Finnish Economic Papers – Volume 12 – Number 1 – Spring 1999

UNCERTAINTY AVERSION IN A SIMPLE INSURANCE MODEL*

FREDRIK ANDERSSON

Department of Economics, Lund University, P.O. Box 7082, S-220 07 Lund, Sweden

A simple insurance model is considered where the distribution of accident probabilities in the population is known, but where the actual probability of each policyholder is unknown to both insurers and the policyholder himself. It is shown that if policyholders are uncertainty averse, deductibles are distorted downwards. A complete view of insurance in such circumstances need thus consider trade in uncertainty as well as risk. (JEL: D81, G22)

1. Introduction

Standardised insurance contracts are generally thought of as homogeneous services with properties that are well known and well understood by all parties. This view overlooks the fact that many policyholders have none or few losses over their entire life-span, and thus may be less well informed about the character of the risk.

In formal terms, standardised insurance is modelled as a risk-sharing problem with asymmetric information, and it is framed in the paradigm of rational expectations; i.e., all probability distributions involved are assumed to be common knowledge.[1] This approach has turned out to be very productive. In the simple accident-type model, for example, it is well understood how the deductible may serve to mitigate adverse selection as well as moral hazard.[2] Puzzles remain, however; for example, observed deductibles seem to be much smaller than existing theories, reasonably parameterised, predict.[3]

It thus seems worth asking to what extent results from existing models depend crucially on the policyholders' complete knowledge about the stochastic properties of the risk. We will provide one answer to this question by analysing a model where the complete knowledge assumptions are relaxed. In this model neither the insurer, nor the policyholders, know the probabilities exactly. Further, the policyholders are *uncertainty averse;* i.e., they prefer knowing probabilities rather than facing *uncertainty* or, synonymously, *ambiguity.*[4]

* *Presented at the Econometric Society Seventh World Congress in Tokyo. The paper has benefited from suggestions made by Anders Borglin, Hans Carlsson, Mattias Ganslandt, Martin Hellwig, Bo Larsson, Lars-Gunnar Svensson, two anonymous referees, and seminar participants at Harvard University. Financial support from the Bank of Sweden Tercentenary Foundation and from the Royal Academy of Sciences is gratefully acknowledged.*

[1] *In the presence of adverse selection, the distribution of probabilities is assumed to be common knowledge.*

[2] *Early insurance models with adverse selection are Stiglitz (1977) (monopoly) and Rothschild and Stiglitz (1976) (competition). An insurance model with moral hazard is Shavell (1979).*

[3] *See Friedman (1974) for an exploration of health insurance, and Stuart (1983) for an investigation of homeowner insurance.*

[4] *In another paper, Andersson (1997), we have analysed a principal-agent model with bilateral asymmetric informa-*

When using uncertainty aversion, we follow the literature developed from the view that genuine uncertainty can make a real difference in decision problems. The argument most often put forward is the so called Ellsberg paradox that we will describe below.[5] Our specification is compatible both with genuine uncertainty and with policyholders' holding probabilities over probabilities.

We show that the policyholders' uncertainty aversion leads to the insurer providing *more extensive insurance coverage* against bad outcomes than in the case without uncertainty aversion. We will make our main point in the simplest possible framework, and then show how it interacts with moral hazard.

Related issues

The general theory of insurance under risk is extensive, and it is surveyed by e.g. Dionne and Harrington (1992). This literature clarifies the role of deductibles as well as experience rating and other aspects of insurance contracts under risk; in particular, it deals with information asymmetries, i.e. with moral hazard and adverse selection.

There is also a considerable literature on insurance with symmetric ambiguity; i.e., insurance in situations where no party knows the probabilities. Examples of such situations may be flooding and nuclear accidents. Contributions include Hogarth and Kunreuther (1989) who investigate experimentally how insurers and policyholders may act when facing ambiguity; they find evidence of aversion to ambiguity, most clearly for small probabilities. Kunreuther *et al.* (1993) focus on the problem of markets being too thin to be viable because of ambiguity. Viscusi (1993) considers pricing of insurance in the presence of ambiguity using data from insurance pricing; he finds weak evidence in favour of insurers' pricing reflecting ambiguity. The asymmetric situation, which is the focus of this paper, has, however, received little attention.

