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BANK MERGERS AND THE FRAGILITY OF LOAN MARKETS*

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We address the question of whether competition makes loan markets more fragile in the sense of increasing the equilibrium bankruptcy risk of firms. This is done using a model of the interaction between the concentration of the banking sector and the investment strategies of imperfectly competitive product market firms. It is shown how a merger between two competing bilateral monopoly banks will typically decrease the interest rate and increase the investment volumes of firms if the investment decisions are strategic complements. Under plausible conditions this implies that a merger will lessen, not aggravate, the fragility of loan markets. (JEL: G21, G33, G34)

1. Introduction

In the 1980s the wave of financial deregulation swept swiftly over many European countries which until then had highly regulated financial markets. The deregulation process was reinforced by one of the main goals of the financial integration within the framework of the European Union, namely to encourage competition in banking.[1] At the same time as the financial integration within the framework of the European Union has encouraged competition in banking, the recent banking crisis in many countries has triggered a wave of substantial bank mergers in the aftermath of financial deregulation. These have been considered as a way for governments to deal with troubled banks and the governments have actively tried

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[1] *Historically, domestic banks have been protected from competition by stringent regulatory requirements. However, according to the prevailing "home country doctrine" of the European Union, a bank which is registered in any European country is eligible to open branches and to offer financial services in any other member country (see, for example, Mayer and Vives, 1993).*

to encourage mergers of sick banks into healthy ones (see, for example, The Economist, 1995).

In recent times we have seen worldwide an unprecedented high level of merger activity in the banking sector (for updated evidence, see Barfield, 1998). The U.S. banking industry has been consolidating rapidly which has led to increased concentration (see e.g. Boyd and Graham, 1996, for details). As is concluded in the study of The European Central Bank, [ECB] (1999), increased competition can be expected to have a significant impact on the risks incurred by banks. In reflection of a view according to which there would be a tradeoff between competition and financial stability, national authorities in many European countries have approved and, in some cases even encouraged, restructuring of the banking industry whereby the national concentration measures of the banking market have increased in most EU-countries (for evidence, see Tables 3.1–3.3 in ECB, 1999). In Europe mergers among commercial banks have so far taken place predominantly within national markets rather than as cross-border consolidations (see Danthine et. al., 1999, in particular their Tables 3.1 and 3.2, for extensive evidence that the degree of cross-border banking penetration in Europe has remained very modest). The introduction of the EMU is expected to further speed up the consolidation process in the European banking markets.

Conventional wisdom suggests that competition eliminates restrictive practices and reduces margins between borrowing and lending rates thereby improving the performance of the banking industry. The banking industry, however, has several idiosyncratic features, making it hard to evaluate the consequences of increased banking competition as it is not possible to rely directly on general insights from the traditional literature in industrial economics.

A number of recent contributions has analyzed aspects of the relationship between lending market structure and performance of the banking industry. Broecker (1990) and Riordan (1993) have studied the consequences of adverse selection due to the unobserved quality of borrowers. They argue that increased competition may make the adverse selection problems more severe when borrowers that have been rejected at one bank can apply for loans at other banks so that the pool of funded projects show lower average quality as the number of banks increases.[2] Shaffer (1998) has extended the analysis of winner's curse problems in lending and reported empirical evidence about the nature and magnitude of these effects. Contrary to the contributions mentioned above, by identifying the intensity of competition with the degree of product differentiation Villas-Boas and Schmidt-Mohr (1999) have demonstrated how banks facing stronger competition may expose credit applicants to more precise screening under asymmetric information. The central mechanism behind the result of Villas-Boas and Schmidt-Mohr relies on the argument that with stronger competition the banks have to compete more aggressively for the profitable projects. Another approach has investigated some important aspects of the relationship between lenders' market power and the agency cost of debt finance. In a credit-rationing model that take a Schumpeterian R&D perspective Petersen and Rajan (1995) have argued that more intense credit market competition may induce banks to reduce their lending in an initial stage (the asymmetric information period) because they are less able to extract borrower surplus at a subsequent stage (the complete information period) so that lenders in a more competitive market may be forced to initially charge higher interest rates than lenders with more market power.

The present study is focused on the question: How does a particular form of competition in the loan market affect the lending rate charged, the investment decisions of firms in the product market and, in consequence, the default risk of loans? We concentrate on a simple economy with two interdependent firms with access to investment projects in the product market and compare the cases of a banking industry with

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[2] Gehrig (1999) extends this approach within the framework of a model making it possible to explore the relationship between the incentives of banks for costly information acquisition based on ex ante monitoring efforts and the market structure of the lending industry.

one and two banks, respectively. Firms are assumed to be tied to their banks in such a way that they are effectively prohibited from switching banks so that our analysis focuses on a pair of bilateral monopolies competing with each other. The effects of a banking merger are identified with the consequences for interest rates, investment volumes and bankruptcy risks of a reduction in the number of banks from two to one.[3]

Even though it might be rare to find banking markets with such a high degree of concentration as exhibited in our analysis, the comparison we carry out captures some crucial qualitative implications of bank mergers. Our model seems particularly realistic for countries with banking industries dominated by main-bank relationships and high switching costs. Indeed, there are recent examples which fit our model almost quite literally. The merger between Union Bank of Finland (UBF) and Kansallis-Osake-Pankki (KOP) in early 1995 created a banking giant holding as much as 45% of Finnish deposits and up to 60% of new loans to Finnish small and mid-size corporations. In light of the recent merger cases involving CIBC and Toronto-Dominion as well as Royal Bank of Canada and Bank of Montreal our model should also be relevant from, for example, a Canadian perspective.

