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THE DISTRIBUTION OF STOCK MARKET RETURNS AND THE MARKET MODEL*

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In this paper the Market Model, estimated for 20 stocks on the Stockholm Stock Exchange, is examined under different assumptions regarding the distribution of the residuals. We find strong evidence that the residuals have a leptokurtic distribution and our results suggest that much of the leptokurticness can be attributed to a jump component in the distribution. Moreover, changes in the assumed distribution of the residuals can sometimes change the beta estimate by 20 percent or more. Our alternative estimators are more robust to extreme observations. (JEL: G12)

1. Introduction

The part of the financial literature dealing with the random character of stock returns is very rich, with the work of Bachelier (1900), as an early milestone. Later Mandelbrot (1963) made an important contribution when he seriously questioned the Gaussian hypothesis and instead pronounced the leptokurtic feature of stock returns. Other significant contributions were made by Cootner (1964), Fama (1965a,b), Press (1968), and Officer (1972). In more recent years attention has been paid to ARCH (originated by Engle, 1982) and GARCH (Bollerslev, 1986) models that appear to describe stock market returns well.

Another branch of the literature is concerned with the problem of finding factors explaining stock returns. A well-known example is the Capital Asset Pricing Model (CAPM) in which an asset's expected risk premium is determined by the expected risk premium of the market portfolio. In empirical applications, however, the pure CAPM is often replaced by the more general Market Model and this is also the approach taken here. In the CAPM or, as well, the Market Model it is often more or less implicitly assumed that asset returns are normally distributed despite the fact that the empirical sup-

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port for this assumption is weak. Theoretically, however, it is possible to justify a linear relationship between asset returns and the market return even if returns are not normally distributed. For instance, CAPM holds if the utility function is additive and quadratic, which may be a good approximation if the length of each period is short and asset returns are driven by processes of diffusion type. Also if we allow for jumps a relationship of CAPM type can be derived under certain circumstances, see Bentzen and Sellin (1994).

It is, however, a priori, not obvious how the estimates of the parameters in the linear Market Model will change when other distributions than the standard normal are considered. Even though the OLS estimator generates consistent beta estimates the presence of outliers are likely to make the beta estimates inefficient, as pointed out by, e.g., Chan and Lakonishok (1992). From an economic point of view, estimates of beta risk nowadays constitute a key input parameter for many financial decision makers. For example, financial managers use estimates of beta as an input parameter in computing a firm's cost of capital, and portfolio managers, as well as ordinary investors, use beta estimates as a measure of non-diversifiable risk[1]. Therefore, it seems relevant to assess how the estimates of beta are affected if other, more appropriate, distributional assumptions than the standard normal are employed in estimating beta.

Thus, the purpose of this paper is to investigate how sensitive the estimates are for changes in the distributional assumptions of asset returns by performing maximum likelihood estimation of the Market Model. Moreover, we will also address the question what kind of distribution is preferable. More precisely, we extend the standard Market Model with normally distributed residuals in three different directions: (i) residuals of ARCH type, (ii) residuals drawn from the general error distribution, and (iii) residuals with a jump component. Since we want to focus on the effects of changes in the distributional assumption of the residuals, we disregard from other more well-known estimation aspects, such as beta's sensitivity to the return interval, see e.g. Hawawini (1983). Furthermore, our focus on the largest and most traded Swedish stocks implies that effects stemming from thin trading, see e.g. Dimson and Marsh (1983), are not considered in this study.

To our knowledge no systematic study has been undertaken that investigates how the estimation of the Market Model is affected by different assumptions regarding the residual distribution. We feel, however, that the study of Chan and Lakonishok (1992) is related to ours in that the effect of outliers is a common main concern. Other aspects of the residual distribution, such as skewness, are a to a lesser extent under consideration.[2]

The paper is organized as follows. In Section 2 the data used is briefly described and in Section 3 the empirical distribution of stock returns is investigated. Section 3 also contains estimates of the standard Market Model equation as well as an investigation of the empirical distribution of the residuals. Estimates of the Market Model under more general assumptions regarding the distribution of the residuals are presented in Section 4. A comparison of the different models is provided in Section 5. On the basis of the insights gained in Sections 3 and 4 we combine and modify different models in section 6, in order to obtain further knowledge of the nature of the return distribution, and possible consequences for the estimates. Section 7 summarizes the paper and concludes the discussion.

2. The Data

Twenty stocks quoted on the Stockholm Stock Exchange will be used in the investigation[3]. Thirteen stocks range from March 1980 to June 1994 which implies 746 weekly obser-

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[1] Other studies that recognize the importance of beta estimates for financial decisions are Cartwright and Lee (1987), and Chan and Lakonishok (1992).

[2] Regarding other empirical studies of Swedish stock returns we refer to Frennberg and Hansson (1993), and Wells (1994).

[3] The data were kindly made available for us by the Department of Financial Economics, Handelshögskolan in Stockholm.

Table 1. Some Summary Statistics

Stock	Abbr.	Industry	Number of obs. (week)	Mean Return (week)	Standard Deviation
MARKET PORTFOLIO			*746*	*0.0034*	*0.0285*
AGA	AGA	chemistry	746	0.0025	0.0396
ASEA BROWN BOVERI	ASEA	engineering, electro tech.	746	0.0049	0.0446
ASTRA	ASTR	drug	602	0.0040	0.0468
ATLAS COPCO	ATCO	engineering, industry tech.	746	0.0033	0.0443
CUSTOS	CUST	investment	746	0.0021	0.0645
ELECTROLUX	ELUX	engineering, white goods	746	0.0030	0.0439
GAMBRO	GAMB	medicine technology	576	0.0004	0.0464
SKÅNE GRIPEN	GRIP	building, construction	503	0.0012	0.0533
HENNES & MAURITZ	H&M	retail trade, clothes	746	0.0056	0.0491
INVESTOR	INVE	investment	746	0.0042	0.0528
ERICSSON	LME	engineering, telecom.	746	0.0046	0.0507
MODO	MODO	forest, pulp	746	0.0026	0.0682
PERSTORP	PERS	chemistry	587	0.0013	0.0475
SANDVIK	SAND	engineering, steel	746	0.0036	0.0448
SCA	SCA	forest, packaging	540	0.0016	0.0460
SKANSKA	SKA	construction, real estate	540	0.0024	0.0497
SKF	SKF	engineering, ball bearing	540	0.0025	0.0484
ESSELTE	SLT	printing, office supplies	746	0.0016	0.0492
STORA	STOR	forest	746	0.0032	0.0504
VOLVO	VOLV	engineering, trucks, cars	746	0.0041	0.0482
			Average	*0.0029*	*0.0494*

vations. The shortest time period for any individual stock, Skåne Gripen (GRIP), is October 1984 to June 1994, or 503 weekly observations. We have computed weekly return series (based on closing prices) for each stock, a market portfolio and a short term asset (which is regarded as risk free)[4]. Dividends are included in the return series the week that incorporates the ex-dividend day. Some summary statistics are presented in Table 1 below.

