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OPTIMAL ENVIRONMENTAL TAXATION, R&D SUBSIDIZATION AND THE ROLE OF MARKET CONDUCT*

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The paper examines the optimal environmental policy in a differentiated goods duopoly with either price- or quantity-setting firms, where firms invest in environmental R&D that reduces emissions. It is shown that in quantity (Cournot) competition, the emission tax is always lower than marginal damages. With price (Bertrand) competition, the emission tax is generally lower than marginal damages. However, for the case of very undifferentiated products, the emission tax is equal to marginal damages, that is, it approaches the first-best tax. Moreover, the Cournot emission tax is always lower than the Bertand emission tax. Concerning the R&D subsidy, the comparison crucially depends on the degree of product differentiation and the initial emissions coefficient. (JEL: Q28, O38, H29, L13)

1. Introduction

In recent years there has been an upsurge in the analysis of environmental policy in the context of a variety of market structures, usually under the assumption that emissions per unit of output are constant. It is well known that in the case of a perfectly competitive industry the optimal tax on emissions should equal the marginal damage that pollution causes (Pigou, 1932, and Baumol and Oates, 1988). For the case of a monopolist, Buchanan (1969) has shown that the second-best policy is to impose a tax that is lower than the marginal environmental damage (see also Barnett, 1980). In the case of oligopolistic competition, recent work by Shaffer (1992), Simpson (1995), Katsoulacos and Xepapadeas (1995) and Lee (1999) generally concludes that the optimal emission tax should be set below marginal damages – this can be readily explained in terms of the market failures present: (i) over-production due to pollution and (ii) under-production due to market power. It is the trade-off between these two distortions that determines the size of the emission tax. This conclusion is generated within the context of a homogeneous good Cournot oligopoly with a fixed number of equally efficient firms. Moreover, the papers cited above examine a situation where firms do not engage in emission reduction (either through R&D that changes their technology of emissions or through end-of-pipe abatement techniques). The purpose of this paper is to examine optimal pollution taxation when firms produce differentiated goods and in addition engage in emission reduction under two different modes of market conduct: quanti-

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ty competition (Cournot) and price competition (Bertrand). In this way it fills a missing niche in the literature.

The case of price competition (Bertrand) with differentiated goods has received little attention, despite the fact that this is probably the most realistic and hence the most relevant market structure, the exception being a recent paper by Lange and Requate (1999). Lange and Requate (1999) examine three models of oligopolistic behavior: a price setting duopoly with differentiated goods, a Dixit-Stiglitz type model and a modified horizontal differentiation model. In the absence of any abatement/R&D activities on the part of the firms, they show that, in general, the second best optimal emission tax is less than marginal damages. In the present paper we build on this work and enrich the literature cited above by introducing emission reduction explicitly, in the form of environmental R&D. In doing so we use as a starting point a paper by Katsoulacos and Xepapadeas (1996) which we suitably adapt to the case of differentiated goods with either Cournot or Bertrand competition. Katsoulacos and Xepapadeas (1996) have analyzed the optimal policy scheme in the case of a pollution generating duopoly, where there are environmental R&D spillovers, firms compete *a la Cournot* and output is homogeneous. The optimal policy scheme they propose takes the form of an emission tax and a simultaneous subsidy towards environmental R&D. In the present paper, we consider the case of a duopoly with differentiated products and examine the optimal policy scheme under both Cournot and Bertrand competition. We are then in a position to further check the result obtained by Lange and Requate (1999) and more importantly to provide a detailed comparison of the two different modes of market conduct.[1]

Before proceeding it would be helpful to discuss briefly the market failures that are at work. First, taking the technology of production including emissions technology as given, because of imperfect competition there will be underproduction relative to the social optimum. Second, pollution, an environmental externality, gives rise to a second market failure making firms to over-produce. In principle, given a fixed number of firms, these two product market failures operate in opposite directions. In these circumstances, it is well known that an emission tax can be set so that the first-best will be achieved (e.g., see Shaffer, 1992). However, the emission tax does depend on the technologies used by firms. Next, suppose that technology can be altered by firms' R&D activities and for the sake of clarity concentrate on emission-reducing or environmental R&D: by engaging in R&D a firm can reduce its emissions and thus face a lower emission tax and consequently lower marginal costs. Suppose further that there are no involuntary leakages of information, i.e., R&D spillovers are absent perhaps as a result of a very effective patent system.[2] There is now an R&D market failure: a firm undertakes too little R&D due to its own output being low because (i) it produces only a fraction of industry output and (ii) industry output is too low due to imperfect competition. In addition, when firms act strategically (i.e., they choose R&D before output/price) they have a stronger incentive to invest in R&D as they attempt to gain an advantage over their rivals, e.g. by grabbing market share. However, in a wide class of cases, social returns to R&D exceed the private returns so that typically there is under-investment in R&D. In this context an R&D subsidy is one appropriate instrument to tackle the R&D market failure.[3]

We demonstrate that the optimal policy would comprise an R&D subsidy to deal with the R&D market failure and an emission tax to deal with the product market failures; this is irrespective of whether firms compete in price or quantity. Regarding the comparison between the two modes of market conduct our results are as follows: In Cournot competition, the emission

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[1] *For recent work in a setting of vertical product differentiation in terms of environmental quality see Lombardini-Riipinen (2002a, b).*

