e^{-m} u_h \geq \left(1 - \delta + \delta e^{-m}\right) c_l$$

Using expressions (4), (5), (6), (7) and (8) the equilibrium in the model can be determined by two equations

(9) $$c_h = u_l = \delta \frac{e^{-m}}{1 - \delta m e^{-m}} u_h + \delta \frac{1 - e^{-m} - m e^{-m}}{1 - \delta m e^{-m}} c_l$$

Although the determination of the equilibrium looks quite simple, it is diffcult to show much more than its existence using non-specific functional forms of utilities and costs. The efficiency comparisons of different trading mechanisms is most certainly an intractable task with general functional forms. We want to emphasise that the purpose of the article is to show that it is no more diffcult to consider other methods of price determination than pairwise bargaining in models of endogenous money, and to demonstrate that it makes a difference which method is chosen. For this purpose we must be able to compare the results which forces us to choose specific functional forms. For the sake of completeness it would be nice to know how general the results are but since we think that the right way to model money is to assume it is divisible it seems of secondary importance to derive the most general results in this setting.

We are interested in investigating what happens in equilibrium with different parameter values for the share of agents that are searching. To be able to answer this we investigate the equilibrium numerically.

2.2. Numerical analysis

We use functional forms that are as easy as possible, and no doubt come to mind first. Our choice is to use linear form for the utility representation which is without loss of generality since one can always use utility as units, and a quadratic cost function. With these assumptions the equilibrium is defined as follows.

(10) $$u\left(q_{l,h}\right) = q_{l,h}, c\left(q_{l,h}\right) = q_{l,h}^2;$$
$$q_h^2 = q_l \text{ and}$$
$$q_l = \frac{\delta e^{-m}}{1 - \delta m e^{-m}} q_h + \delta \frac{1 - e^{-m} - m e^{-m}}{1 - \delta m e^{-m}} q_l^2$$

The produced quantity of high quality good in equilibrium can be calculated from

(11) $$q_h = \frac{\delta e^{-m}}{1 - \delta m e^{-m}} + \delta \frac{1 - e^{-m} - m e^{-m}}{1 - \delta m e^{-m}} q_h^3$$

where δ is the common discount factor and m is the ratio of those who search (buyers) to those who produce (sellers). If there are no money holders in the economy then m takes the value zero. On the other hand if there are no producers in the economy then $m = \infty$. Produced quantities in equilibrium of both types of goods with different parameter values are thus determined by (11) and (10). Results using different discount factors are collected in table 1.

Table 1. Produced quantities of different types of goods with auction markets.

d = 0.7			d = 0.8			d = 0.9		
M	q(h)	q(l)	M	q(h)	q(l)	M	q(h)	q(l)
0.1	0.67	0.46	0.1	0.78	0.61	0.1	0.89	0.79
0.2	0.64	0.41	0.2	0.75	0.56	0.2	0.87	0.75
0.3	0.58	0.33	0.3	0.69	0.48	0.3	0.83	0.69
0.4	0.49	0.24	0.4	0.60	0.36	0.4	0.74	0.55
0.5	0.36	0.13	0.5	0.44	0.20	0.5	0.55	0.30
0.6	0.21	0.04	0.6	0.25	0.06	0.6	0.30	0.10
0.7	0.08	0.01	0.7	0.10	0.01	0.7	0.11	0.01
0.8	0.01	0.00	0.8	0.02	0.00	0.8	0.02	0.00
0.9	0.00	0.00	0.9	0.00	0.00	0.9	0.00	0.00

Proposition 1: With linear utility and quadratic costs there exists a unique equilibrium with production for any value of m. In this equilibrium two types of goods are produced. The relative share of production of high quality good decreases as the share of searching agents increases.