Brander and Lewis (1986) initiated research on the linkages between imperfectly competitive product markets and the debt-equity positions of firms. They showed how, in the presence of limited liability, debt induces the firm to an aggressive output strategy. Poitevin (1989) made use of such a strategic relationship to demonstrate how firms belonging to the same imperfectly competitive industry can noncooperatively sustain some degree of collusion by borrowing from the same bank. In Poitevin's model firms borrow in order to finance production of *fixed* size under circumstances where the production decisions are strategic substitutes and where the firms face firm-specific uncertainty. Our analysis differs from Poitevin's in three respects. Firstly, we allow for *endogenously determined loan sizes*. Secondly our analysis captures strategic interaction where the investment decisions are strategic complements. We compare the situation where firms in a duopoly with interrelated projects facing common uncertainty borrow from the same bank with the configuration where firms in a duopoly borrow from two competing banks. Thirdly, we assume that both the firms and the banks face common uncertainty, which enters the profit function of the firms in a multiplicative way (assumption of symmetric information). We postulate a strategic interaction in two stages such that the banks commit themselves to lending rates in the first stage and given these lending rates the firms subsequently decide on their levels of investments in the second stage. Hence we focus on a subgame perfect equilibrium in a two-stage game.

It is shown that under fairly mild conditions a merger between two bilateral monopoly banks would decrease the interest rate and increase the investments of a downstream industry if the investments are strategic complements. An intuitive explanation goes as follows. An increase in the lending rate has two opposite effects on the bank's profits: a negative one, because it induces a reduction of demand, and a positive one, because of the higher return on each unit money used to finance the projects. If the investment decisions of firms are strategic complements, an increase in the lending rate to one of the projects reduces the investment not only for the project in question, but also for the rival's project. Thus, the strategic complementarity strengthens the negative effect associated with an increase in the lending rate. If the banks merge, the resulting monopoly will internalize this effect by charging a lower interest rate than the competing banks. We also provide plausible sufficient conditions under which a bank merger would decrease the fragility of the loan market in the sense of decreasing the equilibrium bankruptcy risks of firms.

The product market interaction between firms investing in risky projects is introduced in section 2. Section 3 explores the implications

[3] *A substantial part of the empirical research on bank mergers has focused on potential economies of scale and scope as a motive for mergers (see e.g. Berger, Hanweck and Humphrey, 1987). Our analysis can be viewed as a complement to this traditional approach.*

of a bank merger on lending rates as well as on total industry investment and delineates the implications for financial fragility by investigating how such a merger will affect the equilibrium bankruptcy risk of firms. The paper ends up with a brief concluding section.

2. Debt-financed risky investments

2.1 Product market interaction

Consider two identical firms, which compete in the product market and both have access to an investment project with an uncertain return. If firm i (i = 1,2) invests x_i while its competitor j invests x_j the project yields $\theta\pi^i(x_i, x_j)$ for firm i provided that the state of nature turns out to be θ. We assume a continuum of possible states of nature θ distributed over the interval $[\theta_L, \theta_H]$ according to a cumulative distribution function $F(\theta)$ with the corresponding density function $f(\theta)$. Thus the firms face common uncertainty, which enters the profit realization in a multiplicative way.[4] The investment technology is presented in the following assumption.[5]

Assumption A: The revenues (net of the stochastic shock θ) of firm i from the project satisfy

$$(A1)\ \pi^i_i(x_i, x_j) > 0, \quad (A2)\ \pi^i_{ii}(x_i, x_j) < 0,$$
$$(A3)\ \pi^i_i(x_i, x_j) > \left|\pi^i_j(x_i, x_j)\right|.$$

Assumptions (A1) and (A2) indicate that the revenue of firm i's project is an increasing and strictly concave function of its own investment. Assumption (A3) captures the idea that "own effects" dominate over "cross-effects".

Since the goal of the present paper is to analyze how competition between bilateral monopoly banks will affect interest rates and investment decisions, and thereby also the risk exposure, of a representative product market industry, we find it justified to assume that the firms have no capital of their own so that the investment projects have to be fully financed with debt from a bank. We will compare two separate banking regime configurations. In the first situation there are two banks, where the two firms in the product market are not able to switch from one bank to another. In the alternative scenario we focus on the case with a bank monopoly offering loans to both the product market duopolists.

Our restriction of competition to that taking place between two competing bilateral monopoly banks in a situation where two firms in the product market are locked into a bank-client relationship should be thought of as a recognition of high switching costs as a central feature of lending markets. Durable relationships between banks and their clients represent a widely observed phenomenon in credit markets.[6] Also the theoretically oriented literature has offered explanations for such a view. For example, Dell'Ariccia (1998) offers a formal model of how informational asymmetries in lending markets can create barriers to entry even in the absence of exogenous fixed costs. Switching costs also play a central role in the important model by Dasgupta and Titman (1996), which deals with the relationship between capital structure and price decisions. Recently, Bolton and Scharfstein (1996) have offered an incomplete contract explanation for concentrating the loans to one common creditor. The firms tend to borrow from one creditor when default risks and asset complementarities are high. Finally, Detragiache et. al (1997) have argued that multiple banking should be less advantageous in countries, where the banking system is more fragile and where the loan recovery process, in the case of a loan default, is more efficient.

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[4] Dasgupta and Titman (1996) have demonstrated that the consequences of the strategic use of debt might depend on the type and nature of uncertainty facing the product market industry. Since that relationship is outside the focus of our attention, our model exhibits the simplest possible form of common uncertainty.

[5] Partial derivatives are denoted by subscripts.

[6] See e.g. Detragiache, Garella and Guiso (1997) for evidence that U.S. small businesses typically borrow from one bank only. This pattern, however, is not uniform. A striking feature of Italian firms, for instance, is that they ordinarily do business with a variety of different banks.

2.2 Investments with a banking duopoly

In this section we assume that the banks have made their lending rate commitments and we concentrate on the investment decisions of two competing firms maximizing their expected returns. Thus, we analyze the investment equilibrium resulting from the strategic interaction between the firms in the product market, while taking the lending rates as given. Ex ante the firms and the banks face identical aggregate uncertainty, which in our model is separated from the investment technology.

Because the investment is financed with debt, there will be a surplus for the investing firm only when the state of nature is sufficiently good to cover the debt. In a standard debt contract the "breakeven" state of nature η_i, in which firm i is just able to remain solvent, is defined by

$$(1)\quad \eta_i(x_i, x_j, R_i) = \frac{R_i x_i}{\pi^i(x_i, x_j)},$$

where r_i is the interest rate so that $R_i = 1 + r_i$ is the lending factor.