[2] *Introducing spillovers in the analysis adds computational complexity without changing qualitatively the characterization of the optimal design of policy which consists of an emission tax and an R&D subsidy.*

[3] *However, there could be cases where firms over-invest in R&D for strategic reasons, e.g., in Cournot competition and (nearly) homogeneous products, so that it is appropriate to tax R&D.*

tax is always lower than marginal damages, thus strenghtening the general result obtained for the case of homogeneous goods. With Bertrand competition, the emission tax is generally lower than marginal damages. However, for the case of (nearly) homogeneous products, the emission tax is equal to marginal damages, that is, it approaches the first-best tax. In other words, when the capacity constraints of firms are not binding,[4] the optimal emission tax is decreasing in the degree of product differentiation but is never larger than the first-best tax. Moreover, we show that the second-best tax under Bertrand competition always exceeds its counterpart under Cournot competition. With respect to the R&D subsidy we show that the comparison depends crucially on the extent of product differentiation and the emissions technology as captured by the magnitude of the initial emissions coefficient.

The paper is organized as follows. In section 2 we present the elements of the model used; in section 3 we provide the analysis when firms engage in output competition (Cournot) and in section 4 we present the case of price competition (Bertrand). Then in section 5 we carry out a comparison of the two modes of market competition in terms of the emission tax and R&D subsidy. Finally we provide some brief concluding remarks in the final section.

2. *The model*

We consider a duopoly where firms sell differentiated products and compete either by setting quantities or by setting prices. Following Dixit (1979) and Singh and Vives (1984), we assume that there is a representative consumer whose preferences for consumption of the two goods, q_1 and q_2, and the numeraire good, m, are described by a utility function separable and linear in the numeraire good, $U(q_1, q_2) + m$, with

$$U(q_1, q_2) = \alpha(q_1 + q_2) - (q_1^2 + 2\gamma q_1 q_2 + q_2^2)/2$$

where $\alpha > 0$ and $\gamma \in (0,1)$ is the parameter of product differentiation, inversely related to the degree of product differentiation. There are no income effects on the duopoly, and thus we can perform partial equilibrium analysis.[5] Notice that if $\gamma \to 1$ the goods become very similar (in the limit they become homogeneous) and if $\gamma \to 0$ the goods are perfectly differentiated (and independent). Market (inverse)demands are then

$$p_i = \alpha - q_i - \gamma q_j, \tag{1}$$

or, in direct rather than indirect form,

$$q_i = \frac{1}{1-\gamma^2}\left[\alpha(1-\gamma) - p_i + \gamma p_j\right] \tag{2}$$

$i, j = 1, 2$, $i \neq j$. For (2) to be well defined, products cannot be completely homogeneous. To simplify the analysis we assume that unit production costs, c, are constant while there are no fixed costs. Without loss of generality, we normalize unit costs by setting $c = 0$.

Production generates harmful emissions. We assume that emissions per unit of output, e_i, are given by

$$e_i = e_0 - z_i, \tag{3}$$

$i = 1, 2$, $i \neq j$, where z_i represents firm i's R&D effort in emission reduction (which we will refer to as environmental R&D), and e_0 is initial emissions per unit of output. To obtain an emission reduction of z_i a firm has to spend an amount $R_i = vz_i^2/2$, $v > 0$, with the parameter v capturing the relative efficiency of environmental R&D. Total emissions are $E = \sum_i e_i q_i$ while the environmental damage function is represented by $D(E)$ with $D' > 0$ and $D'' \geq 0$.

We model the interaction of the government and firms as a three-stage game; in other words, we are treating the government as an active player with full commitment powers to set the

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[4] *We do not consider capacity constrained firms in this paper. However, as pointed out by a referee, if the firms' capacities were cosntrained, their emissions should be taxed less.*

[5] *There is an important literature on general equilibrium under imperfect competition, e.g., Gabszewicz and Vial (1972), Novshek and Sonnenschein (1978), Mas-Colell (1982) and Myles (1989). It would be interesting to study the questions addressed in this paper within a general equilibrium framework. However, it is outside the scope of the present paper to do so. We leave this as a topic for further research.*

available policy tools: the emission tax and the R&D subsidy. Thus in the first stage of the game the government (or the regulator) taxes emissions at a rate t per unit of emissions and subsidizes (or taxes) R&D at a rate s per unit of R&D investment R_i, anticipating how firms will react to these policies.[6] Next, in the second stage firms simultaneously choose their R&D and then, in the final stage, they set output (Cournot competition) or price (Bertrand competition) taking as given the emission tax and the R&D subsidy set by the government. In the sections that follow we characterize the optimal policy mix that will emerge in the subgame perfect equilibrium of this three-stage game.