Hennes & Mauritz (H&M) shows the highest weekly mean return, 0.56 percent, while Gambro (GAMB) presents the lowest 0.04 percent. The average return of the twenty stocks in the investigation is 0.29 percent which is somewhat less than the mean return of the Market Portfolio, 0.34 percent.

[4] *As an approximation of the market portfolio we have chosen 'Affärsvärldens general index'. The risk free asset is approximated by one month certificates of deposit (bankcertifikat) from March 1980 to December 1982, thereafter one month Treasury bills (Statsskuldväxlar) until June 1994. Data on the risk free asset was provided by Sveriges Riksbank (The Central Bank of Sweden).*

3. *The Empirical Distribution of Raw Returns and the Residuals*

3.1 The distribution of raw returns

Today it is a well-known empirical fact that the distribution of stock market returns are usually not normal but *leptokurtic,* i.e., the empirical distribution has fat tails and a high degree of peakedness as compared to the normal distribution. For example Fama (1965a), Kon (1984), Berglund and Liljeblom (1990), Campbell and Hentschel (1992), Chan and Lakonishok (1992), Frennberg and Hansson (1993) have all found evidence of leptokurtically distributed stock returns in samples covering different stock markets and time periods. A possible explanation to the leptokurtic feature of the empirical distribution of stock returns is offered by Roll (1988). Roll argues that stock market returns are interspersed with outliers that reflect news, events or information released by firms, thereby increasing the kurtosis of the return distribution.

In Table 2 below we examine the empirical distribution of weekly stock returns in our sam-

Table 2. The Distribution of Raw Returns

Stock	N	$E[N_1]$	N_1	$E[N_2]$	N_2	$E[N_3]$	N_3
AGA	746	236.8	161	33.9	38	2.01	15
ASEA	746	236.8	148	33.9	34	2.01	9
ASTR	602	191.1	128	27.4	32	1.63	6
ATCO	746	236.8	172	33.9	28	2.01	9
CUST	746	236.8	132	33.9	28	2.01	8
ELUX	746	236.8	160	33.9	36	2.01	12
GAMB	576	182.8	127	26.2	29	1.56	9
GRIP	503	157.9	112	22.9	28	1.36	7
H&M	746	236.8	143	33.9	42	2.01	10
INVE	746	236.8	161	33.9	31	2.01	8
LME	746	236.8	158	33.9	28	2.01	9
MODO	746	236.8	160	33.9	32	2.01	10
PERS	587	186.3	111	26.7	23	1.58	7
SAND	746	236.8	166	33.9	36	2.01	12
SCA	540	171.4	119	24.6	26	1.46	10
SKA	540	171.4	115	24.6	23	1.46	9
SKF	540	171.4	130	24.6	24	1.46	10
SLT	746	236.8	183	33.9	47	2.01	10
STOR	746	236.8	156	33.9	32	2.01	9
VOLV	746	236.8	154	33.9	38	2.01	6

Note: N = number of weekly observations. $E[N_k]$ = expected numbers of observations exceeding k standard deviations from the mean under the assumption of normally distributed returns. N_k = actual numbers of obs. exceeding k standard deviations from the mean.

ple N_k is defined as the actual number of observations exceeding k standard deviations from the mean return. $E[N_k]$ is the expected number of observations exceeding k standard deviations from the mean under the assumption of normally distributed returns. If the empirical distribution shows a leptokurtic feature, as we have good reasons to believe, then we will observe more returns in the extreme tail areas and near the mean as compared to the normal distribution.

For every stock $E[N_1] > N_1$, i.e., there are more observations near the mean in the empirical distribution than would be the case if returns were normally distributed. Furthermore, $E[N_3] < N_3$ which implies that there are more observations beyond three standard deviations as compared to what we expect in a normal distribution. N_2 seems to be roughly symmetrically distributed around its mean. Thus, we may conclude that for every stock the empirical distribution is more peaked in the center and there are more observations in the extreme tail areas than would be expected if stock returns were distributed normally. That is, we have found clear evidence that raw returns exhibit a leptokurtic distribution.

3.2 The distribution of the residuals

In empirical applications of the Market Model the normal distribution is, as pointed out above, often implicitly assumed. For instance, it is well known that OLS estimates in a linear model, like the Market Model, are identical to the estimates obtained by the maximum likelihood method when assuming normally distributed error terms with constant variance. However, the existence of thick tails may cause considerable difficulties in estimation and inference. If the empirical distribution of error terms shows large quantities of extreme observations the standard OLS procedure will weigh these outliers heavily. Thus, outliers could cause inefficient parameter estimates when using the standard OLS model.

We will henceforth use the standard OLS model (with normally distributed error terms and constant variance) as our reference model and compare it to estimates obtained under alternative distributional assumptions. Table 3 below presents results of the Market Model using OLS. With a few exceptions R^2 is less than 0.5 indicating that most of the variation in re-

Table 3. The Market Model Estimated by OLS $r_{it} = \alpha_i + \beta_i r_{mt} + u_{it}$, $\{u_{it}\}$ *i.i.d.*, $u_{it} \sim N(0,\sigma_i^2)$

Stock	α_i	β_i	σ_i	R^2	$\ln L$	DW
AGA	–0.000775 (0.00116)	0.84297 (0.04064)	0.031587	0.366	1519.9	2.22
ASEA	0.001561 (0.00138)	0.84922 (0.04835)	0.037573	0.293	1390.5	2.09
ASTR	0.001249 (0.00163)	0.82656 (0.05486)	0.039871	0.274	1086.5	2.06
ATCO	–0.000275 (0.00119)	1.06164 (0.04177)	0.032461	0.465	1499.6	2.12
CUST	–0.001842 (0.00192)	1.32414 (0.06748)	0.052441	0.341	1141.7	2.27
ELUX	–0.000621 (0.00114)	1.09133 (0.04004)	0.031117	0.500	1531.1	1.94
GAMB	–0.001895 (0.00173)	0.70145 (0.05877)	0.041599	0.199	1015.2	2.09
GRIP	–0.001324 (0.00199)	0.95709 (0.06499)	0.044606	0.302	851.55	2.35
H&M	0.002499 (0.00166)	0.67029 (0.05819)	0.045228	0.151	1252.1	2.21
INVE	0.001776 (0.00194)	0.11810 (0.06795)	0.052809	0.004	1136.5	2.16
LME	0.000967 (0.00143)	1.13596 (0.05035)	0.039128	0.405	1360.2	1.91
MODO	–0.001642 (0.00190)	1.56310 (0.06668)	0.051826	0.425	1150.5	2.17
PERS	–0.000991 (0.00164)	0.89314 (0.05552)	0.039650	0.307	1062.7	2.16
SAND	0.000166 (0.00128)	0.98017 (0.04512)	0.035065	0.388	1442.0	2.27
SCA	–0.000712 (0.00123)	1.20412 (0.04129)	0.028674	0.613	1152.7	2.21
SKA	0.000116 (0.00152)	1.17546 (0.05076)	0.035256	0.500	1041.2	2.16
SKF	0.000197 (0.00152)	1.10946 (0.05079)	0.035256	0.470	1040.9	2.08
SLT	–0.001662 (0.00159)	0.82129 (0.05583)	0.043388	0.225	1283.1	2.20
STOR	–0.000638 (0.00132)	1.24659 (0.04616)	0.035878	0.495	1424.9	2.23
VOLV	0.000557 (0.00243)	1.26044 (0.04141)	0.032182	0.555	1506.0	1.97

Note: Standard errors are reported in parenthesis. r_{it} = risk premium of asset i and r_{mt} = risk premium of the market portfolio. $\ln L$ is the value of the log likelihood function. DW is the Durbin-Watson statistic.

turns originate from firm specific shocks. The intercept term is insignificant for all stocks and in all but one case the beta estimates lie in the range [0.67, 1.57].