The "breakeven" state of nature depends on the interest rate as well as on the investment level of both the firm itself and of its rival, i.e. $\eta_i = \eta_i(x_i, x_j, R_i)$. The firm remains solvent for states of nature satisfying $\theta \geq \eta_i$, while there is bankruptcy when $\theta < \eta_i$. Consequently, the probability of bankruptcy is given by $F(\eta_i)$. If bank i commits itself to charge a lending factor R_i from firm i, then under the limited liability this firm is protected from negative profits and it will make its investment decision in order to maximize

$$(2)\quad V^i(x_i, x_j) = \int_{\eta_i}^{\theta_H} (\theta \pi^i(x_i, x_j) - R_i x_i)\, dF(\theta).$$

Maximization of (2) (for i=1,2) yields the implicit reaction functions

$$(3)\quad V_i^i(x_i, x_j) = \int_{\eta_i}^{\theta_H} (\theta \pi_i^i(x_i, x_j) - R_i)\, dF(\theta) = 0,$$

which can be rewritten as

$$\int_{\eta_i}^{\theta_H} \theta \pi_i^i(x_i, x_j)\, dF(\theta) = R_i[1 - F(\eta_i)].$$

The left-hand side denotes the marginal revenue increase from an additional unit of investment adjusted to the average among those states of nature in which the firm is solvent. The right-hand side in turn denotes the marginal cost increase in debt from an additional unit of investment for those states of nature, where the firm can fully afford to pay back its debt.

Differentiation of (1) with respect to x_i shows that

$$\frac{\partial \eta_i}{\partial x_i} = \frac{R_i}{\pi^i}(1 - e^{ii}) > 0,$$

because the scale elasticity of investment, $e^{ii}(x_i, x_j)$, satisfies $0 < e^{ii}(x_i, x_j) = \dfrac{x_i \pi_i^i(x_i, x_j)}{\pi^i(x_i, x_j)} < 1$ due to (A2). Analogously, it is easy to show that firm i's "breakeven" state of nature increases with R_i, while the effect of its rival's investment depends on the sign of π_j^i so that we have

$$\frac{\partial \eta_i}{\partial R_i} > 0 \text{ and } sign\left\{\frac{\partial \eta_i}{\partial x_j}\right\} = -sign\,\{\pi_j^i\}$$

Following Clemenz (1986, p. 132) we can rewrite FOC (3) according to

$$(3')\quad \pi_i^i(x_i, x_j)\left[\eta_i + \frac{\int_{\eta_i}^{\theta_H}(1 - F(\theta))d\theta}{1 - F(\eta_i)}\right] - R_i = 0$$

and defining $H(\eta_i) = \dfrac{\int_{\eta_i}^{\theta_H}(1 - F(\theta))d\theta}{1 - F(\eta_i)}$, makes it possible to simplify (3') as

$$(4)\quad R_i(e^{ii}(x_i, x_j) - 1) + \pi_i^i(x_i, x_j) H(\eta_i) = 0.$$

This formulation is convenient as it shows how fairly mild restrictions both on the profit function and on the cumulative distribution function justify the first-order approach.

Lemma 1: FOC (4) is a sufficient condition for optimal investment behavior if *(B1)* $\partial e^{ii}(x_i, x_j)/\partial x_i \leq 0$ and *(B2)* the hazard ratio of F, $f(\eta_i)/[1 - F(\eta_i)]$, is increasing.

For the proof we refer to the Appendix. Assumption (B1) means that the scale elasticity is a non-increasing function of the firm's own investment, which can be regarded as a reasonable assumption. The hazard ratio in Assumption (B2) describes the probability of reaching the breakeven state of nature conditional on not having reached that state earlier.[7]

Corollary 1: Under conditions (A2) and (B2) an increase in the lending rate will decrease the firm's expected marginal return on investment.

Proof: Differentiating (4) with respect to the interest factor gives

$$(5)\quad V^i_{iR_i} = (e^{ii}(x_i, x_j) - 1) + \pi^i_i(x_i, x_j) H'(\eta_i) \frac{\partial \eta_i}{\partial R_i} < 0.$$

The sign of (5) follows from the concavity of the profit function ($e^{ii} < 1$), from (B2) and from the fact that $\partial\eta/\partial R_i > 0$. *QED*

Provided that conditions (B1) and (B2) hold, the Nash equilibrium in investments is characterized by the system of equations

$$(6)\quad V^i_i(x^*_i, x^*_j) = 0$$

$$(7)\quad V^j_j(x^*_j, x^*_i) = 0.$$

(6) and (7) are the equilibrium conditions for given levels of the interest factors R_i and R_j. In what follows our analysis of the industry equilibrium is restricted to situations which satisfy the standard conditions for stability so that there is a unique investment equilibrium. Technically these conditions are $\Delta_V = V^i_{ii} V^j_{jj} - V^i_{ij} V^j_{ji} > 0$, where $V^i_{ii} < 0$ (i=1,2) due to the second-order conditions.

By totally differentiating the system of equations defined by (6) and (7) with respect to R_i we find that

$$(8)\quad \frac{\partial x^*_i}{\partial R_i} = -\frac{V^i_{ii} V^i_{iR_i}}{\Delta_V}$$

and

$$(9)\quad \frac{\partial x^*_j}{\partial R_i} = -\frac{V^j_{ji}}{V^j_{jj}} \frac{\partial x^*_i}{\partial R_i}.$$

For the analysis of (8) and (9) we differentiate the first-order condition (4) with respect to x_j to find that

$$(10)\quad V^i_{ij} = R_i e^{ii}_j + \pi^i_{ij} H(\eta_i) - \pi^i_i H'(\eta_i) \frac{R_i x_i \pi^i_j}{(\pi^i)^2}.$$

We employ the following assumption.

Assumption C: The scale elasticity of firm i is a non-decreasing function of the investment level of the rival firm j so that

$$(C1)\quad e^{ii}_j = \frac{\partial e^{ii}(x_i, x_j)}{\partial x_j} = \frac{x_i(\pi^i \pi^i_{ij} - \pi^i_i \pi^i_j)}{(\pi^i)^2} \geq 0.$$

Condition (C1) means that a firm's relative marginal return on investment does not decrease as the rival's investment increases. For example, a combination of conditions $\pi^i_j \leq 0$ and $\pi^i_{ij} \geq 0$ would be sufficient for (C1) to hold. Also a Cobb-Douglas function of the form $\pi^i(x_i, x_j) = x_i^\beta x_j^{1-\beta}$, where $0 \leq \beta \leq 1$, satisfies condition (C1).