3. *Cournot competition*

In this section we concentrate on the case of Cournot competition in the last (output) stage of the three-stage game. Given the policy scheme (t, s) announced by the government and the choice of R&D effort in emission-reduction, z_i, firm i maximizes

$$\begin{aligned} &\pi_i(q_1, q_2; z_1, z_2, t, s) \\ &= p_i q_i - te_i q_i - (1-s)vz_i^2/2 \\ &= (\alpha - q_i - \gamma q_j)q_i - te_i q_i - (1-s)vz_i^2/2 \end{aligned}$$

which results in the following first-order condition

$$\alpha - 2q_i - \gamma q_j - te_i = 0$$

so that maximum second-stage profit is written as $\pi_i = q_i^2$. The f.o.c. has the natural interpretation that the additional revenue from producing one more unit of output is equal to the emission tax outlay on that unit of output. Solving this f.o.c. results in the Cournot-Nash equilibrium output per firm,

$$(4) \qquad q_i(z_i, z_j) = \frac{(\alpha - te_0)(2-\gamma) + 2tz_i - t\gamma z_j}{4-\gamma^2}.$$

In the preceding stage each firm chooses environmental R&D to maximize its overall profit, π_i (z_i, z_j; t, s)= q_i^2 (z_i, z_j) – (1 – s) vz_i^2. Differentiating this with respect to z_i, setting the result equal to zero, and then solving for the symmetric equilibrium yields the following expression for R&D effort.[7]

$$(5) \qquad z_c = \frac{4t(\alpha - te_0)}{\Delta_c}$$

where $\Delta_c \equiv (4 - \gamma^2)(2 + \gamma)v(1 - s) - 4t^2$.

From (5) note that: (i) if $t = 0$ then $z_c = 0$, i.e., irrespective of the R&D subsidy s, firms only undertake emission reduction if the government taxes emissions (Katsoulacos and Xepapadeas, 1996). Hence the incentive to engage in environmental R&D is present only as long as the environmental tax is positive; (ii) $\frac{\partial z_c}{\partial s} = \frac{1}{\Delta_c^2} 4vt(4-\gamma^2)(2+\gamma)(\alpha - te_0) > 0$, that is, given the emission tax, an increase in the subsidy will lead to an increase in environmental R&D and (iii) $\frac{\partial q_c}{\partial s} = \frac{t(2-\gamma)}{4-\gamma^2}\frac{\partial z_c}{\partial s} > 0$, an increase in the R&D subsidy will result in increased production as expected intuitively.

Next, substituting (5) into (4) we obtain the (symmetric) equilibrium output and price for given s and t

$$(6) \qquad \begin{aligned} q_c^* &= \frac{1}{\Delta_c} v(1-s)(\alpha - te_0)(4-\gamma^2) \\ p_c^* &= \alpha - (1+\gamma)q_c^*. \end{aligned}$$

Having described the behavior of firms for a given policy scheme (s, t) set by the government we now turn to the first stage where the government chooses the optimal policy scheme (in a second-best sense) to maximize social welfare. Welfare is defined in the standard way as total surplus minus production costs (in this case normalized to zero) and R&D costs minus environmental damage,[8]

[6] *In this paper we are concerned with the case of a committed government only. See Poyago-Theotoky and Teerasuwanajak (2002) for the role of commitment on emission taxation in a model of differentiated-goods oligopoly with pollution effects.*

[7] *Note that for given t and s (these are determined in the first stage) the second-order condition requires $v > 8t^2/(1-s)(4-\gamma^2)^2$ while the stability condition is satisfied for $\Delta_c > 0$ or $v > 4t^2/(1-s)(2+\gamma)(4-\gamma^2)$. Further, for positive environmental R&D, $z_c > 0$, we also need $t < \alpha / e_0$ for given t.*

[8] *For example, see Ulph (1996), Petrakis and Xepapadeas (1999) and Petrakis and Poyago-Theotoky (2002). Alternatively, we could have entered the damage function directly into the consumers' utility function in a separable manner without any change in the results.*

$$= U(q_i, q_j) - v/2\sum z_i^2 - D(E)$$
$$= 2\alpha q_c - (1+\gamma)q_c^2 - vz_c^2 - D(2e_c q_c)$$

where $D(\cdot)$ is the environmental damage function, $D' > 0$, $D'' \geq 0$. We are then in a position to determine the optimum tax and subsidy rate. The government maximizes W_c by choosing both t and s. The first-order condition requires that $\frac{\partial W_c}{\partial t} = 0 = \frac{\partial W_c}{\partial s}$, or

$$\frac{\partial W_c}{\partial t} = \alpha\frac{\partial q_c}{\partial t} - (1+\gamma)q_c\frac{\partial q_c}{\partial t} - vz\frac{\partial z_c}{\partial t} - D'e_c\frac{\partial q_c}{\partial t} - D'q\frac{\partial e_c}{\partial z_c}\frac{\partial z_c}{\partial t} = 0$$

and

$$\frac{\partial W_c}{\partial s} = \alpha\frac{\partial q_c}{\partial s} - (1+\gamma)q_c\frac{\partial q_c}{\partial s} - vz_c\frac{\partial q_c}{\partial s} - D'e_c\frac{\partial q_c}{\partial s} - D'q_c\frac{\partial e_c}{\partial z_c}\frac{\partial z_c}{\partial s} = 0.$$

Following a number of steps, which we present in detail in the Appendix, we obtain the optimal policy scheme given by

(7) $$t^* = D' - \frac{q_c}{e_c}$$

(8) $$s^* = 1 - \frac{4t^*}{(4-\gamma^2)D'}.$$

It is evident that the second-best tax is below marginal damages, $t^* < D'$, as expected. Note that the subsidy could be negative[9] (i.e., an R&D tax) depending on the extent of damage, D', and the degree of product differentiation as the term $4 / (4 - \gamma^2)$ is increasing in γ and takes values in the interval (1, 4 / 3).