When applied to interactive decision problems, there is a connection between the theory of uncertainty aversion that we will apply, and the theory of interactions with heterogeneous probability assessments. This is the case for this application too.[6] Because of this, it seems that the same kind of criticism that applies to allowing heterogeneous beliefs applies to allowing uncertainty aversion. That is largely true and, indeed, we do not find the "Harsanyi doctrine", that economic agents having the same information necessarily should have the same beliefs, compelling.[7] We do, however, think that it is desirable to be able to *derive* heterogeneous beliefs from transparent assumptions about preferences; both because it makes the model more easily interpretable and because, relatedly, we get some information about the direction in which beliefs are likely to differ from being homogeneous.

Outline

In Section 2 we review some basics of the theory of uncertainty aversion, and in Section 3 we describe the model. In Section 4 we solve the model; we first consider the monopoly case without moral hazard, and then go on to discussing competition and moral hazard. Section 5 is a conclusion.

2. Non-additive probability measures and uncertainty aversion

It seems fair to say that the notion of uncertainty aversion originates from the celebrated

tion: the agent knows his characteristics but does not know their implications fully unless he learns the principal's private information about the technology. There are then incentives for the principal to induce pooling of agents in order to insure them.

[5] *Good sources are Schmeidler (1989) and Karni and Schmeidler (1991). The approach is, in addition, clearly in accordance with the conception of probability put forward by Keynes (1921).*

[6] *Furthermore, both theories relate to the work on models where the agents' knowledge of events is represented by state spaces over which they have more general information structures than partitions; see Hendon* et al. *(1994) for the case of uncertainty aversion, and Brandenburger* et al. *(1992) for heterogeneous priors.*

[7] *We cannot but cite the critical view expressed by Kreps (1990, p. 110).*

Ellsberg paradox identified by Ellsberg (1961), even though attention was paid considerably earlier to the intrinsic problem – how to handle decision problems with stochastic outcomes and unknown probabilities. The distinction between *uncertainty* in this sense, and randomness with a known probability distribution, *risk*, was introduced by Knight (1921). However, the work of Savage (1954) provided reasons why the distinction could, in effect, be abstracted from. Savage provided a set of axioms that were widely accepted as plausible. The axioms imply that rational choice behaviour can be completely described as maximisation of an expected utility with respect to a unique, *subjective*, probability measure.

One version of the Ellsberg paradox is the following. There are two urns containing a hundred balls each; the first urn, *A*, is known to contain fifty white balls and fifty black ones, while the second urn, *B*, contains black and white balls in unknown proportions. Subjects are asked to report their willingness to pay for the lottery *Aw*, that gives $100 if a white ball is drawn from urn *A*, and also for the lotteries *Ab*, *Bw*, and *Bb* that give the same amount of money in the other events. It turns out that a considerable number of people rank the lotteries $Aw \sim Ab \succ Bw \sim Bb$ which, obviously, is inconsistent with *any* probability assessment of the outcomes.

Recently, axioms have been put forward that accommodate the preferences observed in the Ellsberg example. Schmeidler (1989, first version 1982), employing the two-step framework of Anscombe and Aumann (1963) where both subjective and objective probabilities are at work, shows how a weakening of the *independence axiom* leads to a representation that accommodates the paradox. The weakening amounts to requiring independence only with respect to *comonotonic* acts (an act maps states nature to outcomes); i.e., acts that do not rank states of the world differently.[8] The result is similar to the Anscombe-Aumann representation in that the representation employs a cardinal utility function, while it differs in that the utility function is weighted not by an ordinary probability measure, but by a *non-additive* probability measure, a *capacity.* We will state the representation formally below. To see how it may work on the Ellsberg example, suppose that the subject assigns *probability* one half to each of the events involving urn A, while he assigns *capacity* one fourth to each of the events involving urn *B;* when computing expected values, he puts the remaining probability mass on the worst outcome.