A closer look at the distribution of the residuals in Table 4 below reveals that they inherit the leptokurtic feature that raw returns show. Just as in the case of raw returns $E[N_1] > N_1$, that is, there are more residuals near the mean than would be the case if they were normally distributed. Furthermore, $E[N_3] < N_3$, that is, there are more observations beyond three standard deviations from the mean than would be expected if the residuals were normally distributed. Thus, we conclude that both raw returns and the residuals show a leptokurtic distribution.[5]

4. Extensions of the Standard OLS Model

In this section we consider extensions of the basic OLS model by assuming more general distributions of the residuals. The nature of the distributions to be analyzed is that they are characterized by two parameters and that the normal distribution can be obtained as a special case by setting one of the parameters to a specific value. Thus, it is possible to test the significance of the two-parameter distributions relative the normal distribution by a likelihood ratio test.

4.1 The ARCH model

First we investigate the Auto Regressive Conditional Heteroscedasticity (ARCH) model, where the error term, u_t, is given by

$$(1)\quad u_t \sim N(0,\sigma_t^2), \qquad \sigma_t^2 = h_0 + h_1 u_{t-1}^2$$

ARCH type models have attained increasing interest since it seems to capture the leptokurtic feature observed in asset returns. More precisely, it captures changes in variance over time, which appears as thick tails in the density function[6]. Notice that the standard OLS model (i.e.,

[5] *Blume (1968) reaches the same conclusion, i.e., the residuals from estimating betas by OLS have approximately the same distribution as raw returns.*

[6] *A comprehensive survey of ARCH and GARCH type models is given by Bera and Higgins (1993).*

Table 4. Distribution of Residuals from the Standard OLS Regression of the Market Model

Stock	N	$E[N_1]$	N_1	$E[N_2]$	N_2	$E[N_3]$	N_3
AGA	746	236.8	184	33.9	41	2.01	10
ASEA	746	236.8	178	33.9	28	2.01	13
ASTR	602	191.1	149	27.4	34	1.63	7
ATCO	746	236.8	199	33.9	29	2.01	7
CUST	746	236.8	127	33.9	30	2.01	9
ELUX	746	236.8	195	33.9	38	2.01	10
GAMB	576	182.8	138	26.2	25	1.56	7
GRIP	503	157.9	133	22.9	33	1.36	3
H&M	746	236.8	168	33.9	39	2.01	14
INVE	746	236.8	183	33.9	36	2.01	9
LME	746	236.8	191	33.9	37	2.01	11
MODO	746	236.8	179	33.9	39	2.01	9
PERS	587	186.3	123	26.7	26	1.58	8
SAND	746	236.8	199	33.9	33	2.01	8
SCA	540	171.4	144	24.6	30	1.46	5
SKA	540	171.4	118	24.6	21	1.46	8
SKF	540	171.4	141	24.6	21	1.46	9
SLT	746	236.8	192	33.9	41	2.01	7
STOR	746	236.8	206	33.9	38	2.01	6
VOLV	746	236.8	191	33.9	38	2.01	9

Note: N = number of weekly observations. $E[N_k]$ = expected numbers of observations exceeding k standard deviations from the mean under the assumption of normally distributed returns. N_k = actual numbers of obs. exceeding k standard deviations from the mean.

the case when error terms are normally distributed with constant variance) is obtained as a special case by setting h_1 equal to zero.

The ARCH estimate of the Market Model is presented in Table 5 below. For almost all stocks the coefficient h_1 is significant but small, indicating that some ARCH effects are present. With exception of GAMB, SKF and VOLV the increase in the log of likelihood function is large enough to reject the hypothesis of normally distributed residuals with constant variance, i.e., that h_1 is zero.[7]

The intercept term alpha is still insignificant, with one exception, and the change of beta is modest in most cases. But, for CUST and INVE the change in beta is remarkable. The main impression is that the ARCH model is a little improvement in that it captures the leptokurtic feature of the residuals to some extent. However, inspection of normalized ARCH-residuals tells us that leptokurtosis still is present in the data for all stocks[8].

4.2 The theta model

Next we will consider the case when the density of the error term is given by the general error distribution

$$(2) \quad f(u;\theta,\sigma) = \frac{e^{-|u/\sigma|^{\theta}}}{2\sigma\Gamma(1 + 1/\theta)}$$

where Γ denotes the gamma function[9]. Notice that the normal density function is obtained when the parameter theta, θ, is set to 2. For values of theta lower (higher) than two the distribution becomes more (less) leptokurtic than the normal distribution. For the double exponential distribution θ = 1, and when theta tends to infinity, the distribution tends to the rectangular distribution. Due to the importance of theta for

[7] *It should be noticed, however, that the log of likelihood function for the ARCH model is based on one observation less. A closer inspection that takes this aspect into account indicates that one should probably reject the hypothesis of a constant variance in the case of SKF.*

[8] *The measure of excess kurtosis is larger than zero for all stocks, typically in the range 1–6 with occasionally larger values.*

[9] *This probability density function for the error terms was suggested by Zeckhauser and Thompson (1970).*

Table 5. Estimate of the Market Model: the ARCH-model $r_{it} = \alpha_i + \beta_i r_{mt} + u_{it}$, $u_t \sim N(0,\sigma_t^2)$, $\sigma_t^2 = h_0 + h_1 u_{t-1}^2$