Because of imperfect competition in the product market, firms will produce at a lower level relative to the social optimum so that the emission tax is set below marginal damage (e.g., Buchanan, 1969, Barnett, 1980); however, because of the R&D market failure firms invest the wrong amount in environmental R&D. Typically, there is underinvestment in R&D as social returns exceed private returns; in this instance an R&D subsidy would be optimal. However, there are cases where firms, acting in their own interest, may over-invest in R&D relative to the social optimum for strategic reasons so that in this case a negative R&D subsidy may be necessary;[10] this is more likely in the case of Cournot competition for relatively small environmental damage and less differentiated products (i.e., large γ). Thus, in general, the optimal policy scheme consists of an emission tax which is set below marginal damages and an R&D subsidy that induces firms to undertake the socially optimal environmental R&D (cf. Katsoulacos and Xepapadeas, 1996).

4. *Bertrand competition*

In this section we consider the case where firms choose prices instead of quantities. Recall that inverting the demand system (1) results in direct demands

$$q_i = \frac{1}{1-\gamma^2}\left[\alpha(1-\gamma) - p_i + \gamma p_j\right].$$

To facilitate comparisons with the Cournot case in the following sections let τ be the emissions tax and σ the R&D subsidy. Hence, profit per firm is given as

$$\pi_i(p_1, p_2; z_1, z_2, \tau) = p_i q_i - \tau e_i q_i - (1-\sigma)vz_i^2/2$$
$$= \frac{1}{1-\gamma^2}(p_i - \tau e_i)\left[\alpha(1-\gamma) - p_i + \gamma p_j\right] - (1-\sigma)vz_i^2/2.$$

[9] *Because of imperfect competition, firms do not take into account consumers' surplus so that there is a tendency for firms to under-invest in R&D, and hence this points to a positive R&D subsidy. This output-distortion effect is reflected in the fact that $t^* < D'$. However, when firms evaluate the effects of environmental R&D on their profits they take into account a strategic effect, not perceived by the regulator, which is positive and tends to make firms over-invest in R&D. This is captured by the term $4 / (4 - \gamma^2)$. When the strategic effect is quite important it will dominate the output-distortion effect and the subsidy will be negative.*

[10] *For example, in the context of international R&D competition (patent race) and a homogeneous good, Spencer and Brander (1983) show that an R&D tax is optimal. However, this result has been challenged by Beath et al. (1989).*

Maximizing this with respect to p_i yields

$$p_i(z_i, z_j; \tau, \sigma) = \frac{[(\alpha(1-\gamma) + e_0\tau)(2+\gamma) - \tau(2+\gamma\delta)z_i - (\gamma+2\delta)z_j]}{4-\gamma^2}. \quad (9)$$

In the preceding stage, each firm chooses its R&D, z_i, to maximize overall profit, which can be written as $\pi_i = \frac{(p_i - \tau e_i)^2}{1-\gamma^2} - (1-\sigma)vz_i^2/2$ by using the first-order condition from the price stage.[11] Maximizing overall profit with respect to z_i results in equilibrium environmental R&D[12]

$$z_b = \frac{2\tau(\alpha - e_0\tau)(2-\gamma^2)}{\Delta_b} \quad (10)$$

where $\Delta_b = (1-\sigma)\, v\, (1+\gamma)\, (2-\gamma)\, (4-\gamma^2) - 2\tau^2\, (2-\gamma^2)$.

In line with the Cournot case, from (10) we can deduce that (i) if $\tau = 0$ then $z_b = 0$, (this is the same as in the Cournot case: unless emissions are taxed firms will not invest in environmental R&D), (ii) $\frac{\partial z_b}{\partial \sigma} = \frac{2v\tau(\alpha - e_0\tau)(2-\gamma^2)(1+\gamma)(2-\gamma)(4-\gamma^2)}{\Delta_b^2} > 0$, that is, the effect of an increase in the subsidy is to increase R&D and (iii) $\frac{\partial p_b}{\partial \sigma} = -\frac{\tau(2+\gamma)}{4-\gamma^2}\frac{\partial z_b}{\partial \sigma} < 0$, i.e., the exact opposite of the Cournot case but having a similar effect, as expected.

Finally, substituting (10) into (9), we obtain the (symmetric) equilibrium price for given σ and τ

$$p_b^* = \frac{1}{\Delta_b}\left[v(1-\sigma)(\alpha - \alpha\gamma + e_0\tau)\right.$$
$$\left.(1+\gamma)(4-\gamma^2) - 2\tau^2\alpha(2-\gamma^2)\right] \quad (11)$$

while equilibrium output is given by

$$q_b = \frac{\alpha - p_b}{1+\gamma} = \frac{1}{\Delta_b}v(1-\sigma)(4-\gamma^2)(\alpha - e_0\tau). \quad (12)$$

In the first stage the government sets the emission tax, τ, and the R&D subsidy, σ, to maximize welfare, as defined in section 3, and given by

$$W_b = U(q_i, q_j) - v/2\sum z_i^2 - D(E)$$
$$= 2\alpha q_b - (1+\gamma)q_b^2 - vz_b^2 - D(2e_bq_b).$$