Lemma 2: If the investments are strategic complements ($\pi^i_{ij} > 0$) and if conditions (B2) and (C1) hold, firm i's expected marginal return on investment is an increasing function of its rival's investment ($V^i_{ij} > 0$).

Proof: Follows directly from combining (10) and Lemma 1. *QED*

Consequently, conditions (B2) and (C1) are sufficient to make sure that we can extend the property of the investments being strategic complements in the ordinary sense to hold also in situations where these investments are financed by debt under limited liability.

One can distinguish between technological complementarities and complementarities induced by to aggregate demand externalities. The standard duopoly R&D race discussed e.g. in Harris and Vickers (1987) is a good example of a model where the investment decisions are strategic complements due to technological externalities[8] (see also Bagwell and Staiger, 1994,

[7] *Assumption (B2) is commonly used, for example, in the principal-agent literature as the monotone likelihood-ratio condition (see Rogerson, 1985), and as Jorgensen, McCall and Radner (1967) show, it is satisfied for a wide range of distribution functions.*

[8] *In a paper slightly related to ours, Clemenz (1991) has focused on some aspects which are of relevance for projects with interrelated returns. His article has a particular emphasis on R&D projects.*

for other characterizations). A potentially even more convincing justification for the assumption of strategic complementarities is based on aggregate demand externalities, whereby an expansion of firm i's investments would benefit firm j. Such aggregate demand externalities are particularly relevant under circumstances with imperfectly competitive firms which are large relative to the size of the economy (see Matsuyama, 1995). One can consider our model to capture the aggregate demand externalities in a reduced form. In a recent study, controlling for industry effects and time effects, Stenbacka and Tombak (1999) offer empirical evidence across a large number of industries which supports the notion that the investment decisions of large competing firms are strategic complements.

We are now able to evaluate (8) and (9). Based on Lemma 2 and the assumption of stability it must hold that

$$(11)\quad \frac{\partial x_i^*}{\partial R_i} < \frac{\partial x_j^*}{\partial R_i} < 0.$$

Thus, an increase in the lending rate levied on firm i will reduce the investment of both firm i and its rival because of strategic complementarity between their investments. But, as a result of the stability condition, the reduction in firm i's own investment will be larger than that of its rival.

2.3 Investments with a banking monopoly

Since we intend to compare the investment sensitivity to lending rate changes between a banking monopoly and duopoly we now turn to analyze a bank monopoly. A banking monopoly finds it optimal to charge the identical duopolists in the product market a common lending factor R. Keeping the notation otherwise unchanged, firm i will then decide on its investment level in order to maximize

$$(12)\quad V^i(x_i, x_j) = \int_{\eta_i}^{\theta_H} (\theta \pi^i(x_i, x_j) - Rx_i)\, dF(\theta).$$

Following the approach of the previous section we can now characterize the impact of the lending rate on the equilibrium investment decisions. Total differentiation of the first-order conditions for the investments reveals that

$$(13)\quad \frac{\partial x_i^*}{\partial R} = -\frac{V_{iR}^i (V_{ii}^i - V_{ji}^j)}{\Delta_V} < 0,$$

where we have made use of (5) together with Lemmas 1 and 2. The terms in the numerator of (13) refer to second order partial derivatives of the objective function (12) and their properties are analogous to those in the bank duopoly case. The difference factor $V_{ii}^i - V_{ji}^j$ in (13) captures that a monopoly bank internalizes the strategic externalities between the two product market duopolists it finances and we know that under the assumptions made this difference is negative. The denominator Δ_V refers to the sufficient second-order and stability conditions for the maximization of (12) (i = 1,2) and it satisfies that $\Delta_V > 0$.

Under the assumptions prevailing, comparison of (8), (9) and (13) makes it possible to formulate the relationship

$$(14)\quad \frac{\partial x_i^*}{\partial R} < \frac{\partial x_i^*}{\partial R_i} < \frac{\partial x_j^*}{\partial R_i} < 0,$$

if the investment decisions are strategic complements. Thus, an increase in the monopoly lending rate will reduce industry investment to a greater extent than a corresponding increase in the lending rate charged by the banks in a duopolistic industry. This can be explained as follows. The lending rate charged by a monopoly bank affects investment behavior of the firms directly and symmetrically. But, when a bank in a duopolistic industry increases its lending rate it will also affect the investment behavior of its customer's rival. An increase in the lending rate confronting firm i, however, generates a smaller investment change for the rival firm j than for firm i.

3. Interest rate decisions

Having analyzed the investment decisions of firms and their reactions to changes in the lending rates, we now turn to consider the first stage of the game, i.e. the lending rate decisions of banks. We start by looking at a banking duopoly and then move on to the case with a bank monopoly. Finally, we examine the implications of a bank merger on the fragility of the banking industry by investigating how such a merger will affect the equilibrium bankruptcy risk of firms.

3.1 Interest rate decisions in duopoly and monopoly banking

The banks make the lending rate commitments taking into account how the interest rates will affect the investments of the firms in the product market. In the previous section we delineated the investment equilibrium for given lending rates which results from the strategic interaction between the firms in the product market. Given this equilibrium, bank i commits itself to lend to firm i at an interest factor R_i which maximizes the expected value of the debt contract from bank i

$$\text{(15)}\quad \Gamma^i(R_i, R_j) = \pi^i(x_i^*, x_j^*)\int_{\theta_L}^{\eta_i}\theta dF(\theta) + x_i^*[(1-F(\eta_i))R_i - R_0],$$

where $R_0 = 1 + r_0$ denotes the opportunity cost factor of granting loans, which is assumed to be constant. The objective function (15) reflects the assumption that the bank knows the nature of the investment technology available. The first term on the right hand side of (15) describes the bank's profit in those states of nature where firm i faces bankruptcy. The second term expresses the bank's profits net of the opportunity cost of granting loans in those states of nature where the firm remains solvent.