Stock	α_i	β_i	h_0	h_1	ln*L*
AGA	–0.0001741	0.83529 (0.02761)	0.000721	0.27384	1547.2*
ASEA	0.0014053	0.84350 (0.03751)	0.001299	0.07563	1395.2*
ASTR	0.0017955	0.82685 (0.04769)	0.001349	0.12932	1097.9*
ATCO	–0.0002422	1.04442 (0.03115)	0.000873	0.17619	1506.2*
CUST	–0.0023290■	1.11351 (0.31443)	0.001233	0.95466	1224.7*
ELUX	–0.0005146	1.09091 (0.02748)	0.000852	0.12566	1533.9*
GAMB	–0.0013507	0.67962 (0.05170)	0.001589	0.08837	1015.7
GRIP	0.0000502	0.94666 (0.05446)	0.001685	0.14493	856.16*
H&M	0.0020925	0.61634 (0.03617)	0.001609	0.24899	1262.7*
INVE	0.0014411	0.47908 (0.03009)	0.001912	0.40860	1150.4*
LME	0.0009403	1.13062 (0.03899)	0.001313	0.14086	1368.3*
MODO	0.0007274	1.51838 (0.04079)	0.001908	0.27725	1185.7*
PERS	–0.0014822	0.84578 (0.03251)	0.001065	0.21941	1114.6*
SAND	–0.0001473	1.01850 (0.03455)	0.000970	0.22306	1453.7*
SCA	–0.0008280	1.21310 (0.03205)	0.000723	0.11696	1154.9*
SKA	0.0006556	1.23266 (0.04806)	0.000836	0.36368	1066.5*
SKF	0.0008773	1.13667 (0.03950)	0.001091	0.01298	1042.3
SLT	–0.0019607	0.80734 (0.04116)	0.001525	0.19960	1293.0*
STOR	–0.0004734	1.26365 (0.03326)	0.001038	0.16691	1444.5*
VOLV	0.0004165	1.25772 (0.03337)	0.000960	0.07457	1505.8

Note: Standard errors are reported in parenthesis. A ■ indicate that the intercept term is significant on the 5% level. A * implies that the hypothesis of the standard OLS model (normally distributed residuals with constant variance) is rejected in a likelihood ratio test on the 5% level. ln*L* is the value of the log likelihood function.

characterizing the degree of kurtosis of the distribution we will refer to the regression model based on the general error distribution as the theta model. Zeckhauser and Thompson (1970) finds in different examples that the residuals of linear regressions often implies an estimate of theta much smaller than two, and that the change in the parameter estimates (relative the reference case of normally distributed residuals) can be substantial.

The parameter estimates of the Market Model in this case are presented in Table 6 below. The most striking observation is that the estimate of theta in all cases is closer to one than to two. Indeed, the increase in the log likelihood function leads to a rejection of the hypothesis of normally distributed residuals for all stocks. With the exception of AGA and PERS the theta model also outperforms the ARCH model in terms of the likelihood function indicating that the leptokurtic feature of the returns is more accurately captured in the theta model than in the ARCH model.[10] [11]

It is noteworthy that the intercept term seems to be significant for some stocks (GAMB, GRIP and SLT). This is an important observation since a significant intercept term contradicts the CAPM. Moreover, there is a tendency that the theta model generates lower estimates of beta compared to the normal distribution. For GAMB, PERS and H&M the reduction in the beta estimate is substantial. Our conclusion is that the theta model in most cases fits the residual distribution better than the previously studied models do.

4.3 The jump diffusion model

Now we will consider the case when the error term contains a jump component. A jump is

[10] *This is a somewhat doubtful statement in the case of STOR if one takes into account the fact that the log likelihood function in the ARCH model is based on one observation less.*

[11] *It can be hazardous to rank different distributional assumptions in terms of the log likelihood value but this device can be justified in the following sense. It is possible to combine the different distributions considered in this study and perform likelihood ratio tests whether the restricted distributions can be rejected relative the combined distribution. (An example of a test of this type can be found in Section 6). Thus, it will be harder to reject the distribution with the higher log likelihood value in such a test and it is only rejected when the distribution with the lower log likelihood value is rejected.*

Table 6. Estimate of the Market Model: the theta-model $r_{it} = \alpha_i + \beta_i r_{mt} + u_{it}$, $\{u_t\}$ i.i.d, and distributed according to (2)

Stock	α_i	β_i	σ_i	θ	ln*L*
AGA	–0.0013748	0.81093 (0.03265)	0.03049	1.25551	1546.3*
ASEA	0.0015924	0.79144 (0.03475)	0.02742	1.02796	1448.4*
ASTR	0.0011906	0.81181 (0.04083)	0.03052	1.05491	1125.3*
ATCO	–0.0009234	1.08911 (0.03392)	0.03431	1.35555	1513.6*
CUST	–0.0006284	1.07298 (0.03451)	0.02259	0.81639	1314.3*
ELUX	–0.0010989	1.02519 (0.03129)	0.02741	1.16154	1562.9*
GAMB	–0.0032795■	0.57050 (0.04672)	0.03719	1.18410	1043.5*
GRIP	–0.0040123■	0.90204 (0.05488)	0.04491	1.29454	863.27*
H&M	–0.0001374	0.46680 (0.04077)	0.03040	0.97145	1311.1*
INVE	0.0024938	0.14443 (0.05293)	0.04520	1.15040	1181.6*
LME	–0.0000361	1.11424 (0.03992)	0.03466	1.17207	1395.5*
MODO	–0.0026450	1.44491 (0.05167)	0.04401	1.14092	1194.3*
PERS	–0.0010859	0.70406 (0.04210)	0.03042	1.07206	1111.4*
SAND	–0.0002494	0.91695 (0.03799)	0.03574	1.31861	1463.8*
SCA	–0.0013799	1.18201 (0.03308)	0.02947	1.32319	1165.6*
SKA	–0.0002540	1.16368 (0.03632)	0.03388	1.05632	1095.8*
SKF	0.0012338	1.11703 (0.04061)	0.03388	1.25370	1061.7*
SLT	–0.0030568■	0.74536 (0.04403)	0.03846	1.16624	1313.6*
STOR	–0.0009815	1.25934 (0.03742)	0.03659	1.31870	1446.3*
VOLV	–0.0000250	1.25000 (0.03521)	0.03456	1.38230	1521.3*

Note: Standard errors are reported in parenthesis. A ■ indicate that the intercept term is significant on the 5% level. A * means that the hypothesis of the standard OLS model is rejected in a likelihood ratio test on the 5% level. ln*L* is the value of the log likelihood function.

an infrequent but drastic movement in the stock price that is not likely to be the outcome of the previous studied processes. It is, however, a nontrivial matter to identify a jump when one is dealing with a sample in discrete time. There is no obvious procedure that can sort out jumps in this case, and it is always troublesome to distinguish jumps from drastic movements in the diffusion component. It is also a delicate task to characterize the jump size, which in general can be random. Moreover, we want to accomplish this by only adding one additional parameter, in order to make comparisons within the group of two parameter distributions. We will address these problems in a rather primitive way by assuming that the error term can be written as

(3) $u_t = u_{1t} + u_{2t}$, where

(4) $u_{1t} \sim N\,0{,}\sigma^2$, and

(5) $$u_{2t} = \begin{cases} \varphi & \text{with probability } p \\ -\varphi & \text{with probability } q \\ \varphi\,(q-p)/(1-p-q) & \text{with probability } 1-p-q \end{cases}$$

and where φ, p and q are non-negative constants. In (5) it is understood that p and q are the probabilities of an upward jump of size φ and a downward jump of size $-\varphi$, respectively and that these probabilities are small ($p + q << 1$). It remains to identify a jump, and we will adopt the following identification rule:

(6) A jump of size φ ($-\varphi$) is identified at date t if $r_{it} - r_{mt} > (<)\ 0.1$.