In line with the previous section, we just give the two first-order conditions for the maximization of W_b and present the optimal policy rule for the case of Bertrand competition, leaving the detailed derivations in the Appendix. The first-order condition for the maximization of W_b with respect to the emissions tax, τ, is

$$\frac{\partial W_b}{\partial \tau} = \alpha\frac{\partial q_b}{\partial \tau} - (1+\gamma)q_b\frac{\partial q_b}{\partial \tau} - vz\frac{\partial z_b}{\partial \tau} -$$
$$D'e_b\frac{\partial q_b}{\partial \tau} - D'q_b\frac{\partial e_b}{\partial z_b}\frac{\partial z_b}{\partial \tau} = 0$$

where from (12), $\frac{\partial q_b}{\partial \tau} = -\frac{1}{1+\gamma}\frac{\partial p_b}{\partial \tau}$, while the first-order condition with respect to the R&D subsidy, σ, is

$$\frac{\partial W_b}{\partial \sigma} = \alpha\frac{\partial q_b}{\partial \sigma} - (1+\gamma)q_b\frac{\partial q_b}{\partial \sigma} - vz_b\frac{\partial z_b}{\partial \sigma} -$$
$$D'e_b\frac{\partial q_b}{\partial \sigma} - D'q_b\frac{\partial e_b}{\partial z_b}\frac{\partial z_b}{\partial \sigma} = 0$$

and from (12) $\frac{\partial q_b}{\partial \sigma} = -\frac{1}{1+\gamma}\frac{\partial p_b}{\partial \sigma}$. As shown in the Appendix, the optimal policy mix for this case, in line with the case of quantity competition, consists of an emission tax

$$\tau^* = D' - \frac{(1-\gamma^2)q_b}{e_b} \quad (13)$$

and, an R&D subsidy

$$\sigma^* = 1 - \frac{2\tau^*(2-\gamma^2)}{(4-\gamma^2)D'} \quad (14)$$

Notice, that the second-best tax in the case of price competition is directly related to the extent of product differentiation, γ. Interestingly the emission tax is lower than marginal damages unless product differentiation is minimal,

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[11] The first-order condition is given by $p_i = \frac{1}{2}[\alpha\,(1-\gamma) + \gamma p_j + \tau e_i]$.

[12] Note that for given τ and σ the second order condition requires $v > \frac{2\tau^2(2-\gamma^2)}{(1-\sigma)(1-\gamma^2)(4-\gamma^2)}$ while the stability condition is satisfied for $\Delta_b > 0$ or $v > \frac{2\tau^2(2-\gamma^2)}{(1-\sigma)(1+\gamma)(2-\gamma)(4-\gamma^2)}$, Further, for positive environmental R&D, $z_b > 0$, it is also necessary that $\tau < \alpha / e_0$.

Note that when γ is close to one, v must be large in order to satisfy the second-order condition for an interior solution. In this instance the goods are close substitutes and competition is intense, as firms try to cut prices and capture market share. Suppose that v is small so that environmental R&D is not very costly. Then, a large R&D effort will give a firm a large price advantage so that it can win the price war. Thus v must be large to avoid such corner solutions.

i.e., for γ close to one, in which case it is approximately equal to marginal damages, or the first-best emission tax, $\tau^* \approx D'$.[13] Notice that this is also the case for a perfectly competitive market (Baumol and Oates, 1988). This novel result on the value of the emission tax is reminiscent of the 'Bertrand Paradox' in industrial organization, i.e., price being equal to marginal cost when the product is homogeneous despite firms having market power.[14]

In contrast to Lange and Requate (1999) we do not get their result that in case firms are very different the optimal tax will exceed marginal damages. In fact, that can never happen in our model which predicts that in this event the tax will be below marginal damages. This is because their result is obtained for very heterogeneous firms (in terms of cost and emission coefficients) in the context of a specific example whereas our firms are heterogeneous in terms of the degree of product differentiation and the result we obtain is rather general within the setting of a linear demand system. We would expect that in the case of convex or concave demand the results could be perhaps different and that we could obtain situations were the optimal tax exceeds marginal damages (e.g., see Lee, 1999) – however, within the present setting such an extension would be rather complex and lies outside the scope of the present paper.

Regarding the optimal R&D subsidy, contrary to the Cournot case, it cannot take negative values. To see this notice that the term $\frac{2(2-\gamma^2)}{(4-\gamma^2)}$ is decreasing in γ and takes values in the interval (2/3, 1) which taken together with $\tau^* \leq D'$ implies that the right-hand side of (14) is never greater than one. However, for γ close to one, i.e., when products are nearly homogeneous, the R&D subsidy becomes zero. In this case, recall that the emission tax is set equal to marginal damage so that social returns are equal to private returns; as a result there is no R&D market failure and hence no need for an R&D subsidy.

[13] *Notice that in this case v has to be large enough to satisfy the second-order condition, see footnote 10.*

[14] *When $\gamma \to 0$, the goods are independent so that each firm has a sort of local monopoly. It should then be no surprise that in both Cournot and Bertand competition the emission tax and the R&D subsidy are the same.*

5. A comparison

To make further progress in comparing the two modes of market conduct, i.e., price versus quantity competition, in terms of the emission tax and the R&D subsidy it is convenient to simplify and assume $D' = d$, i.e., to concentrate on the case of linear damages, for the remainder of the paper.