Proof: Consider equation (11). Recalling that $q_h^2 = q_l$ this expression unambiguously determines q_l once q_h is known. Thus, if there exists an equilibrium, two types of goods are produced (second claim in the proposition). The relevant values satisfy $q_h \in (0, 1)$. Evaluate both sides of equation (11) at 0 and 1. $LHS\ |q_{h=0} = 0$, $RHS\ |q_{h=0} > 0$ and $LHS\ |q_{h=1} = 1$, $RHS\ |q_{h=1} < 1$. Consider next the derivatives of LHS and RHS: $\frac{\partial}{\partial q_h} LHS = 1$ and $\frac{\partial}{\partial q_h} RHS = \delta 3 \frac{1-e^{-m}-me^{-m}}{1-\delta me^{-m}} q_h^2$. Thus, there must exist an equilibrium $\widehat{q}_h$ with properties $\widehat{q}_h < 1$ and $\frac{\partial}{\partial q_h} RHS = \delta 3 \frac{1-e^{-m}-me^{-m}}{1-\delta me^{-m}} \widehat{q}_h^2 < 1$. Assume now that there exists another equilibrium. The sufficient condition for this is that for some $\tilde{q}_h \in (0, 1)$, $\delta 3 \frac{1-e^{-m}-me^{-m}}{1-\delta me^{-m}} \widetilde{q}_h^2 > 1$. But then because of $LHS\ |q_{h=0} = 0$, $RHS\ |q_{h=0} > 0$ and $LHS\ |q_{h=1} = 1$, $RHS\ |q_{h=1} < 1$ there would have to exist a third equilibrium $\overline{q}_h > \widetilde{q}_h$ with $\delta 3 \frac{1-e^{-m}-me^{-m}}{1-\delta me^{-m}} \overline{q}_h^2 < 1$. This is a contradiction which proves the first part of the proposition. Consider finally the relative shares of different types of goods produced. q_h is produced with probability me^{-m} and q_l with probability $(1 - e^{-m} - me^{-m})$. The relative share of production of q_h decreases in m if $\frac{me^{-m}}{(1-e^{-m}-me^{-m})}$ decreases in m. $\frac{\partial}{\partial m} \frac{me^{-m}}{(1-e^{-m}-me^{-m})} < 0$ thus the relative share of production of q_h is decreasing as the share of searching agents increases. ■

As the share of searching agents in equilibrium decreases the share of factors of production devoted to production increases thus increasing the total production in the economy. The good associated with higher costs of production is produced only if a single buyer meets a particular seller. If there is a meeting with multiple buyers the good with lower production costs is being produced. As the share of buyers relative to sellers decreases the likelihood of a bilateral meeting between a single buyer and a particular seller increases compared to the likelihood of a multiple buyer meeting. This means that the relative amount of the high cost production increases. The total production in the economy is also increasing with the patience of agents measured by the discount factor.

2.3. Bilateral meetings

Contrary to auction markets we have investigated so far, in standard search models meetings are assumed to be of a bilateral nature. A single seller and a single buyer meet pairwise to decide the conditions of the transaction.

In a discrete time setting every buyer that arrives to a particular seller has an equal chance of being selected by the seller. Life time utilities are determined by the following equations where $c \equiv c(q)$ and $u \equiv u(q)$, where we have anticipated that a unique quality is produced (see proposition 2).

$$U_s = e^{-m}\delta U_s + \left(1 - e^{-m}\right)\left(-c + \delta U_b\right) \tag{12}$$

$$U_b = \frac{e^{-m}}{m}\left\{\frac{m}{1!} + \frac{m^2}{2!} + ...\right\}(u + \delta U_s) + \left(1 - \frac{e^{-m}}{m}\left\{\frac{m}{1!} + \frac{m^2}{2!} + ...\right\}\right)\delta U_b \tag{13}$$

Expressions (12) and (13) determine the utilities of agents in an economy with bilateral meetings without specifying the terms of trade. The terms are the same in all meetings and are determined later. From (12) it can be seen that if none of the buyers come to a particular seller, this seller receives his reservation utility. In every other case this seller produces with the associated costs and utilities. Similarly it is clear from (13) that if a particular buyer is chosen to be the bilateral partner this buyer gets to transact. If not, he receives his reservation utility. Solving the life time utilities from (12) and (13) yields.

$$U_s = \frac{\delta\left(1 - e^{-m}\right)^2}{(1-\delta)\left[\left(1 - \delta e^{-m}\right)m + \delta\left(1 - e^{-m}\right)\right]}u - \frac{(1-\delta)\left(1 - e^{-m}\right)m + \delta\left(1 - e^{-m}\right)^2}{(1-\delta)\left[\left(1 - \delta e^{-m}\right)m + \delta\left(1 - e^{-m}\right)\right]}c \tag{14}$$

$$U_b = \frac{\left(1 - e^{-m}\right)\left(1 - \delta e^{-m}\right)}{(1-\delta)\left[\left(1 - \delta e^{-m}\right)m + \delta\left(1 - e^{-m}\right)\right]}u - \frac{\delta\left(1 - e^{-m}\right)^2}{(1-\delta)\left[\left(1 - \delta e^{-m}\right)m + \delta\left(1 - e^{-m}\right)\right]}c \tag{15}$$

For future reference it is usefull to calculate $U_b - U_s$ from (14) and (15).