Maximization of (15) (for i = 1,2) yields the implicit reaction functions for duopoly banks

(16)

$$-\left[\frac{\partial \pi^i(x_i^*, x_j^*)}{\partial x_i}\frac{\partial x_i^*}{\partial R_i} + \frac{\partial \pi^i(x_i^*, x_j^*)}{\partial x_j}\frac{\partial x_j^*}{\partial R_i}\right]\int_{\theta_L}^{\eta_i}\theta dF(\theta)$$

$$= x_i^*[1-F(\eta_i)] + \frac{\partial x_i^*}{\partial R_i}[(1-F(\eta_i))R_i - R_0],$$

where the first-order condition has been simplified based on (1). The first term in squared brackets on the left-hand side of (16) represents a direct negative effect of an increase in the interest rate factor, because a lending rate increase reduces the gross return of investment and thereby the profit of the lender in those states of nature where the firm defaults. The second term in squared brackets on the left-hand side of (16) captures the indirect effect due to the strategic investment interaction between the projects. By combining Assumption (A3) with (11) we can conclude that this indirect effect cannot dominate the direct effect. Hence, the left-hand side of (16) is positive. The first term on the right-hand side of (16) describes the direct interest rate effect in those states of nature where the firm remains solvent. The second term on the right-hand side of (16) expresses the indirect effect induced by changes in investments in those states of nature where the firm remains solvent. The intersection between the reaction functions defined by (16) will constitute the lending rate equilibrium between two competing bilateral monopoly banks.

In order to distinguish the interest rate equilibrium in a banking duopoly from the optimal lending rate of a monopoly bank we have to study also the lending rate determination for the monopoly bank. A monopoly bank will choose R so as to maximize

$$\Psi(R) = \pi^i(x_i^*, x_j^*)\int_{\theta_L}^{\eta_i}\theta dF(\theta) + x_i^*[(1-F(\eta_i))R - R_0].$$

Since an increase in the interest rate will have a symmetric effect on the investments of the competing firms, it directly follows that it is optimal for such a monopoly bank to charge the same lending rate from the firms. Therefore the first-order condition can be written as

(17)

$$-\left[\frac{\partial \pi^i(x_i^*, x_j^*)}{\partial x_i}\frac{\partial x_i^*}{\partial R} + \frac{\partial \pi^i(x_i^*, x_j^*)}{\partial x_j}\frac{\partial x_j^*}{\partial R}\right]\int_{\theta_L}^{\eta_i}\theta dF(\theta)$$

$$= x_i^*[1-F(\eta_i)] + \frac{\partial x_i^*}{\partial R}[(1-F(\eta_i))R - R_0],$$

The interpretation of (17) is analogous to that of (16) except for the fact that (17) is an ordinary first-order condition while (16) defines the reaction functions. Conditions (16) and (17) allow us to compare the optimal lending rate of a bank monopoly with the interest rate equilibrium prevailing in a banking duopoly.[9] Now one can establish

Proposition 1: When the investment decisions are strategic complements, a merger of duopoly banks into a monopoly bank will decrease the lending rate.

Proof: For the proof, see the appendix[10].

An explanation goes as follows. A rise in the lending rate has two opposing effects on the bank's profits: a negative one, because it induces a reduction in the investment volumes, and a positive one, because of the higher return on each unit money used to finance the projects. If the investment decisions of firms are strategic complements, an increase in the lending rate to one of the projects reduces the investment not only for the project in question, but also for the rival's project. Thus, the strategic complementarity strengthens the negative effect associated with an increase in the lending rate. If the banks merge, the resulting monopoly will internalize this effect by charging a lower interest rate than the competing banks.

Proposition 1 has an immediate corollary as one can see from equation (14).

Corollary 2: A bank merger will imply an expansion of the investment programs.

Proposition 1 and Corollary 2 show potential gains for interdependent firms from concentrating their borrowing activities to a single monopoly bank. We have identified competition with the number of banks. This could be justified by arguing that more competition is plausibly related to a higher degree of fragmentation of the banking industry and hence it could conceivably happen that interdependent firms are financed by different lenders with a higher probability when the degree of competition is high.[11]

Throughout the analysis we have focused on the consequences of a merger whereby the banking industry is transformed from a duopoly to a monopoly. However, the same arguments would hold in response to any change in the banking structure from n to n–1 banks ($n \geq 2$). Clearly, the impact of such a change is most dramatic, and can therefore be demonstrated in the strongest and simplest possible way, under the scenario which is exhibited in our analysis. Mechanisms whereby informational asymmetries in lending markets can create a barrier to entry for new lending institutions even in the absence of exogenous fixed costs offer an additional justification for why we formally focus on lending market structures which are very concentrated. Dell'Ariccia (1998) characterize one such mechanism, where creditworthy borrowers, unable to signal their quality to competing lenders, are locked into bank-credit relationships of the type considered in our model.

Proposition 1 is restricted to hold only when the investment decisions are strategic complements. In order to see that it cannot be generalized to hold when the investment decisions are strategic substitutes we consider the following simple example. Assume that the firms make production investments within the framework of a Cournot model with linear inverse demand $p = 1 - x_i - x_j$, where x_i denotes the production investment of firm i. Suppose further for simplicity that there is no uncertainty. In such a case the profit of firm i is given by $\pi^i(x_i, x_j) = x_i\,[1 - x_i - x_j - R_i]$. For given lending factors, R_i and R_j, the investment equilibrium (the Cournot equilibrium) can then be calculated to be $x_i^* = \frac{1}{3}[1 - 2R_i + R_j]$. Let us further normalize so

[9] *It is assumed that the complicated sufficient second-order and stability conditions are satisfied in a way analogous to the conditions presented in section 2.2.*

[10] *Our proof for this is different and applies under much more general conditions than the proof carried out by Poitevin.*

[11] *Berger, Kashyap and Scalise (1995) have studied what has happened to the size distribution of bank loans in U.S. after bank mergers due to the relaxation of interstate banking. According to their results there has been a loss of market share by small banks in favour of large banks. This finding lies in conformity with our analysis.*

that the opportunity cost of loans is zero, i.e. $R_0=0$. In the duopoly case bank i then determines the lending rate in order to maximize $\Gamma^i(R_i, R_j)=\frac{1}{3}[1-2R_i+R_j]R_i$. Straightforward calculations show that the lending rate equilibrium is then $R_i^*=\frac{1}{3}$. In case a bank merger takes place the resulting lending market monopoly would find it optimal to raise the interest factor to $R^m=\frac{1}{2}$. Consequently, when the investment decisions are strategic substitutes the merged bank would internalize the strategic interaction between the interdependent loan projects by increasing the lending rates.