Using the relevant expressions for the optimal emission tax, i.e., (13) and (7), we find after some algebraic manipulation that

$$(15) \quad \tau^* - t^* = \frac{v(\alpha - de_0)\gamma^2}{e_0(1+\gamma)v - \alpha d}.$$

In deriving both τ^* and t^* we ensure that the relevant stability conditions are satisfied and that environmental R&D and emissions are positive; this is equivalent to the following two conditions being satisfied: (i) $\frac{\alpha}{e_o} > d$ and (ii) $v > \frac{\alpha d}{e_0(1+\gamma)}$, in addition to the relevant second-order conditions. Note that conditions (i) and (ii) also guarantee that (15) is positive, i.e., $\tau^* - t^* > 0$, for $\gamma \in (0, 1)$.[15]

The following proposition follows immediately:

Proposition 1: For linear marginal damages, $D' = d$, the second-best emission tax under Bertrand competition always exceeds the second-best emission tax under Cournot competition, $\tau^* > t^*$.

Recall that in the absence of taxation there is no incentive for firms to undertake emission-reducing R&D activities. Also, it is well known that, *ceteris paribus* (given the same level of R&D subsidy), firms produce more output in Bertrand competition than in Cournot competition so that emissions would be higher in the case of Bertrand competition. Hence the under-production market failure is more pronounced in the case of Cournot competition. However, there is also the market failure associated with the pollution externality so that the emission tax would be higher in this case. The emission tax

[15] *Note that when $\gamma \to 0$, $\tau \approx t$, i.e., when goods are very different so that firms become local monopolists, the tax rate is the same, as expected.*

is geared also towards correcting the pollution externality and thus, given that output (and hence emissions) is larger in the case of Bertrand competition, the emission tax is higher relative to Cournot competition. Recall also that except for the case of nearly homogeneous products and Bertrand competition the emission tax is always lower than the first-best which would necessitate the emission tax to be set equal to marginal damages.

Regarding the R&D subsidy the results are summarized in the following proposition (the proof is relegated in the Appendix).

Proposition 2: In the case of linear marginal damages, $D' = d$ and given an initial emissions coefficient, e_0, there exists a critical value of the product differentiation parameter, $\overline{\gamma}$, such that if $\gamma > \overline{\gamma}$, $\sigma^* > s^*$, i.e., the R&D subsidy under Bertrand competition exceeds the R&D subsidy under Cournot competition, and if $\gamma < \overline{\gamma}$, $\sigma^* < s^*$, i.e., the opposite is true. The critical product differentiation parameter, $\overline{\gamma}$, is decreasing in the initial emissions coefficient, e_0.[16]

Proposition 2 indicates that the R&D subsidy can be higher or lower in the two different market conduct regimes depending on the degree of product differentiation. Our intuitive explanation is as follows: There are two effects at work here. First, given that $\tau^* > t^*$ we would expect firms in Bertrand competition to undertake more environmental R&D as the emission tax acts as a mechanism to induce emission-reducing R&D (and thus lower marginal costs) and hence a reduced need for an R&D subsidy, so that $\sigma^* < s^*$. Second, when γ is high there is increased competition between firms as they offer more similar products and this competition is more intense in the Bertrand case; output (and hence emissions) generally rises so that a further incentive is needed for environmental R&D giving rise to $\sigma^* > s^*$. The first effect dominates for relatively low values of γ while the second effect dominates for high values of γ. Further, the higher the initial emissions coefficient, e_0, the smaller the critical product differentiation parameter, $\overline{\gamma}$, so that it becomes more and more the case for the R&D subsidy under Bertrand competition to exceed the R&D subsidy under Cournot competition.

[16] *The exact solution for* $\overline{\gamma}$ *is given by* $\overline{\gamma} = \frac{1}{2v(\alpha - de_0)} \left\{-de_0v + \sqrt{v}\sqrt{[v(de_0)^2 + 4(\alpha - de_0)[-4de_0v + a(3v + d^2)]]}\right\}$ *(see Appendix).*

6. *Concluding remarks*

In this paper we have examined the optimal policy rules (in a second-best sense) under conditions of imperfect competition and product differentiation where firms engage either in quantity competition (Cournot) or in price competition (Bertrand) and undertake environmental R&D to reduce their emissions.

The optimal policy mix consists of an emission tax and an R&D subsidy. We have found that in general the emission tax is lower that marginal damages in line with previous results for the case of homogeneous products. However, for the case of Bertrand competition and relatively undifferentiated products we have shown that the emission tax is equal to marginal damages, i.e., the first-best tax. In addition, by way of an example, where we restrict the analysis to linear marginal damages, we have shown that the second-best tax under Bertrand competition always exceeds the second-best tax under Cournot competition. A similar comparison for the case of the R&D subsidy reveals that whether the R&D subsidy is higher or lower in quantity or price competition depends crucially on the degree of product differentiation and on initial emissions.