Even though we discuss the Ellsberg paradox, the deviation from the expected utility axioms is much less conspicuous in this model since the situation is inherently asymmetric and the deviation is unambiguously in one direction. Here, the agent will be pessimistic in the sense that when uncertain about the probability of a favourable event versus a harmful one, he assigns a "probability equivalent" to the good event that is smaller than the expected probability.

Definitions and basic properties

We will start by defining a capacity and the notion of uncertainty aversion. Let Ω be an outcome space, and let Σ be an algebra on Ω.

Definition 1. A capacity is a set function K: $\Sigma \to [0,1]$ that satisfies $K(\emptyset) = 0$, $K(\Omega) = 1$, and that $E \subset F \Rightarrow K(E) \leq K(F)$.

That is, additivity is replaced by monotonicity in the definition of a probability measure. Further, we follow Dow and Werlang (1992) in defining eventwise uncertainty aversion quantitatively.

Definition 2. The *uncertainty aversion* of K at an event A is $c(K,A) = 1 - K(A) - K(\Omega \backslash A)$.

We will only consider $c \geq 0$, but the opposite is clearly conceivable. Expectations with respect to a capacity are defined by the Choquet integral which generalises the ordinary expectation.[9] The representation of preferences derived by Schmeidler (1989, theorem) is, letting

[8] *See Schmeidler (1989) or Karni and Schmeidler (1991) for a more precise account.*

[9] *The definition is originally due to Choquet (1954); see Schmeidler (1989), or Karni and Schmeidler (1991).*

the random variable X take only two values, $x_1 < x_2$, on $\Omega = \{\omega_1, \omega_2\}$,

$$u(X) = K(\Omega)u(x_1) + K(\omega_2)[u(x_2) - u(x_1)]$$
$$= K(\omega_2)u(x_2) + [1 - K(\omega_2)]u(x_1),$$

where the last expression uses $K(\Omega) = 1$. K is a uniquely determined capacity and u is a utility function determined up to positive affine transformations. The capacity reflects the relative (subjective) likelihoods of the various outcomes, as well as the attitude towards uncertainty. When $c(K,A) > 0$, the definition is equivalent to putting the "left-over" probability mass on the *smallest* value of $u(x)$ over the outcome space; this is the theory's notion of pessimism. Note that risk aversion (determined by the shape of u) and uncertainty aversion (determined by K) are completely separable within the framework. Note also that in the last expression the weights are non-negative and sum to one; we will refer to these objects as *probability equivalents.*

3. The model

We will consider a large number of policyholders (or, as we will call them interchangeably, *agents;* they will sometimes be referred to by the pronoun *he*), each of whom, for exogenous reasons, must participate in a two-outcome project. The project either fails or succeeds, and a failure results in a loss for the policyholder. There exists a risk-neutral party who may provide insurance; below we will consider also the case where there are many such parties competing.

The extensive form of the market game is simple. The insurer initially offers a contract which each policyholder may accept or reject. If an agent accepts, the outcome of the project – success or failure – is fully contractible.

The policyholder

The outcome, *Failure* or *Success*, will be denoted $A \in \{F, S\}$, and the probability of success is denoted q. If an agent undertakes the project by himself, he suffers a monetary loss of L units if he fails. Without loss of generality, we may assume that his monetary wealth in the two cases is w if the succeeds, and $w - L$ if he fails. Each agent has a cardinal utility-of-wealth function, v, that is strictly increasing and strictly concave.

Further, an additional component of his utility function, $s(\cdot)$, that enters additively, depends solely on A. This amounts to allowing the agent's *utility level,* but not his marginal utility, to be *state dependent.* It may also be interpreted as a utility cost due to the loss that is not subsumed by the monetary loss and which does not affect the marginal utility of money; e.g. the time cost of filing the claim. The reason for assuming such state dependence is that it highlights a potential discontinuity in agents' beliefs – we will discuss the assumption at some length below. We assume that Failure, F, is the bad state since this is the natural case (the other case is straightforward, but allowing it would burden the analysis unduly); without further loss of generality we let $s(S) = 0$ and $s(F) \leq 0$. Hence, an agent's utility from a certain outcome is,

$$(1) \qquad u(x, A) = v(x) + s(A).$$

An agent's reservation utility is defined by the absence of insurance; i.e., by the utility obtained by undertaking the project by himself. Before describing the agent's preferences over uncertain outcomes, we will describe the technology and the information structure.