A problem with this rule is that drastic outcomes in the diffusion component will wrongly be identified as a jump[12]. However, this bias problem should be small if the true return generating process actually contains a jump component. In other words we think it is plausible that investors regard a deviation of more than

[12] *Notice that there is a potential bias towards identifying jumps for high beta stocks. For instance, if we observe a (weekly) market return of 20 percent then we expect a stock with a beta of 1.6 to generate a return of 12 percentage points over the market return, which falsely is interpreted as jump according to the identification rule. This kind of misidentification could be avoided if we had used an identification rule base on the difference* $r_{it} - \beta_i r_{mt}$ *instead. However, since beta is a parameter to be estimated we find it inappropriate to base the identification rule upon it.*

10 percentage points from the weekly market return as a drastic and infrequent event, that is not generated by the "normal" return process, but by the jump component. Instead, we feel that the identification rule above is too sharp in that it fails to identify jumps that are not so drastic. On the other hand, we are aware of the possibility that the identification rule will tend to generate too good fit since it can be seen as a device, where outliers are controlled by a dummy variable. Thus, we expect that the values of the maximized log of likelihood functions will be relatively high.

Estimates of the probabilities p and q are given by

$$p = \frac{\#\mathit{upward\ jumps\ according\ to}\ (6)}{\mathit{total\#\ of\ observations}},$$

$$q = \frac{\#\mathit{downward\ jumps\ according\ to}\ (6)}{\mathit{total\#\ of\ observations}},$$

respectively. Notice that p and q can be determined separately from the identification rule above, thus, they will not be estimated in the ML procedure. Besides the fact that the existence of jumps can explain the observed fat tails, and thus leptokurtosis, the proposed model can also capture some degree of skewness in that p may differ from q. Moreover, when we impose the restriction $\varphi=0$ then the log likelihood function simplifies to the case of a normal distribution. Thus, it is possible to evaluate the significance of a jump component by imposing the constraint $\varphi=0$, and then perform a likelihood ratio test.

The estimates of the parameters of the Market Model under the jump diffusion assumption are presented in Table 7 below. The changes of the beta estimates are in some cases (CUST, INVE, LME, MODO and PERS) substantial in comparison to the OLS estimates of beta. There is also a tendency that the jump diffusion model does not generate so many extreme beta estimates as the ARCH and theta models. One can say that the beta estimates generated by the jump diffusion model seem to be quite stable over stocks. The estimates of the jump size, φ are with a few exceptions typically in the range 0.11–0.14. In addition, the probability of an upward jump does not differ much from the probability of a downward jump for any stock and the expected effect on stock return that the jump component gives rise to is small for all stocks[13]. This symmetry between upward and downward jumps strengthens our belief that the jump component is an important characteristic of stock returns. The probability of a jump $(p+q)$ is normally less than 0.04 implying that jumps typically occur once or twice every year. Compared to the previous models the jump diffusion model exhibits a better fit for the residuals in terms of the log likelihood function for all stocks. This is in line with our expectations due to the dummy-effect mentioned earlier, and it is premature at this stage to conclude that the jump-diffusion model is preferable to the other models.

5. *A Comparison of the Different Models*

Since our main purpose is to assess how the estimates of the Market Model (in particular beta) are affected by different distributional assumptions of the residuals, we will first compare the beta estimates generated by the ARCH, theta and jump diffusion models to our reference model OLS. Thereafter we perform a stability test of the beta estimates in the different models. Finally, we conduct Monte Carlo simulations, using a bootstrapping technique, in order to examine the robustness of the estimates.

5.1 Comparison of beta estimates

Estimates of beta risk have become readily accessible and constitute important information to investors. However, taking the empirical distribution of stock market returns into serious consideration, our results indicate that one should be careful not to 'over interpret' the beta figures generated by the standard OLS model. Inspection of the results reported in the previous sections shows that beta estimates are sen-

[13] *The expected jump size, calculated as $\varphi(p-q)$, is in the range ±0.1 percent for all stocks with the exception of MODO for which the expected jump size is 0.11 percent.*

Table 7. Estimate of the Market Model: the jump-diffusion model $r_{it} = \alpha_i + \beta_i r_{mt} + u_{it}$, $u_{it} = u_{1t} + u_{2t}$, $u_{1t} \sim N(0,\sigma^2)$ and u_{2t} is distributed according to (5)

Stock	α_1	β_i	φ	σ_i	p	q	ln$L$
AGA	–0.000732	0.80707 (0.03135)	0.12337	0.02853	0.0040	0.0080	1595.0*
ASEA	0.001548	0.85095 (0.03542)	0.13384	0.03085	0.0134	0.0121	1536.3*
ASTR	0.001206	0.87949 (0.04801)	0.12599	0.03333	0.0183	0.0116	1193.2*
ATCO	–0.000285	1.07449 (0.03189)	0.10877	0.03091	0.0027	0.0054	1535.2*
CUST	–0.001650	1.17680 (0.04610)	0.15667	0.04001	0.0215	0.0255	1341.7*
ELUX	–0.000567	1.04964 (0.02701)	0.11934	0.02818	0.0054	0.0067	1604.2*
GAMB	–0.001916	0.72464 (0.03691)	0.13162	0.03483	0.0174	0.0122	1116.5*
GRIP	–0.001321	0.93071 (0.04670)	0.12097	0.03747	0.0239	0.0159	938.05*
H&M	0.002424	0.73939 (0.03586)	0.13465	0.03489	0.0255	0.0201	1444.5*
INVE	0.001308	0.49246 (0.04424)	0.12494	0.04008	0.0442	0.0375	1339.4*
LME	0.001098	1.03061 (0.04439)	0.14581	0.03370	0.0107	0.0080	1470.6*
MODO	–0.001403	1.37332 (0.04293)	0.13760	0.03917	0.0349	0.0268	1357.4*
PERS	–0.000964	0.77261 (0.04775)	0.15155	0.03325	0.0102	0.0102	1164.7*
SAND	0.000193	0.95432 (0.03442)	0.13038	0.03158	0.0080	0.0054	1518.8*
SCA	–0.000717	1.22171 (0.03532)	0.12119	0.02767	0.0037	0.0000	1171.2*
SKA	0.000135	1.12417 (0.03718)	0.13975	0.02959	0.0074	0.0111	1134.4*
SKF	0.000198	1.11744 (0.03907)	0.12774	0.03013	0.0074	0.0129	1124.9*
SLT	–0.001644	0.80867 (0.04206)	0.12672	0.03600	0.0201	0.0161	1421.3*
STOR	–0.000564	1.18632 (0.03438)	0.14093	0.03317	0.0054	0.0040	1482.4*
VOLV	0.000338	1.23653 (0.03224)	0.11178	0.03029	0.0040	0.0054	1550.9*

Note: Standard errors are reported in parenthesis. A * means that the hypothesis of the standard OLS model is rejected in a likelihood ratio test on the 5% level. ln$L$ is the value of the log likelihood function.

sitive to the assumed residual distribution. For example, in the jump diffusion model beta estimates for five stocks (CUST, H&M, INVE, MODO, PERS) deviate more than ten percent from the standard OLS estimates. The theta model shows similar figures compared to OLS while the ARCH estimates deviates somewhat less. For about half of the stocks examined, the largest beta estimate exceeds the smallest by 10 percent or more. For CUST, GAMB, H&M, INVE, and PERS, the largest beta estimate exceeds the smallest by 20 percent or (far) more.