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7. Appendix

7.1 Derivation of expressions (7) and (8)

The first order condition for the maximization of social welfare, W_c, requires that $\frac{\partial W_c}{\partial t} = 0$ and $\frac{\partial W_c}{\partial s} = 0$. In particular,

$$\text{(A1)} \quad \frac{\partial W_c}{\partial t} = \alpha \frac{\partial q_c}{\partial t} - (1+\gamma) q_c \frac{\partial q_c}{\partial t} - vz \frac{\partial z_c}{\partial t} - D' e_c \frac{\partial q_c}{\partial t} - D' q \frac{\partial e_c}{\partial z_c} \frac{\partial z_c}{\partial t} = 0$$

where from (4)

$$\frac{\partial q_c}{\partial t} = -\frac{(2-\gamma) e_c}{4-\gamma^2} + \frac{(2-\gamma)t}{4-\gamma^2} \frac{\partial z_c}{\partial t}.$$

Substituting the above into (A1), noting that $p_c = \alpha - (1 + \gamma)\, q_c$ and collecting terms yields

$$\text{A2)} \quad \frac{\partial W_c}{\partial t} = -\frac{(2-\gamma) e_c}{4-\gamma^2}(p_c - D' e_c) + \left\{ (1+\delta) \left[(p_c - D' e_c) \frac{(2-\gamma)t}{4-\gamma^2} + D' q_c \right] - v z_c \right\} \frac{\partial z_c}{\partial t} = 0.$$

Moreover

$$\text{(A3)} \quad \frac{\partial W_c}{\partial s} = \alpha \frac{\partial q_c}{\partial s} - (1+\gamma) q_c \frac{\partial q_c}{\partial s} - v z_c \frac{\partial q_c}{\partial s} - D' e_c \frac{\partial q_c}{\partial s} - D' q_c \frac{\partial e_c}{\partial z_c} \frac{\partial z_c}{\partial s} = 0$$

where from (4),

$$\frac{\partial q_c}{\partial s} = \frac{t(2-\gamma)}{4-\gamma^2} \frac{\partial z_c}{\partial s}.$$

Using this in (A3) and simplifying yields

$$\text{(A4)} \quad \frac{\partial W_c}{\partial s} = \left\{ \left[p_c - D' e_c) \frac{t(2-\gamma)}{4-\gamma^2} + D' q_c \right] - v z_c \right\} \frac{\partial z_c}{\partial s} = 0.$$

Recall that $\frac{\partial z_c}{\partial s} > 0$, so that from (A4) it must be true that

$$\text{(A5)} \quad \left[(p_c - D' e_c) \frac{t(2-\gamma)}{4-\gamma^2} + D' q_c \right] - v z_c = 0.$$

From (A5) and (A2) we then have

$$\text{(A6)} \quad p_c - D' e_c = 0 \Rightarrow p_c = D' e_c$$

and from (A6) and (A5) we obtain

$$\text{(A7)} \quad D' q_c - v z_c = 0 \Rightarrow D' q_c = v z_c$$

Equations (A6) and (A7) determine the structure of the optimal policy scheme: equation (A6) says that at the optimum, price must be equal to marginal environmental damage per unit of output and equation (A7) says that at the optimum, the marginal R&D cost must equal marginal savings in environmental damage from R&D since $D'q_c = -(\partial e_c/\partial z)\, D'q_c$. The optimal policy entails a combination of an emissions tax and an R&D subsidy/tax, (t^*, s^*); at the optimal policy combination, the firms' first-order conditions must satisfy the optimum conditions identified above. Applying these[17] we have (at the symmetric equilibrium)

$$\text{(A8)} \quad q_c = \alpha - (1+\gamma) q_c - t e_c \Rightarrow p_c = q_c + t e_c$$
$$\frac{4t}{4-\gamma^2} q_c - (1-s) v z_c = 0.$$

From the optimal conditions, equations (A6), (A7) and (A8), we obtain the optimal emissions tax,

$$t^* = D' - \frac{q_c}{e_c}$$

and by using (A7), the optimal subsidy

$$s^* = 1 - \frac{4t^*}{(4-\gamma^2) D'},$$

which are expressions (7) and (8) in the main text respectively.

7.2 Derivation of expressions (13) and (14)

The first-order condition for the maximization of W_b with respect to the emissions tax, τ, is as follows:

$$\text{(A9)} \quad \frac{\partial W_b}{\partial \tau} = \alpha \frac{\partial q_b}{\partial \tau} - (1+\gamma) q_b \frac{\partial q_b}{\partial \tau} - vz \frac{\partial z_b}{\partial \tau} - D' e_b \frac{\partial q_b}{\partial \tau} - D' q_b \frac{\partial e_b}{\partial z_b} \frac{\partial z_b}{\partial \tau} = 0$$

[17] *In detail these are:* $\partial \pi_i / \partial q_i = \alpha - 2q_i - \gamma q_j - t e_i$ *and* $\partial \pi_i / \partial z_i = 2 q_i \frac{\partial q_i}{\partial z_i} - (1-s)\, v z_i$.

where from (12) we have

$$\frac{\partial q_b}{\partial \tau} = -\frac{1}{1+\gamma}\frac{\partial p_b}{\partial \tau}$$

and

$$\frac{\partial p_b}{\partial \tau} = \frac{(2+\gamma)e_b}{4-\gamma^2} - \frac{\tau(1+\delta)(2+\gamma)}{4-\gamma^2}\frac{\partial z_b}{\partial \tau}$$