Technology and information

The technology of the project is given by *success probabilities,* that vary across agents. There are no hidden actions. The success probabilities satisfy $q \in I = [\underline{q}, \bar{q}] \subseteq [0, 1]$.

There is symmetric information in the sense that neither the insurer, nor the agent himself, knows the success probability. The insurer, however, is assumed to know the population distribution and thus holds beliefs over q that are given by a probability measure, P, on I. The *expected probability* with respect to P is

$$(2) \qquad Q = \int_I q\,dP(q).$$

The specification reflects an environment where the population heterogeneity causing success probabilities to differ is *de facto* unobserved; i.e., an environment where policyholders cannot identify a subgroup to which they belong and thereby get a better estimate of their success probabilities. Even though such an assumption may seem strong taken literally, it is arguably a very plausible approximation of reality.

Uncertainty preferences

An agent initially perceives the environment from the point where he finds himself in the absence of a contract, and hence unambiguously considers F the bad state. We will assume that the agent holds a capacity, K, over the probability, q. Letting $\tilde{q}$ denote this random variable on I, we will further assume that,

$$(3) \qquad K(\tilde{q} \in O) \leq P(\tilde{q} \in O),\ O \subseteq I;$$

i.e., that K exhibits uncertainty aversion or uncertainty neutrality, and, importantly, that if there is no uncertainty aversion, there are homogeneous beliefs. The assumpion implies that an agent's *expected probability equivalent*, $\hat{q} \equiv E_K \tilde{q}$, satisfies $\hat{q} \leq Q$. This can be interpreted in two ways; either as saying that an agent knows the distribution of $\tilde{q}$ induced by P but is uncertainty averse, or as his being ignorant. In particular, we *allow for* the simplest case, that where agents know only I and apply the maximin criterion, i.e., ascribe the worst case, $\underline{q}$, probability equivalent one, but we also allow intermediate cases (i.e., we allow $\underline{q} \leq \hat{q} \leq Q$). In either case, the assumption captures the intuitive notion of the agents' being pessimistic due to uncertainty aversion.[10] We will, further, assume that if informed about the true value of q, each agent would be an expected-utility maximiser with respect to the *known* accident risk.

Assuming uncertainty *aversion* is well in accordance with experimental evidence. Viscusi *et al.* (1991) find that agents facing uncertainty generally will put more weight on unfavourable indications of the odds. In particular, the *worst case* seems to be taken into account and to be given some weight by itself (pp. 170–71).

As noted in the introduction, the difference between $\hat{q}$ and Q could have been generated also by assuming the parties to hold differing beliefs in the sense of differing probability measures for $\tilde{q}$. Such a model would seem economically rather empty, however, since the reason for the difference would be unexplained. It is also important to note that the difference between $\hat{q}$ and Q is not an instance of asymmetric information since parties have the same information but different probability assessments; moreover, the different assessments are commonly known.

An epistemic problem

The definition of the expectation with respect to a capacity as a Choquet integral can be said to state that the probability mass that lacks due to uncertainty aversion is moved to the worst outcome. An insurance contract may well *reverse* the monetary payoffs of the two outcomes. If the agents' utility function is completely described by v, such a contract then potentially leads to a jump in the probability equivalent of the agents' expectation, from $\hat{q} \leq Q$ to a $\hat{\hat{q}} \geq Q$ defined by

$$(4) \qquad 1 - \hat{\hat{q}} = E_K\,[1 - \tilde{q}];$$

i.e., defined by the lacking probability mass being assigned to the Success state.