(16) $$U_b - U_s = \frac{1-e^{-m}}{(1-\delta e^{-m})\,m + \delta(1-e^{-m})}u + \frac{(1-e^{-m})\,m}{(1-\delta e^{-m})\,m + \delta(1-e^{-m})}c$$

At this point it is clear that the trading mechanism assumed has important consequences for the equilibrium in the economy. Recall that when the meetings are multilateral and the terms of trade are determined in auction the heterogeneity originates from the production side; two types of goods are produced in equilibrium. Since the consumers have identical preferences no matter how the surplus is divided, the bilateral trading mechanism in itself implies that only one type of good is being produced in the equilibrium. We state this as a proposition.

Proposition 2: Bilateral meetings result in an equilibrium with a homogenous good.

The actual division of surplus from the transaction is yet to be determined. Bargaining between the meeting agents, price posting and social welfare maximising planner are three natural possibilities for this purpose.

2.3.1. Bargaining

Following the standard practice bargaining is assumed to take a simple nonstrategic form although a strategic game can easily be described. The division is obtained by a Nash-bargaining solution determined by the following maximisation problem.

(17) $$\max_q\,[u(q) + \delta U_s - \delta U_b]^{\theta}\,[-c(q) + \delta U_b - \delta U_s]^{1-\theta}$$

where U_s and U_b are determined by (14) and (15) and θ is the parameter associated with the bargaining power of the respective bargaining partners. First order condition for this maximisation problem is

(18) $$\theta\,[u(q) + \delta U_s - \delta U_b]^{-1}\,[-c(q) + \delta U_b - \delta U_s]\,u'(q) - (1-\theta)\,c'(q) = 0$$

Assuming linear utility, quadratic cost functions, identical bargaining power between agents and using (18), the equilibrium is described by the following equation (19).

(19) $$q_B = \frac{m(1-\delta) + \delta(1-e^{-m}) + 2m(1-\delta e^{-m}) \pm \sqrt{\left[\begin{array}{c} m(1-\delta) + \delta(1-e^{-m}) + \\ 2m(1-\delta e^{-m})\end{array}\right]^2 - 8\delta^2 m(1-e^{-m})^2}}{4\delta m(1-e^{-m})}$$

where δ is again the common discount factor and *m* is the share of those who search (buyers) to those who produce (sellers). Produced quantities from (19) with different discount factors are collected in table 2.

Table 2. Produced quantities with bargaining.

d = 0.7		d = 0.8		d = 0.9	
M	q(B)	M	q(B)	M	q(B)
0.1	0.40	0.1	0.53	0.1	0.71
0.2	0.36	0.2	0.47	0.2	0.64
0.3	0.32	0.3	0.41	0.3	0.56
0.4	0.27	0.4	0.34	0.4	0.46
0.5	0.22	0.5	0.27	0.5	0.35
0.6	0.17	0.6	0.21	0.6	0.26
0.7	0.12	0.7	0.14	0.7	0.18
0.8	0.07	0.8	0.09	0.8	0.11
0.9	0.03	0.9	0.04	0.9	0.05

As can be seen from table 2 total production with a particular share of agents searching and producing is increasing with the patience of agents measured by the discount factor.

2.3.2. Price posting

It is also possible to think that the sellers announce a price, i.e., how much they produce, the buyers observe this price and based on this choose who to visit. One should notice that markets with price posting differ from the two previous cases since here the agents are not randomly matched but this is an example of a market with directed search. Of course, in equilibrium every seller posts the same price, and the meetings look just like in random matching markets. When multiple buyers arrive at a seller all of them have an equal chance of trading with the seller at the announced terms.