Substituting the above into the (A9) and collecting terms results in

$$\text{(A10)}\quad \frac{\partial W_b}{\partial \tau} = -\frac{(2+\gamma)e_b}{(1+\gamma)(4-\gamma^2)}(p_b - D'e_b) + \left\{(1+\delta)\left[(p_b - D'e_b)\frac{(2+\gamma)\tau}{(1+\gamma)(4-\gamma^2)} + D'q_b\right] - vz_b\right\}\frac{\partial z_b}{\partial \tau} = 0$$

Next, consider the f.o.c. with respect to the R&D subsidy, σ,

$$\text{(A11)}\quad \frac{\partial W_b}{\partial \sigma} = \alpha\frac{\partial q_b}{\partial \sigma} - (1+\gamma)q_b\frac{\partial q_b}{\partial \sigma} - vz_b\frac{\partial z_b}{\partial \sigma} - D'e_b\frac{\partial q_b}{\partial \sigma} - D'q_b\frac{\partial e_b}{\partial z_b}\frac{\partial z_b}{\partial \sigma} = 0$$

where from (12) we have

$$\frac{\partial q_b}{\partial \sigma} = -\frac{1}{1+\gamma}\frac{\partial p_b}{\partial \sigma} = -\frac{\tau(2+\gamma)}{4-\gamma^2}\frac{\partial z_b}{\partial \sigma} < 0$$

so that the f.o.c. becomes

$$\text{(A12)}\quad \frac{\partial W_b}{\partial \sigma} = \left\{\left[(p_b - D'e_b)\frac{\tau(2+\gamma)}{(1+\gamma)(4-\gamma^2)} + D'q_b\right] - vz_b\right\} = 0$$

Since $\frac{\partial z_b}{\partial \sigma} > 0$, (A12) implies that the expression in brackets is zero, i.e.,

$$\text{(A13)}\quad \left[(p_b - D'e_b)\frac{\tau(2+\gamma)}{(1+\gamma)(4-\gamma^2)} + D'q_b\right] - vz_b = 0$$

Next, from (A13) and (A10) we have

(A14) $p_b - D'e_b = 0 \Rightarrow p_b = D'e_b$

and from (A14) and (A13)

(A15) $D'q_b - vz_b = 0 \Rightarrow D'q_b = vz_b$

In line with the Cournot case, (A14) and (A15) describe the optimal policy mix. To proceed further, notice that from the first-order condition for the price-setting stage ($\alpha(1-\gamma) - 2p_i + \gamma p_j + \tau e_i = 0$) and the expression for the demand function, at the symmetric equilibrium we have

$$\text{(A16)}\quad p_b = \alpha(1-\gamma) - p_b(1-\gamma) + \tau e_b = (1-\gamma^2)\,q_b$$

while profits are $\pi = (p_b - \tau e_b)^2/(1-\gamma^2) - (1-\sigma)\,vz^2/2$, so that, after some manipulation, the first-order condition for the R&D stage reduces to

$$\text{(A17)}\quad \frac{2\tau(2-\gamma^2)}{4-\gamma^2}q - (1-\sigma)vz = 0.$$

Thus, from the optimal conditions (A14), (A15) and (A16), (A17), we obtain the optimal emission tax

$$\tau^* = D' - \frac{(1-\gamma^2)q_b}{e_b}$$

and the optimal R&D subsidy

$$\sigma^* = 1 - \frac{2\tau^*(2-\gamma^2)}{(4-\gamma^2)D'},$$

i.e., expressions (13) and (14) respectively in the main text.

7.3 Proof of Proposition 2

Using the relevant expressions for the optimal R&D subsidy, i.e., (14) and (8), we find after some algebraic manipulation that

$$\sigma^* - s^* = \frac{-\{2\gamma^2[de_0v(-4+\gamma(\gamma-1)) + a(d^2 - v(-3+\gamma^2))]\}}{d(4-\gamma^2)[e_0v(1+\gamma) - \alpha d]}$$

where the denominator is positive by condition (ii) in the main text. Hence

$$\sigma^* - s^* \gtrless 0 \quad iff$$

$$de_0v(4+\gamma-\gamma^2) - \alpha[d^2 - v(\gamma^2-3)] \gtrless 0.$$

Note that the solution to $\sigma^* - s^* = 0$ gives two solutions for γ, one of which is negative and hence discarded the other is the critical product differentiation parameter, $\overline{\gamma}$, given by

$$\overline{\gamma} = \frac{1}{2v(\alpha - de_0)} \Big\{ -de_0 v + \sqrt{v}\sqrt{[v(de_0)^2 + 4(\alpha - de_0)[-4de_0 v + \alpha(3v + d^2)]]} \Big\}$$

Further, notice that $\lim_{\gamma \to 0} \sigma^* - s^* = 0$ while $\lim_{\gamma \to 1} \sigma^* - s^* = \frac{2(\alpha d^2 + 2\alpha v - 4de_0 v)}{3d(\alpha d - 2ve_0)} \gtrless 0$. Hence, by continuity there exists $\overline{\gamma}$ such that if $\gamma > \overline{\gamma}$ then $\sigma^* > s^*$ and if $\gamma < \overline{\gamma}$ then $\sigma^* < s^*$. Further, it can be established that $\frac{\partial \overline{\gamma}}{\partial e_0} < 0$. ■