This possibility is not as unnatural as it may seem since utility is constant at the point where the probability equivalent jumps, and the possibility is allowed below. Nevertheless, the discontinuity is unlikely to be important in practice since most insurance contracts involve some kind of moral hazard which rules out that failures be rewarded.

Our specification, $u(x, A) = v(x) + s(A)$, turns out to imply that the resulting contract has the non-standard feature of a negative deductible whenever $s(F) < 0$; moreover, the natural analogue – that the deductible is distorted down-

[10] *Note that $E_K[1 - \tilde{q}] \leq 1 - \hat{q}$, but that the additional weight is put on the worst outcome so that $(\hat{q}, 1 - \hat{q})$ is the agent's probability equivalent resulting from the capacity over $\tilde{q}$.*

wards – holds in the presence of moral hazard, or in the presence of a "risk loading". We would like to remark that for some applications the assumed state dependence, though somewhat *ad hoc,* is very reasonable. Those applications include insurance contracts where a policyholder may prefer not to have an accident or theft even if he were to be slightly over-compensated monetarily, and they include cases with a non-negligible non-monetary cost of having an accident or filing a claim.

The contract

All insurance contracts for a two-outcome project are subsumed by the formulation where the insurer charges the policyholder p for taking on the risk, and punishes him with a deductible D in case of failure. The indirect utility for an agent from such a contract is thus

(5) $$u(p,D) = \hat{q}v(w-p) + (1-\hat{q})v(w-p-D) + (1-\hat{q})\,s(F),$$

and the reservation utility is $\underline{u} = \hat{q}v(w+(1-\hat{q})v(w-L)+(1-\hat{q})\,s(F)$. Note that the reservation utility depends on beliefs, but not on the contract offered (F is unambiguously the bad state). This feature seems perfectly reasonable in the environment considered where all agents are offered the same contract; in an environment with adverse selection, one would need to address how uncertainty preferences would be affected by information revealed by contracts.

4. Results

The analysis will start from the simplest case by avoiding issues of moral hazard. Although unrealistic, we strongly believe that this makes the analysis as transparent as possible; we then go on to discuss generalisations to more realistic environments.

It is clear that the strongest notion of first-best solution would be defined by complete information, i.e., by all parties *knowing q.* At that solution, policyholders would have constant wealth across states of nature.

Solving the model

The problem for a monopoly insurer is simply to maximise profits,

(6) $$\pi = p-(1-Q)\cdot(L-D),$$

subject to the policyholder attaining his reservation utility; i.e., subject to (where $\hat{q} \leq Q \leq \hat{\hat{q}}$)

(7) $$\hat{q}v(w-p) + (1-\hat{q})[v(w-p-D) + s(F)] \geq \underline{u}$$
$$\text{if } v(w-p-D) + s(F) \leq v(w-p),$$
$$\hat{\hat{q}}v(w-p) + (1-\hat{\hat{q}})[v(w-p-D) + s(F)] \geq \underline{u}$$
$$\text{if } v(w-p-D) + s(F) \geq v(w-p),$$

Note that there is no ambiguity at the border case since utility is independent of q there.

Suppose first that $s(F)$ is negative enough for the first case to apply. Then the first-order conditions (which are sufficient) are easily seen to imply that

(8) $$v'(w-p-D) = \frac{\hat{q}\cdot(1-Q)}{(1-\hat{q})\cdot Q}v'(w-p);$$

since $\hat{q} \leq Q$ by assumption and since v is concave, we see that whenever the inequality is strict, $D < 0$, and the agent is monetarily better off in case of failure. Consider now an arbitrary value of $s(F) \leq 0$ and consider the two indifference curves defined in $p-D$-space (which are downward sloping and concave); the first case in (7) applies whenever D is greater than the cut-off value D^* defined by $v(w-p-D^*) + s(F) = v(w-p)$, and the second case applies when the opposite is true. Note that $D^* < 0$ whenever $s(F) < 0$. The situation is illustrated