The equilibrium price is determined the same way as in Kultti (1999)[1]. Assume that in equi-

[1] *Since there is an infinite (or undetermined) number of agents it is not reasonable to use Nash equilibrium as the equilibrium concept. However, one can determine an*

librium the sellers announce price q. Further, assume that proportion z of sellers deviate for one period and announce price $\tilde{q}$. The utility of these sellers is $\tilde{u} \equiv u(\tilde{q})$. Since the number of agents is indefinite or infinite the standard Nashtest of one agent deviating does not work here but we get the analogous result by letting z approach zero once the equilibrium price is determined as a function of z. Proportion σ of the buyers go to the deviating sellers while the rest of the buyers go to non-deviating sellers.

The Poisson-rate for the deviators is $\widetilde{m} = \frac{\sigma}{z}m$ and the rate for the non-deviators is $\widehat{m} = \frac{1-\sigma}{1-z}m$. The utilities for buyers that go to deviators and non-deviators are

$$(20) \quad U_b^D = \frac{1-e^{-\widetilde{m}}}{\widetilde{m}}\left(\widetilde{u} + \delta U_s\right) + \left(1 - \frac{1-e^{-\widetilde{m}}}{\widetilde{m}}\right)\delta U_b$$

$$(21) \quad U_b^N = \frac{1-e^{-\widehat{m}}}{\widehat{m}}\left(u + \delta U_s\right) + \left(1 - \frac{1-e^{-\widehat{m}}}{\widehat{m}}\right)\delta U_b$$

In equilibrium the buyers have to be indifferent between going to the deviators and non-deviators meaning that $U_b^D = U_b^N$ which is equivalent to

$$(22) \quad \frac{1-e^{-\widehat{m}}}{\widehat{m}}\left(u + \delta U_s - \delta U_b\right) = \frac{1-e^{-\widetilde{m}}}{\widetilde{m}}\left(\widetilde{u} + \delta U_s - \delta U_b\right)$$

The deviating sellers' problem is $\max_{\tilde{q}} e^{-\tilde{m}}\delta U_s + (1 - e^{-\tilde{m}})(-\tilde{c} + \delta U_b)$ and the first order condition to this problem is

$$(23) \quad -e^{-\widetilde{m}}\frac{m}{z}\frac{d\sigma}{d\widetilde{q}}\left(-\widetilde{c} + \delta U_s - \delta U_b\right) - \left(1 - e^{-\widetilde{m}}\right)c'\left(\widetilde{q}\right) = 0$$

Of course, this only holds in equilibrium namely when $\tilde{q} = q$ in which case $\tilde{m} = \hat{m} = m$. The only thing to figure out is $\frac{d\sigma}{d\tilde{q}}$. One gets this by totally differentiating the equilibrium condition (22) which yields

$$(24) \quad \frac{d\sigma}{d\widetilde{q}} = \frac{\left[\frac{1-e^{-\widetilde{m}}}{\widetilde{m}}\right]}{\left[\begin{array}{l}\frac{\frac{1}{1-z}m\left(1-e^{-\widehat{m}}-\widehat{m}e^{-\widehat{m}}\right)}{\widehat{m}^2}\left(q + \delta U_s - \delta U_b\right) + \\ \frac{\frac{1}{z}m\left(1-e^{-\widetilde{m}}-\widetilde{m}e^{-\widetilde{m}}\right)}{\widetilde{m}^2}\left(\widetilde{q} + \delta U_s - \delta U_b\right)\end{array}\right]}$$

Inserting this into the first order condition, evaluating at $\tilde{q} = q$, and letting z converge to zero one gets the following equation that determines the equilibrium posted price

$$(25) \quad \begin{aligned} &q^2\left[2\delta\left(1-e^{-m}\right)\left(1-e^{-m}-me^{-m}\right)\right] - \\ &q\left[\left(1-\delta e^{-m}\right)\left(1-e^{-m}-me^{-m}\right) + \right. \\ &\left.\left(1-\delta m e^{-m}\right)\left(1-e^{-m}\right)\right] + \\ &\delta e^{-m}\left(1-e^{-m}\right) = 0 \end{aligned}$$

The notation in (25) is the same as before. Produced quantities with different shares of money holders are collected in table 3.