In Table 8 average cross-firm beta estimates and cross-firm variances of beta estimates are reported. The average cross-firm beta estimate is fairly close to unity for OLS, ARCH and the jump diffusion model, while the average beta estimate of the theta model is as low as 0.929, or seven percent lower than the standard OLS average beta estimate. We have no reasonable explanation for this result[14]. In relation to other considerations, e.g. the return interval (Hawawini, 1983), the assumption regarding the distribution of the residuals appears to be an additional aspect that is crucial for the estimate of beta. The effects for individual stocks of changing the distributional assumption are often as large as the substantial effects reported by Hawawini. Moreover, estimation effects originating from small samples or thin trading are probably not a main concern in this study since we consider the largest Swedish stocks over a period of about ten years or more.

The cross-firm variance of beta estimates is significantly lower in the jump diffusion mod-

Table 8. A Comparison of beta estimates in the different models

Stock	OLS	ARCH	theta	jump diff.
Min.	0.11810	0.47908	0.14443	0.49246
Max.	1.56310	1.51838	1.44491	1.37332
Average	0.99833	0.99503	0.92935	0.97757
Variance	0.09279	0.06471	0.09518	0.04784
Mean $\lvert\beta-\beta^{OLS}\rvert$	0	0.04760	0.06969	0.07003
Average standard error of beta	0.05234	0.03478	0.04088	0.03866

[14] *We note, however, that the low average beta estimate in the theta model to some extent is due to three exceptionally low beta estimates (GAMB, H&M, PERS) relative the other models.*

el (0.047) compared to the OLS (0.093) and theta (0.095) models. In the ARCH model cross-firm variance of the estimated betas takes on an intermediate value (0.065). Hence, according to these results, the jump diffusion model seems to generate beta estimates that are more stable across stocks than the other models. In a similar study on NYSE firms Chan and Lakonishok (1992) found that the cross-section variance of beta estimates was 13 percent higher in the OLS model compared to their proposed robust estimation method; *Trimmed Regression Quantile* (TRQ)[15]. Our results indicate a more substantial reduction in variance; cross-firm variance of beta estimates is 98 percent higher in OLS compared to the jump diffusion model.

Notice that the ARCH model produces the lowest average cross firm standard errors of beta estimates whereas OLS produces the highest. It is well known that robust standard errors of estimates obtained in ARCH regressions often are much higher and inspection of the normalized residuals indicates that this is likely to be the case (see Section 4.1, footnote 7). The average cross firm standard errors of the theta and jump diffusion models are somewhat higher, but the reported numbers should also in these cases be interpreted with large caution. The robustness of the beta estimates will be addressed more rigorously in Section 5.3.

5.2 Stability test of Beta Estimates

We perform a test of parameter stability in the different models by using a dummy variable variant of the Chow test (given that each sample is split in two sub-samples of equal size). The unconstrained version of the Market Model is then written as

$$(7)\quad r_{it} = \alpha_1 + (\alpha_2 - \alpha_1)\, D1 + \beta_1 r_{mt} + (\beta_2 - \beta_1)\, D_2 + u_{it}$$

where subindex on alpha and beta indicate period and

$$D1 = \begin{cases} 0 & \text{in subsample 1} \\ 1 & \text{in subsample 2} \end{cases} \quad D2 = \begin{cases} 0 & \text{in subsample 1} \\ r_{mt} & \text{in subsample 2} \end{cases}.$$

Imposing the null hypothesis $\beta_1 = \beta_2$ in the restricted version of the model makes it possible to perform a likelihood ratio test. In Table 9 below we present the results of the stability test. Seven stocks in the ARCH-model show significantly different beta estimates in the two periods. In the standard OLS and in the theta model six stocks show significantly different beta estimates, while only three stocks show significantly different beta estimates in the jump diffusion model. Thus, unless betas actually have changed, these results indicate that the jump diffusion model generates more stable beta estimates than the other models, that is, estimates that are less sensitive to the sample period chosen.

Notice that for the stocks AGA, GAMB, and STOR it is probable that the betas actually have changed since this is suggested by all models. Finally, we have computed the average $|\Delta\beta_i|$ in the last row of Table 9. The least average deviation for both $|\Delta\beta_1|$ and $|\Delta\beta_2|$ is shown by the jump diffusion model, hence, further supporting the view that beta estimates in the jump diffusion model seem to be more robust compared to the other models.

Our interpretation of these results is that the relatively more frequent "extreme" beta estimates generated by the OLS, ARCH, and theta models, as compared to the jump diffusion model, are due to a few outliers in the distribution – outliers that weigh heavily in the estimation procedure when not explicitly recognized as a jump.

5.3 Monte Carlo Simulations and the Robustness of the Beta Estimates

In order to assess the robustness of the different estimation methods we employ Monte-Carlo simulations based on a bootstrap strategy. More precisely, we construct a sample that consists of the residuals from all stocks that was generated when we estimate the market model by OLS in Section 3. This sample, consisting of 13,584 residuals, is then representing the residual distribution of an average stock of the

[15] *Chan and Lakonishok (1992) describes their estimation method (TRQ) on pp. 268–271, while the variance of beta estimates is discussed on pp. 276–277.*