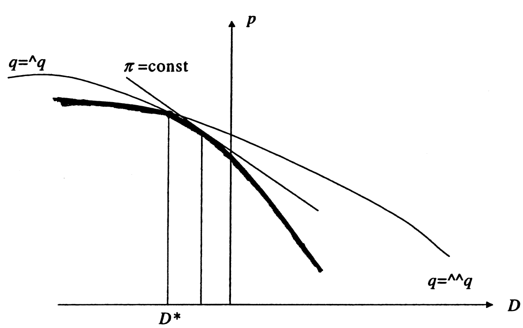

Figure 1. The indifference curve.

in figure 1, the actual indifference curve being the lower envelope of the two. The figure also displays an isoprofit line; note that the slope of the isoprofit line is $(-(1-Q))$ which is in between the slopes of the two indifference curves at $D = 0$.

It is clear that the solution is at the point defined by the first-order condition (8) whenever this point is to the right of D^* as in the figure; if that is not the case, the solution is at the kink at D^* (since the "$\hat{\hat{q}}$-indifference curve" is flatter than the isoprofit line for $D \leq 0$). We have thus proved (where the last part is immediate from (8)):

Proposition 1. If the agent is uncertainty averse and if $s(F) < 0$, the optimal contract will entail a negative deductible that continuously approaches zero with $Q - \hat{q}$.

A consequence of this theory is that agents will demand full insurance also if the insurer (letting the policyholders choose coverage) charges a (small enough) *risk loading;* i.e., a profit margin, α, proportional to payouts, $L - D$.[11] Moreover, this is true also in the absence of state dependence. To see this, consider $s(F) = 0$ and hence $D^* = 0$, and note that the first-order condition for a solution on the "$\hat{q}$-indifference curve" is[12]

$$(9) \qquad v'(w-p-D) = \frac{\hat{q} \cdot (1 - Q + \alpha)}{(1 - \hat{q}) \cdot (Q - \alpha)} v'(w-p).$$

Whenever $\hat{q} \cdot (1 - Q + \alpha) < (1 - \hat{q}) \cdot (Q - \alpha)$ – i.e. if $\hat{q} < Q$ and $\alpha > 0$ is small enough – this defines $D < 0$, and hence the solution is at the kink, $D^* = 0$. Thus:

Claim 2. Whenever there is strict uncertainty aversion (i.e. $\hat{q} < Q$), there is an $\alpha^* > 0$ such that the solution entails full insurance if the risk loading, α, satisfies $0 < \alpha < \alpha^*$.

The interpretation of the fact that one-sided uncertainty aversion leads to over-insurance is that there is a *demand for protection against uncertainty.* This demand, as well as its distorting the deductible downwards, are general features of uncertainty-aversion insurance models, but over-insurance depends on the absence of moral hazard – we will come back to this below. The over-insurance result may seem less conspicuous if one, for comparison, keeps in mind that in the case of pure risk, full insurance equalises *marginal utilities*, not utility levels; even with solid probabilities and no transaction costs, the full-insurance intuition relies on state-independence of marginal utility.

The analysis is made for the case where all policyholders are identical. If policyholders do not have the same degree of uncertainty aversion, and if the degree of uncertainty aversion is observable by the insurer, it is optimal for the insurer to offer a menu of contracts with the strength of the distortion increasing with uncertainty aversion. Less obviously, the same qualitative properties hold for the menu of contracts that is optimal if the degree of uncertainty aversion is private information and policyholders select contracts from the menu.[13]

It is worth noting that there would be no incentive for the insurer to reveal the true success probability to each policyholder even if this was possible, this because pessimism benefits the insurer. Note also that mathematically, the argument is related to the reason for over-insurance in the presence of heterogeneous beliefs. One may also note from expression (8) that whenever there is uncertainty aversion, D is larger in magnitude the less concave is v, and if the policyholders were risk neutral, both they and the insurer would prefer to make p and D arbitrarily large in magnitude. This is an obvious consequence of pessimism, and in our opinion it points to the implausibility of an agent being risk neutral but strictly uncertainty averse.