Table 3. Produced quantities with price posting.

d = 0.7		d = 0.8		d = 0.9	
M	q(P)	M	q(P)	M	q(P)
0.1	0.67	0.1	0.78	0.1	0.88
0.2	0.61	0.2	0.73	0.2	0.85
0.3	0.53	0.3	0.65	0.3	0.80
0.4	0.41	0.4	0.53	0.4	0.68
0.5	0.27	0.5	0.34	0.5	0.45
0.6	0.13	0.6	0.17	0.6	0.20
0.7	0.05	0.7	0.05	0.7	0.06
0.8	0.01	0.8	0.01	0.8	0.01
0.9	0.00	0.9	0.00	0.9	0.00

An interesting difference between bargaining and posted prices is the fact that produced quantities are higher with lower shares of money holders when the equilibrium is determined by posted prices but the opposite is true with high shares of money holders in the economy.

2.3.3. Social planner

The social planner's objective is to maximise the total welfare of the agents in the economy. In our model this is equivalent to maximising the total welfare of buyers and sellers. The

equilibrium price in pretty much the standard fashion by thinking that deviations by a positive proportion of sellers should not be beneficial in equilibrium. Of course, the price is then a function of the proportion of sellers that deviate. Letting this proportion go to zero one gets a price that corresponds very closely to the equilibrium price that one would get if there were a finite number of agents. Working with a finite number of agents is so cumbersome that the gain in exactness is rarely warranted.

welfare measure is defined to be the weighted average of utilities of these two types where the weights used are the respective shares of both types.

$$\max_{q} mU_b + U_s \tag{26}$$

Again using the previous assumptions about linear utility and quadratic costs it is straightforward to show that the production chosen by the social planner is determined by equation (27)

$$q_* = \frac{\delta\,(1-e^{-m}) + m\,(1-\delta e^{-m})}{2\,[m - \delta m e^{-m} + \delta\,(1-e^{-m})]} \tag{27}$$

which yields a simple result. Irrespective of the share of money holders in the economy or the common discount factor, the optimal quantity in the economy is always $q_* = \frac{1}{2}$. It is interesting to compare these methods of dividing the surplus among the agents in the economy associated with bilateral meetings. Aggregate welfare of the economy is maximised when the produced quantity is $\frac{1}{2}$. The produced quantities determined by bargaining or price posting can be either higher or lower than $\frac{1}{2}$ depending on the parameter values but in general they are different from $\frac{1}{2}$.

3. *Utility and efficiency under different trading mechanisms*

Of course, the ultimate question in economics is not the produced quantity of different products but the welfare of the agents in the economy. It is interesting to compare utilities when the terms of trade are determined using auction markets, bargaining and posted prices. There exist two types of agents in our model, buyers and sellers. Utilities of the two types of agents are determined under the assumption of auction market by life-time utility equations (6) and (7). The corresponding equations with bargaining, price posting and the social planner are (14) and (15). Results with different discount factors and shares of money holders (buyers) are collected in table 4[2].

[2] *Utility is, of course, not zero with any of the parameter values. Utility appears to be zero with large shares of money holders due to rounding of results.*

Table 4. Utility under different trading mechanisms.

M	Auction	Bargaining	Posted	Planner
d = 0.7				
0.1	0.08	0.08	0.08	0.09
0.2	0.17	0.17	0.18	0.18
0.3	0.28	0.25	0.29	0.29
0.4	0.38	0.32	0.39	0.41
0.5	0.38	0.36	0.42	0.53
0.6	0.25	0.37	0.29	0.65
0.7	0.07	0.32	0.14	0.75
0.8	0.00	0.21	0.03	0.82
0.9	0.00	0.10	0.00	0.83
d = 0.8				
0.1	0.09	0.13	0.09	0.13
0.2	0.22	0.28	0.22	0.28
0.3	0.38	0.42	0.40	0.44
0.4	0.58	0.55	0.61	0.61
0.5	0.66	0.62	0.71	0.79
0.6	0.38	0.64	0.55	0.97
0.7	0.33	0.54	0.21	1.13
0.8	0.01	0.40	0.05	1.23
0.9	0.00	0.19	0.00	1.25
d = 0.9				
0.1	0.11	0.21	0.11	0.26
0.2	0.27	0.51	0.28	0.55
0.3	0.54	0.86	0.56	0.87
0.4	1.01	1.21	1.07	1.22
0.5	1.47	1.44	1.56	1.58
0.6	1.06	1.50	1.24	1.94
0.7	0.30	1.33	0.51	2.26
0.8	0.02	0.96	0.10	2.45
0.9	0.00	0.47	0.00	2.50

A benevolent social planner would always choose $\frac{1}{2}$ as the produced quantity no matter what the division into money holders and sellers in the economy is. For illustrative purposes the utility that this level of production would yield is also presented in table 4. The difference between optimal utility and what different trading mechanisms imply is increasing in the share of money holders in the economy.