The feature with a common lender deciding on a lower interest rate is present also in Poitevin's (1989) analysis of the impact of lending rates on production decisions which are strategic substitutes. He explains the mechanism by referring to a lower interest rate as a way for a bank monopoly to limit firm's incentives to choose risky output. In our model with endogenously determined loan volume the interpretation of the interaction between financial structure and imperfect product market competition is different, because Poitevin's analysis is restricted to the case where firms have to debt-finance a fixed expenditure (normalized to one) in order to produce.

Next we will turn to investigate the relationship between the lending market structure and the fragility.

3.2 Bank merger and equilibrium bankruptcy risk

In the case of the Scandinavian banking crisis in the early 90's it has often been pointed out that the crisis was preceded by deregulation of the banking market. For that reason many observers have used the Scandinavian experience as evidence of how increased competition in the bank loan market will generate higher instability in the financial sector.[12] When evaluating the impact of bank mergers on financial stability the crucial issue is how such a consolidation of the banking industry will affect the failure rates. In this section these issues are investigated from a restricted point of view where we make no attempt to explicitly model the mechanisms whereby borrowers' project failures generate states of bank insolvency. In this respect we simply subscribe to the view expressed by Bhattacharya, Boot and Thakor (1998) insofar as "business activity measures, such as small business failure rates, are very useful in predicting bank runs" (p. 752). As long as the borrowers are homogeneous the identification of credit market fragility with failure rates of borrowing firms can hardly be seen as very restrictive. Of course, the transmission mechanism from business failure rates to bank solvency is more complex with heterogeneous borrowers. Our model, however, is too complicated for a detailed analysis of this interesting mechanism.

In the previous subsection we established how under certain conditions a bank merger would decrease the interest rate and increase the level of investments. This suggests that the effect of a bank merger on the fragility of loan markets, in the sense of equilibrium bankruptcy risks of borrowers, is not a priori clear. While a fall in the interest rate decreases the default risk, the resulting increase in investment does the reverse.

To study this issue more closely we consider the equation

$$(18)\quad \eta_i^*\pi^i(x_i^*, x_j^*)-R_i x_i^*=0,$$

where η_i^* denotes the "breakeven" state of nature generated by equilibrium investments and interest rates. Differentiating η_i^* as defined by (18), we prove in the Appendix that

$$(19)\quad \frac{\partial\eta_i^*}{\partial R_i}=\frac{x_i^*}{\pi^i}\,[1-\varepsilon^*(1-(e^{ii}+\lambda e^{ij}))],$$

in which $\varepsilon^*=-\frac{\partial x_i^*/x_i^*}{\partial R_i/R_i}>0$ denotes the interest rate elasticity of investment, while $e^{ii}+\lambda e^{ij}$ is the adjusted scale elasticity with respect to industry investments. The adjusted scale elastici-

[12] According to Honkapohja and Koskela (1999) the credit losses of the Finnish banks amounted on average to 4.1% of the amount of lending during the period 1991–93, while the corresponding figure for Sweden was 4.3%.

ty with respect to industry investments is defined by $e^{ij}=\frac{\pi_j^i x_j}{\pi^i}$ together with $\lambda=\frac{\partial x_j^*/\partial R_i}{\partial x_i^*/\partial R_i} = -\frac{V_{ji}^j}{V_{jj}^j}$ so that $0 < \lambda < 1$ based on Lemma 2. The adjustment factor λ accounts for the fact that an increase in the lending rate for firm i will change its investment to a larger extent than that of its rival. We can thus conclude that

$$\text{(20a)}\quad \frac{\partial \eta_i^*}{\partial R_i} > 0 \text{ for all } \varepsilon^* \text{ when } e^{ii}+\lambda e^{ij} \geq 1$$

and

$$\text{(20b)}\quad \frac{\partial \eta_i^*}{\partial R_i} > 0 \text{ iff } \varepsilon^* < \frac{1}{1-(e^{ii}+\lambda e^{ij})} \quad \text{when } e^{ii}+\lambda e^{ij} < 1.$$

Consequently, (20a) and (20b) delineate the conditions under which the equilibrium "breakeven" state of nature η^* depends positively on the lending rate. Under such conditions a lending rate increase will raise the equilibrium probability of default because $F'(\eta)>0$.

Equation (19) decomposes the impact of the lending rate on bankruptcy into three different effects. Firstly, if the investment volume does not respond to a lending rate change, as in Poitevin (1989), only the first term would be relevant and a merger would automatically stabilize the loan market, since then the lending rates would directly determine the equilibrium probability of firm bankruptcy. However, a change in the interest rate facing firm i will typically also generate effects on the investment volumes of both firms i and j. In the absence of strategic interaction between the funded projects we would have an investment response working in an opposite direction to the 'direct effect', but as long as $\varepsilon^* < [1-e^{ii}]^{-1}$ the 'direct effect' would dominate. The presence of strategic interaction between the firms will further modify the investment response to interest rate changes. The investment response to a merger will be strengthened (weakened) when $\lambda e^{ij} < 0$ ($\lambda e^{ij} > 0$).

As a merger between two bilateral monopoly banks will decrease the equilibrium interest rate according to Proposition 1 we can formulate

Proposition 2: A merger between two bilateral monopoly banks will lessen the fragility of loan markets by reducing the bankruptcy risk of firms (a) always when the adjusted scale elasticity with respect to industry investments weakly exceeds one, (b) if and only if the interest rate elasticity of investment is "sufficiently small", when the adjusted scale elasticity with respect to industry investments is below one.

An intuitive interpretation is as follows. When the response of the industry investment is greater than or equal to one, i.e. when $e^{ii} + \lambda e^{ij} \geq 1$, the investment effect will add on to (or does not counteraffect) the 'direct' interest rate effect in stabilizing the loan market, because an interest rate increase will add to the bank's revenues (or leave them unchanged) via the generated investment increase. Consequently, a merger between duopoly banks will reduce the fragility of the credit market.