Table 9. Test of Stability of Beta Estimates

	OLS		ARCH		theta		jump diffusion	
stock	$\Delta\beta_1$	$\Delta\beta_2$	$\Delta\beta_1$	$\Delta\beta_2$	$\Delta\beta_1$	$\Delta\beta_2$	$\Delta\beta_1$	$\Delta\beta_2$
AGA	0.1782	–0.1088*	0.1576	–0.0806*	0.1563	–0.1138*	0.1435	–0.0864*
ASEA	0.2053	–0.1254*	0.2013	–0.1120*	0.1694	–0.0980*	0.0730	–0.0450
ASTR	–0.0081	0.0088	0.0028	–0.0025	–0.0873	0.0505	0.0560	–0.0583
ATCO	0.0089	0.0054	–0.0134	0.0087	–0.0167	0.0142	–0.0574	0.0374
CUST	–0.1649	0.1007	0.0241	–0.0670	–0.1891	0.1068*	–0.1058	0.0662
ELUX	–0.0179	0.0109	–0.0424	0.0296	–0.0532	0.0850	–0.0079	0.0083
GAMB	0.2777	–0.2804*	0.3222	–0.2586*	0.3235	–0.2176*	0.2003	–0.2023*
GRIP0.0934	0.0934	–0.0891	0.0921	–0.0983	0.0501	–0.0929	0.0072	–0.0080
H & M	0.0757	–0.0462	0.0693	–0.0464	0.0428	–0.0920	0.0261	–0.0204
INVE	–0.0245	0.0150	–0.2582	0.0743*	0.0166	–0.0049	–0.0451	0.0277
LME	–0.1470	0.0898*	–0.1364	0.0915*	–0.1310	0.0834	–0.0560	0.0380
MODO	–0.0914	0.0558	–0.0022	0.0004	0.0101	–0.0533	–0.0466	0.0311
PERS	0.1737	–0.1791*	0.1133	–0.1250*	0.0057	–0.0317	0.0783	–0.0789
SAND	–0.0553	0.0338	–0.0549	0.0270	–0.1245	0.0538*	–0.0567	0.0356
SCA	0.0035	–0.0034	0.0141	–0.0190	0.0213	–0.0425	0.0607	–0.0565
SKA	–0.0461	0.0455	–0.0610	0.0698	–0.0265	0.0250	–0.0751	0.0747
SKF	–0.0144	0.0142	–0.0473	0.0598	0.0068	–0.0208	–0.0208	0.0200
SLT	–0.0122	–0.0074	–0.0070	0.0039	0.0990	–0.0740	0.0331	–0.0210
STOR	–0.1667	0.1018*	–0.1624	0.0919*	–0.2238	0.1714*	–0.1721	0.1052*
VOLV	0.0430	–0.0263	0.0407	–0.0259	–0.0029	0.0012	0.0390	–0.0281
|Average|	*0.0904*	*0.0674*	*0.0911*	*0.0646*	*0.0878*	*0.0716*	*0.0682*	*0.0525*

Note: A * indicates that H_0: $\beta_1 = \beta_2$ is rejected at the 5% level. $\Delta\beta_i$ shows the difference between the unconstrained β_i (period i = 1,2) and the restricted beta estimate, i.e., $\Delta\beta_1 = \beta_i - \beta$.

given sample of stocks. In each replication in the Monte Carlo simulation we randomly draw 746 residuals from the sample of residuals, and 746 observations from our sample of excess market returns. Then we add these two random sequences to obtain a time series of excess returns for an imaginary stock, for which beta equals unity and the intercept term (alpha) equals zero. The market model is then estimated with the different estimation methods using the constructed sample of excess returns. The drawback with this approach is that it fails to capture potential ARCH effects, and the result of the ARCH model can not be presented.

We estimate the market model 10,000 times for each model, and the result of this Monte Carlo exercise is summarized in Table 10 A below. First, the estimates seem to be consistent although the intercept term for the theta model is relatively large. The jump diffusion model produces the most efficient estimates, which strengthens the impression that this model generates the most reliable estimates. Notice, however, that the jump diffusion model only marginally outperforms the theta model in terms of efficiency. Moreover, as can be seen in the last row of Table 10 A, the probability of obtaining an estimate that deviates more than 10 percent from the true value is much smaller for the jump diffusion model and the theta model compared to the standard OLS model. Also in this case, the jump diffusion model marginally outperforms the theta model. Thus, the result of the Monte Carlo simulations reinforce the feeling from the stability test reported in Table 8, i.e. the jump diffusion model produces more stable beta estimates than the other models.[16,17]

[16] *One may argue, however, that the device of using residuals that represent an average stock does not fully reflect the ability of jump diffusion to capture certain characteristics of a single stock's residual, such as skewness. We have experimented with simulations, for which each replication is based on residuals from a single but random stock. The results of this exercise do not differ much from those reported in Table 10 A even though the standard errors of the OLS and theta estimates for beta are somewhat higher, 0.0533 and 0.0486 respectively. (The standard error of the jump diffusion model is 0.0437.)*

[17] *Notice that the standard errors generated by the theta and jump diffusion model are somewhat higher than the average cross firm standard errors of beta reported in Table 8.*

Table 10. Monte Carlo Simulations (10,000 simulations)

	A. (746 obs.)		
	OLS	theta	jump diff.
beta	1.00046 (0.052021)	1.00057 (0.046004)	0.999947 (0.043351)
alpha	0.474924E-04 (0.001463)	–0.641242E-03 (0.001317)	–0.134342E-04 (0.001459)
no. of beta outliers	558	298	241
	B. (200 obs.)		
	OLS	theta	jump diff.
beta	0.999321 (0.101043)	1.00055 (0.091000)	0.999225 (0.083758)
alpha	–0.194145E-04 (0.006783)	–0.594938E-03 (0.002509)	–0.146126E-04 (0.002843)
no. of beta outliers	3101	2659	2286

Note: *No. of beta outliers* shows the number of beta estimates that deviate more than 10 percent from the true value in 10,000 simulations.

The efficiency gains of using the theta or jump diffusion models relative the OLS model is, according to Table 10 A, about 12 and 17 percent respectively. However, the sample size in each replication is relatively large (746 weekly observations) and in practice, beta estimates are based on much shorter samples. Therefore we perform simulations using a sample size of 200, but, as seen from Table 10 B, the efficiency gains for the alternative models are of the same magnitude as earlier.

6. A Further Extension of the Models

6.1 A combination of the jump diffusion- and the theta model

It is possible to combine the different models studied in Section 4. This will lead to models where the distribution of the error terms is characterized by three parameters as compared to the “two parameter models” in Section 4. Since leptokurticness seem to be of great importance in estimation and, furthermore, it is interesting to find out how much of the leptokurticness that comes from jumps, we find it most natural to combine the jump diffusion model and the theta model. In this case, the assumption of normality of the error component, u_{1t}, see (4), is replaced by the theta distribution according to (2). The result of the maximum likelihood estimation of the Market Model in this extended jump diffusion case is given in Table 11 below.[18]

The extended model generates estimates that are close to the ones obtained in the jump diffusion model. Notice, however, there is still a tendency that the beta estimates become smaller when the parameter theta is free to deviate from 2[19]. For CUST the reduction of the beta estimate is substantial. In comparison to the pure theta model the estimates often change considerably, especially for GAMB, H&M and INVE for which the extended jump diffusion model generates significantly higher estimates

[18] *We have also estimated a GARCH(1,1)-model, for which the residual variance is of the form: $\sigma_t^2 = h_0 + h_1 u_{t-1}^2 + g\sigma_{t-1}^2$. For most of the stocks the change of the beta estimates (relative the ARCH-model) is small (less than 0.03 in absolute values). For CUST, GAMB, INVE and SKA the changes are +0.10, –0.07, –0.35 and –0.08 respectively. The log likelihood values are smaller for all stocks in comparison with the jump diffusion model, whereas the theta model outperformed (in terms of log likelihood values) the GARCH model for 10 stocks. These results reinforce the impression that ARCH effects are not of great importance for explaining the leptokurticness of the residuals.*

[19] *The average cross-firm beta estimate is 0.95930 as compared to 0.97757 in the “pure” jump diffusion model analyzed in Section 4.3.*

Table 11. Estimate of the Market Model: Extended Jump Diffusion Model $r_{it} = \alpha_i + \beta_i r_{mt} + u_{it}$, where $u_{it} = u_{1t} + u_{2t}$, and u_{1t}, u_{2t}, are distributed according to (2) and (5) respectively.