A couple of other interesting remarks are clear from table 4. Auction market yields higher total utility than bargaining with relatively small patience of agents measured by the discount factor and small share of buyers in the economy. With high shares of money holders and higher level of patience the total utility of the economy is larger under bargaining. The difference in favor of bargaining is largest when the economy is populated almost entirely by agents seeking for transaction possibilities. Posted prices as a mechanism to divide the surplus can lead to higher utility than bargaining but this, too, depends on the model parameters. The two most important observations are that there is no hope for any equivalence results between the trading mechanisms, and that none of them dominates or is dominated. The first observation should not be too surprising since it is well known that the equivalence of auctions and posted prices only holds with linear utility (e.g., Kultti and Riipinen, 2001).

Finally the results from table 4 are illustrated in figure 1. The remarks about the results concerning utilities under different trading mechanisms made above can be clearly seen from this figure which is constructed using value 0.8 for the discount factor.

Above the focus of interest has been on the total added life-time utilities of buyers and sellers under different trading mechanisms and model parameters. However, it should be remembered that total life-time utilities of agents and the efficiency of a particular trading mechanism are not synonyms in this setting. One way to consider the efficiency of a trading mechanism is to calculate the periodic utility net of production costs that it generates. In auction market the net utility is determined by the share of meetings with one buyer and the share of multiple buyer meetings. In bargaining and posted price markets every meeting, regardless of how many buyers meet a seller, results in the same trade, and the efficiency is determined by the number of meetings. Equation (28) gives the production of utility under auction markets while equations (29) and (30) determine the production in bargaining and posted price markets. Thus, the efficiency measures for auction, bargaining and posted price markets are

$$(28)\quad 100 * \left[(1-M)\, me^{-m}\, [u_h - c_h] + (1-M)\,(1 - e^{-m} - me^{-m})\,[u_l - c_l]\right]$$

$$(29)\quad 100 * \left[(1-M)\,(1 - e^{-m})\,[u_B - c_B]\right]$$

$$(30)\quad 100 * \left[(1-M)\,(1 - e^{-m})\,[u_p - c_p]\right]$$

The multiplification in (28)–(30) is done in order to increase the illutrativeness of the results collected in table 5.

As can be seen from table 5 there is no equivalence between different trading mechanisms when efficiency is considered. The efficiency results from, say, labour search models (e.g., Jansen, 2000), associated with posted prices or auction do not hold in this setting where the amount produced is a choice variable.

Results concerning the efficiencies of different trading mechanisms are again illustrated in figure 2 using value 0.8 for the common discount factor.

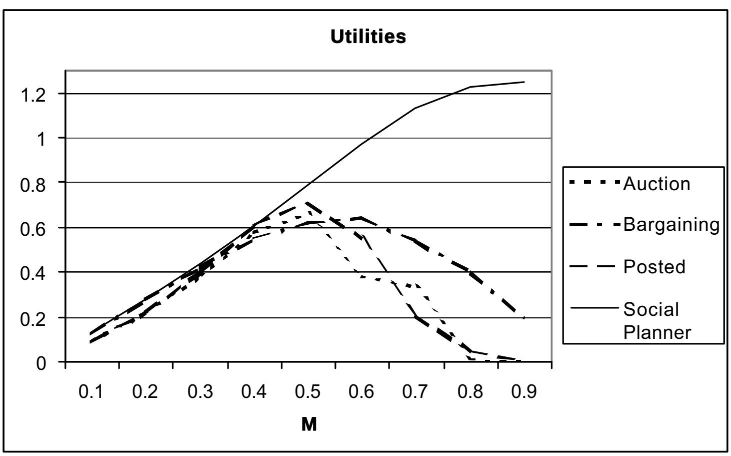

Figure 1. Utility under different trading mechanisms.

Table 5. Efficiency under different trading mechanisms.