On the other hand, when $e^{ii} + \lambda e^{ij} < 1$ the generated investment increase will decrease the bank's revenues via the generated investment growth. The literature offers some guidance regarding the quantitative size of the response of firms' investment decisions to changes in interest rates. With a production function with capital and labor of Cobb-Douglas type, the interest rate elasticity would be equal to one. This is regarded as an upper bound for the interest rate elasticity (see Abel, 1990, p. 762–63, for more details). In fact, the huge empirical literature on investment functions suggests that the elasticity is well below one (see Chirinko, 1993, for a recent survey). Thus as an empirical matter we can safely expect the condition in Proposition 2 (b) to hold even when $e^{ii} + \lambda e^{ij} < 1$.

The present section has demonstrated that a bank merger does not increase the fragility of the credit market despite the fact that such a merger will generate a reduction in the lending rate under the circumstances considered. In this respect the present model reinforces the central argument in Koskela and Stenbacka (2000), which exhibits a completely different mechanism for the absence of a tradeoff between low lending rates and high failure rates of projects funded with debt. In Koskela and Stenbacka

(2000) uncertainty is generated conditional on the investments, making thus uncertainty an endogenous feature of that model.

Propositions 1 and 2 can be used to explore some important welfare implications of a bank merger within the context of our model with investments being strategic complements. As a benchmark for a welfare evaluation we formulate the following social objective function:

$$(20)\quad W(x_i(R),x_j(R)) = \big(\pi^i(x_i(R),x_j(R)) + \pi^j(x_j(R),x_i(R))\big)\int_0^{\theta_H}\theta dF(\theta) - R\big(x_i+x_j\big) - \big(F(\eta^i(x_i,x_j,R)) + F(\eta^j(x_j,x_i,R))\big)K,$$

where K denotes a fixed social cost of bankruptcy and where we consider a symmetric equilibrium ($R = R_i = R_j$). The social objective function (20) captures the idea that society is indifferent regarding the distribution of resources between lenders and borrowers and also incorporates social costs of bankruptcy in the simplest possible way. In order to find out the welfare consequences of a bank merger which according to Propositions 1 and 2 both decreases the lending rate and the failure rate of borrowers, we totally differentiate (20) with respect to R. Proceeding with such an approach we find that

$$(21)\quad \frac{dW(x_i,x_j)}{dR} = 2\left[E(\theta)(\pi_i^i + \pi_i^j)\frac{dx_i}{dR} - x_i(1-\varepsilon) - \frac{dF(\eta^i)}{dR}\mathrm{K}\right] < 0,$$

where $E(\theta) = \int_0^{\theta_H}\theta dF(\theta) > 0$, $\pi_i^i + \pi_i^j < 0$ by (A3), $\varepsilon \leq 1$, as previously, denotes the interest rate elasticity of investment and where it holds that $dF(\eta^i)/dR > 0$ by Proposition 2. Consequently, the feature that a bank merger reduces failure rates of borrowers adds on to the welfare gains, which are created by expanded investments due to a lower interest rate.

4. Concluding discussion

We have modelled the interaction between the concentration of the banking sector and the investment strategies of imperfectly competitive firms in the product market to address the question of whether competition makes loan markets more fragile by increasing the equilibrium bankruptcy risk of firms. It has been shown how under fairly mild conditions a merger between two bilateral monopoly banks would decrease the interest rate and increase the investment volume of imperfectly competitive firms if the investment decisions are strategic complements. Under quite plausible conditions our model implies that a merger would lessen the fragility of loan markets. With fixed social costs of bankcruptcy reduced failure rates add on to the welfare gains which are created by expanded investments due to a lower lending rate in equilibrium.

This paper has delineated the advantages for interdependent firms of borrowing from the same monopoly bank rather than from duopolists in the banking market. These advantages could be compared with those of a common agent as identified by Bernheim and Whinston (1985). However, as in Poitevin (1989), our results are different from theirs, because a common monopoly bank will not be able to dictate the investment decisions of the borrowing firms and this weakens the collusive power of a common lender in the banking market.

Throughout our analysis we have restricted our attention to lending markets by assuming that the bank's opportunity cost of granting loans is constant. Clearly, as financial intermediaries banks compete simultaneously both in the output (loan) and input (deposit) markets. Within such a general framework the opportunity cost of funds is itself endogenous (as elaborated in the contribution by, for example, Yanelle, 1997). In order to more extensively evaluate the consequences of bank mergers it is also important to explore their implications with respect to the performance of the deposit market. The present model has delineated circumstances under which a merger between two duopoly banks will decrease the lending rate without increasing the fragility of loan markets in the sense of increasing the bankruptcy risks of firms in equilibrium. Insofar as the circumstances outlined imply that the bankruptcy risks of a merged bank are reduced, such a merger is likely to generate a comparative advantage also with respect to the opportunity cost of loan funding. Namely, it is plausible that lower bankruptcy risks make the merged bank able to attract risk averse depositors at lower deposit rates. In this respect the main results of the present analysis seem to be robust relative to generalizations incorporating deposit markets.

Our model has formally been restricted to the case of common uncertainty meaning that the risks facing the investments are strictly correlated across the different projects. However, when evaluating the implications of assuming that the risks of the projects are strictly correlated it should be emphasized that it is an assumption which actually is in disfavour of a bank merger because in the presence of such an assumption a merger does not achieve risk diversification. Consequently, in this respect our assumptions rather underestimate than overestimate the potential gains from bank mergers.

The present analysis has abstracted from investigating how introduction of competition might affect the diversification incentives of banks. In light of the recent study by Shy and Stenbacka (1999) we have reasons to conjecture that our results are robust to generalizations making the diversification decisions of the banks an endogenous variable. Shy and Stenbacka demonstrate how, in the context of a differentiated banking industry, the banks' incentives for diversification are invariant to a change in the banking market structure from duopoly to monopoly.