Stock	α_i	β_i	φ	σ_i	θ	ln*L*
AGA	–0.001288	0.79887 (0.03191)	0.12149	0.03383	1.53341	1601.1*
ASEA	0.001631	0.82614 (0.03599)	0.12630	0.03536	1.46628	1542.3*
ASTR	0.001631	0.86782 (0.04317)	0.11935	0.03711	1.42194	1199.9*
ATCO	–0.000800	1.08957 (0.03381)	0.10779	0.03542	1.47325	1543.0*
CUST	–0.001143	1.05897 (0.03446)	0.11950	0.02606	0.98268	1436.8*
ELUX	–0.000993	1.03037 (0.02972)	0.11941	0.03088	1.40126	1614.1*
GAMB	–0.002187	0.69454 (0.04162)	0.12830	0.04530	1.73590	1117.6
GRIP	–0.002403	0.92039 (0.04814)	0.11741	0.04681	1.63535	939.8
H&M	0.001665	0.69274 (0.03980)	0.13140	0.04226	1.57126	1448.5*
INVE	0.001851	0.48013 (0.04682)	0.11623	0.04532	1.45279	1350.4*
LME	0.000667	1.04002 (0.04231)	0.14319	0.04192	1.63201	1473.6
MODO	–0.001356	1.35857 (0.04468)	0.13235	0.04994	1.68505	1359.9
PERS	–0.000890	0.71870 (0.04412)	0.13875	0.03688	1.42188	1173.4*
SAND	0.000081	0.93698 (0.03544)	0.12690	0.04073	1.71219	1520.5
SCA	–0.000994	1.18787 (0.03492)	0.11982	0.03062	1.41825	1178.8*
SKA	–0.000151	1.13639 (0.03728)	0.12417	0.03114	1.35006	1146.2*
SKF	0.000216	1.11900 (0.04011)	0.12645	0.04068	1.84007	1125.2*
SLT	0.002279	0.78664 (0.04245)	0.12476	0.04325	1.55377	1425.5*
STOR	–0.000754	1.20361 (0.03749)	0.13622	0.04160	1.64752	1485.2
VOLV	–0.000085	1.23875 (0.03424)	0.11094	0.03592	1.53725	1557.7*

Note: Standard errors are reported in parenthesis. A * means that the hypothesis of the simple jump diffusion model ($\theta = 2$) is rejected in a likelihood ratio test on the 5% level. ln*L* is the value of the log likelihood function.

Table 12. Monte Carlo Simulations (10,000 simulations)

	Combined Model (746 obs.)	Combined Model (200 obs.)
beta	0.999377 (0.041818)	0.999589 (0.082868)
alpha	–0.000250 (0.001395)	–0.000233 (0.002776)
no. of beta outliers	189	2262

Note: *No. of beta outliers* shows the number of beta estimates that deviate more than 10 percent from the true value in 10,000 simulations.

of beta. However, there is still a reduction in the average cross-firm beta estimate compared to the OLS average beta estimate, but the reduction is not as large as in the pure theta model in Section 4.2. The reduction is 4 percent, which is smaller than when no jump component was included (7 percent). The most striking observation, though, is that the theta estimate is considerably higher when a jump component is included suggesting that some of the leptokurticness, that a low value of theta indicates, is caused by the jump component. Notice, however, that the theta estimates are all less than two and, for most stocks, the hypothesis of a normally distributed component (u_{1t}) is rejected in a likelihood ratio test on the 5 percent level. However, in some cases (GAMB, GRIP, LME, MODO, SAND, and STOR) we can not reject the hypothesis $\theta = 2$.

Finally, we have also examined the robustness of the estimates for the combined model. As seen from Table 12 the efficiency has improved somewhat. Moreover, the probability of obtaining a beta estimate that deviates more than ten percent from the true value is further reduced. (Cf. estimates in Table 10).

7. Summary and Conclusions

This paper analyzes maximum likelihood estimates of the Market Model under various assumptions regarding the distribution of the residuals, viz.: (i) Normally distributed residuals. (ii) Residuals of ARCH(1) type. (iii) Residuals drawn from the general error distribution, where the density is proportional to $\exp(-|u/\sigma|^{\theta})$. A value of theta ($\theta$) less than two implies a leptokurtic distribution and we refer to this model as the theta model. (iv) Residuals with a jump component. The main findings are:

(I) The empirical distribution of both raw returns and the residuals (generated from the standard OLS procedure of the Market Model) are leptokurtic for twenty stocks quoted on the Stockholm Stock Exchange. On the basis of the results adherent to different model specifications we believe that leptokurtosis is a relatively more important aspect of the distribution than skewness.

(II) When we perform maximum likelihood estimation of the Market Model the choice of distribution of the residuals does matter, and sometimes beta estimates differ substantially between different specifications. For instance, in the theta model the average beta estimate across stocks is seven percent lower than the standard OLS average of beta. For individual stocks the effect on the beta estimate of changing the assumed residual distribution can be 20 percent or more. Moreover, beta estimates seem to be very sensitive to outliers in the distribution.

(III) The standard OLS model, for which the residuals are normally distributed with constant variance, is rejected in favor of the alternative models for almost all stocks. Moreover, in terms of the log likelihood function, the ARCH model is outperformed by the theta model, which in turn is outperformed by the jump diffusion model.

(IV) The jump diffusion model generates more stable estimates of beta than the other models in the sense that it seems to be less sensitive to changes in the sample considered.

(V) The jump diffusion model is also more stable across stocks in comparison to the other models and the beta estimate is seldom very far from unity.

(VI) Monte Carlo simulations based on a boot strapping technique indicate that there are efficiency gains in the range 10–20 percent by using the alternative distributions. More important, though, is that the probability of obtaining a beta estimate that deviate more than 10 percent from the true value is significantly lower in the jump diffusion model as compared to the standard OLS case.

(VII) Estimation of a combination of the theta model and the jump diffusion model indicates that much of the leptokurticness of the residuals comes from the jumps. The hypothesis of a normally distributed non-jump component is, however, in most cases rejected.

We feel there is fairly strong evidence that there exist drastic observations, i.e. jumps, that affect beta estimates substantially and give rise to instability, and we find the proposed jump diffusion model an interesting device for reducing these problems and obtain more robust estimates. This is not only of interest for economic statisticians, but also for investors, since beta risk constitutes important information in financial decisions. However, we do not maintain that the proposed jump diffusion model is the ideal one, and the relatively high log of likelihood function values are probably, to some extent, a dummy effect that the rule that identifies jumps generates. Thus, it is desirable in future work to develop a more realistic jump diffusion model. Such a model should also take into account the leptokurticness of the diffusion component. It is also too early to rule out ARCH and GARCH type models. Such models are useful in that they are capable of capturing how uncertainty varies over time. The results of this study suggest, however, that the random variable underlying this uncertainty is not normally distributed and it sometimes takes drastic values.

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