M	Auction	Bargaining	Posted	Planner
d = 0.7				
0.1	2.09	2.25	2.10	2.34
0.2	4.10	4.08	4.21	4.42
0.3	5.86	5.32	6.09	6.12
0.4	6.73	5.77	7.09	7.32
0.5	5.73	5.42	6.23	7.90
0.6	2.97	4.38	3.51	7.77
0.7	0.63	2.86	1.29	6.77
0.8	0.02	1.28	0.14	4.91
0.9	0.00	0.29	0.00	2.50
d = 0.8				
0.1	1.64	2.34	1.61	2.34
0.2	3.44	4.41	3.49	4.42
0.3	5.41	5.92	5.57	6.12
0.4	6.95	6.57	7.30	7.32
0.5	6.59	6.23	7.09	7.90
0.6	3.55	5.16	4.18	7.77
0.7	0.82	3.26	1.29	6.77
0.8	0.04	1.61	0.16	4.91
0.9	0.00	0.38	0.00	2.50
d = 0.9				
0.1	0.95	1.93	0.99	2.34
0.2	2.15	4.08	2.26	4.42
0.3	3.81	6.03	3.91	6.12
0.4	6.12	7.28	6.38	7.32
0.5	7.34	7.19	7.82	7.90
0.6	4.26	5.98	4.97	7.77
0.7	0.91	4.00	1.53	6.78
0.8	0.04	1.92	0.18	4.91
0.9	0.00	0.47	0.00	2.50

4. Conclusions

We investigate a search-model of endogenous money when money is indivisible while production possibilities are divisible. Depending on the trading mechanism there may exist different types of goods in equilibrium. The model is solved with three different trading mechanisms. Multiple meetings of agents and auction markets imply results very different from those of pairwise meetings of agents with bargaining or posted prices. With auction there is price distribution while with the other two mechanisms associated with pairwise meetings or trading there is only one type of good produced. This is a difference that arises solely from the difference in the assumptions about the nature of trading.

Even though our results are got by using special functional forms and we solve the model numerically we think that there are some worthwhile things to be learned from this exercise. First, there are other possibilities than pairwise meetings and bargaining to advance the models of endogenous money. Multiple meetings and auctions or posted prices may be of great help in solving the problem of the distribution of money holdings when money is divisible. Presently we work on the problem using auctions as the trading mechanism. Secondly, the equivalence results between auctions and price posting do not hold basically since in models

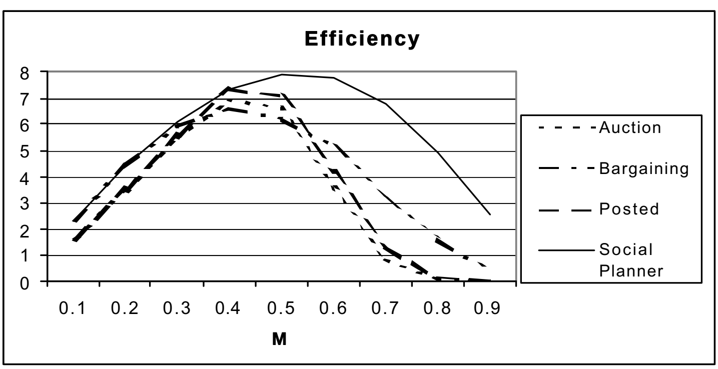

Figure 2. Efficiency under different trading mechanisms.

with endogenous money and production the utility functions cannot be linear. Thirdly, the results about the efficiency of auctions and posted prices do not hold when the object for sale is variable, i.e., when its quantity or quality depends on the trading mechanism.

It would, perhaps, be nice to know to what extent the results hold for more general functional forms. This seems, however, quite a diffcult task; it is, for instance, known that even in the simplest unit supply and unit demand model the equivalence of auctions and price posting does not hold, and either one can be preferred by buyers or sellers depending on the functional forms of the utility functions. Besides the main point of this article is to show that there are alternative ways to generate prices than pairwise bargaining in models of endogenous money. These models do not feature particularly general utility functions.

Finally, we would like to emphasise that one should not take this model of endogenous money too seriously; it is only a medium, and a simplest one we could come up with, to make the above points. If one seriously wants to do work with endogenous money, we think that the primary features of the model should be divisible money and a tractable mechanism of trade.

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