A central conflict of interests between shareholders and bondholders is a typical feature of debt contracts. The shareholders place emphasis only on states of nature that are solvent, while in bankrupt states the shareholders' losses are truncated at zero due to limited liability. Debtholders, on the other hand, place emphasis only on bankrupt states, which would distort them to favor investment strategies which are too conservative relative to the first-best level. These observations led Stiglitz (1985) to suggest debtholder representation in the boards of borrowing firms as a mechanism of implementing first-best investment levels. Later on Brander and Poitevin (1992) have presented a much more detailed model of the agency costs of debt, in which they showed how the terms of the compensation contract offered to outside management by shareholders can reduce these agency costs substantially. Our results suggest that the agency costs of debt are dependent on the market structure in the banking industry, which is an important subject to further research.

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Appendix

Proof of Lemma 1: Differentiation of (4) yields

$$V^i_{ii}(x_i, x_j) = R\frac{\partial e^{ii}}{\partial x_i} + \pi_{ii}(x_i, x_j)H(\eta_i) + \pi^i_i(x_i, x_j)H'(\eta_i)\frac{\partial \eta_i}{\partial x_i}.$$

As it can be shown, an increasing hazard ratio of F is sufficient for the property $H'(\eta_i) < 0$. Since $\pi^i_{ii} < 0$, $\partial\eta_i/\partial x_i > 0$ and $\pi^i_i > 0$ it follows that (B1) and (B2) are sufficient to justify the first-order approach. *QED*

Proof of Proposition 1: Let us define the function $g^i(R_i, R_j)$ according to

$$g^i_i(R_i, R_j) = \frac{\partial x^*_i}{\partial R_i}[(1-F(\eta_i))R_i - R_0] + x^*_i[1-F(\eta_i)].$$

Consider the system of equations

$$(21) \quad g^i_i(R_i, R_j) + \alpha\left[\frac{\partial\pi^i(x^*_i, x^*_j)}{\partial x_i} + \frac{\partial\pi^i(x^*_i, x^*_j)}{\partial x_j}\right]\int_{\theta_L}^{\eta_i}\theta dF(\theta) = 0.$$

In a symmetric equilibrium we get that $R_i = R_j$, and, consequently, (16) is also an equilibrium condition. However, (16) can also formally be interpreted as the optimality condition of a merged bank charging the same interest rate to both borrowers. From the relationships (14) we know that a bank merger corresponds to a decrease in α if the investment decisions are strategic complements and if Assumption (A3) holds. In order to find out the impact of a bank merger on the interest rate we totally differentiate the system of equations (21) with respect to α. Leaving out the arguments from the function g^i, total differentiation yields

$$g^j_{jj}\frac{\partial R_j}{\partial\alpha} + g^j_{ji}\frac{\partial R_i}{\partial\alpha} + \left[\frac{\partial\pi^j(x^*_j, x^*_i)}{\partial x_j} + \frac{\partial\pi^j(x^*_j, x^*_i)}{\partial x_i}\right]\int_{\theta_L}^{\eta_i}\theta dF(\theta) = 0$$

and, similarly with respect to firm j,

$$g^j_{jj}\frac{\partial R_j}{\partial\alpha} + g^j_{ji}\frac{\partial R_i}{\partial\alpha} + \left[\frac{\partial\pi^j(x^*_j, x^*_i)}{\partial x_j} + \frac{\partial\pi^j(x^*_j, x^*_i)}{\partial x_i}\right]\int_{\theta_L}^{\eta_i}\theta dF(\theta) = 0.$$

Solution of this system of equations shows that

$$\frac{\partial R_i}{\partial\alpha} = -\frac{g^j_{jj} + g^i_{ij}}{g^i_{ii}g^j_{jj} - g^i_{ij}g^j_{ji}}\left[\frac{\partial\pi_i(x^*_i, x^*_j)}{\partial x_i} + \frac{\partial\pi^i(x^*_i, x^*_j)}{\partial x_j}\right]\int_{\theta_L}^{\eta_i}\theta dF(\theta) > 0.$$

This conclusion is based on a combination of ordinary sufficient second order conditions ($g^i_{ii} < 0$), stability conditions ($g^i_{ii}g^j_{jj} - g^i_{ij}g^j_{ij} > 0$) and on the assumption that "own effects" dominate over "cross-effects" (A3) (implying that $g^i_{ii} - g^i_{ij} < 0$). Consequently, we have proved that $\partial R_i/\partial\alpha > 0$, from which the conclusion of the proposition follows. *QED*

Proof of (19): Differentiating (18) we find that

$$(22)\qquad \frac{\partial \eta_i^*}{\partial R_i} = \frac{x_i^* + R_i \dfrac{\partial x_i^*}{\partial R_i} - R_i x_i^* \left(\dfrac{\pi_i^i}{\pi^i} \dfrac{\partial x_i^*}{\partial R_i} + \dfrac{\pi_j^i}{\pi^i} \dfrac{\partial x_j^*}{\partial R_i} \right)}{\pi^i}.$$

Defining the scale elasticity with respect to the rival's investment according to $e^{ij} = \dfrac{\pi_j^i x_j}{\pi^i}$ and making use of symmetry we can reformulate (22) as

$$(23)\qquad \frac{\partial \eta_i^*}{\partial R_i} = \frac{x_i^*}{\pi^i} \left[1 + \frac{R_i}{x_i^*} \frac{\partial x_i^*}{\partial R_i} - \frac{R_i}{x_i^*} \left(\frac{\partial x_i^*}{\partial R_j} e^{ii} + \frac{\partial x_j^*}{\partial R_i} e^{ij} \right) \right].$$

Making use of the interest rate elasticity of investment, $\varepsilon^* = -\dfrac{\partial x_i^*/x_i^*}{\partial R_i/R_i} > 0$, and (9) we define $\lambda = \dfrac{\partial x_j^*/\partial R_i}{\partial x_i^*/\partial R_i} = -\dfrac{V_{ji}^j}{V_{jj}^j}$ so that $0 < \lambda < 1$ if the investments are strategic complements and $e_j^{ii} \geq 0$, while $-1 < \lambda < 0$ if they are strategic substitutes and $e_j^{ii} \leq 0$. Making use of this notation we can express (23) according to

$$\frac{\partial \eta_i^*}{\partial R_i} = \frac{x_i^*}{\pi^i} \left[1 - \varepsilon^* (1 - (e^{ii} + \lambda e^{ij}))\right]. \qquad QED$$