---

[11] *In the standard model without uncertainty aversion, a policyholder prefers a strictly positive deductible whenever there is a risk loading.*

[12] *This problem is defined by the policyholder choosing coverage, $L - D$, subject to a unit price which, with risk loading α, is $1 - Q + \alpha$. Formally thus, the policyholder maximises $\hat{q}v(\omega - (1 - Q + \alpha)(L - D)) + (1 - \hat{q})v(w - L + (1 - (1 - Q + \alpha))(L - D))$ over D. Monopoly rents can be extracted by a fixed fee which is subsumed in ω. The problem is formally identical to the original monopoly problem, and the first-order condition is (9).*

[13] *The analysis is straightforward but a bit messy; the argument is provided in the appendix for the case with two types.*

The result of this paper has a flavour similar to that of the theory of *first-order risk aversion.* According to this theory the willingness to pay for insurance does not become negligible relative to the risk as the magnitude of the risk goes to zero (as is true in the standard expected-utility model as long as indifference curves are smooth).[14] One consequence of this theory is the analogue of Claim 2, namely that agents will demand full insurance also if the insurer charges a (small enough) risk loading. That theory is thus similar to that of this paper in implying that more insurance is demanded, but different in that it does not predict a demand for more than full insurance (this is true also with the kind of state dependence considered in this model).

Market conditions

We will now examine how our model is affected by competition among insurers. Suppose that insurers bid for agents in a competitive fashion. It is then clear that any contract that either is inefficient, or makes strictly positive profits, cannot be viable since in both cases there exists a contract which is strictly more attractive to agents, and which makes positive profits as well. The only contract making non-negative profits for which this is not true is the efficient one making zero profits; that is, a contract where (p, D) are determined by the appropriate first-order condition (i.e., possibly at the kink) and the zero-profit condition. This proves an analogue to Proposition 1: that if the agent is uncertainty averse, the equilibrium contract under competition will make zero profit and entail a negative deductible that continuously approaches zero with $Q - \hat{q}$.

Our model thus turns out to be robust with respect to an important feature of the environment. The reason is chiefly the absence of asymmetric information. Sensitivity to the specification of competition is generally a problem in agency models.[15]

case.

[14] *See e.g. Segal and Spivak (1990) for a precise statement of assumptions and results.*

[15] *The issue is discussed in an adverse selection framework by Rothschild and Stiglitz (1976) and in a moral hazard framework by Arnott and Stiglitz (1991).*

Moral hazard

Even though we will not elaborate on moral hazard, it is useful to discuss a simple case, thereby pointing out in what ways the results arrived at so far depend on the absence of moral hazard.

We assume that *effort, e,* exerted by a policyholder affects his success probability in a specific fashion. The success probability is $q + r(e)$, $e \in [0, \infty)$, where q is distributed in the population precisely as in Section 3; i.e., the ordinary expectation is Q, and each policyholder holds a capacity that renders the expected probability equivalent $\hat{q} \leq Q$. His perceived success probability is thus $\hat{q} + r(e)$. The extra probability resulting from effort satisfies $r'(e) > 0$ and $r''(e) < 0$.[16] Furthermore, each agent's utility function is assumed to be, for wealth level x, $u(x, e) = v(x) - e$.

With these requisites, we can – in the absence of corner solutions – prove the following:

Proposition 3. Under the assumptions stated, the deductible is distorted downwards by uncertainty aversion

Proof. See the Appendix.

It is conceivable that the problem has a corner solution; such a solution entails the lowest possible effort, i.e. $e = 0$, being optimal. At a corner solution, $e = 0$ and the analysis of Section 4 applies; it is ruled out by effort being sufficiently valuable (e.g. if $r'(0)$ is large enough).

The theory thus predicts *small* deductibles. The result is intuitively very simple; D performs two tasks, it provides insurance and incentives, and the tradeoff leads to an intermediate optimum which is pushed downward by the fact that there is trade in ambiguity.

5. Conclusion

We have developed a simple model that explicitly recognises the asymmetry between insurers and policyholders that arises if the *insurer* has superior knowledge of the nature of

[16] *The q and r must, obviously, staisfy that $0 \leq q + r(e) \leq 1$ for all possible realisations of q and effort levels e.*

the risk involved. In more precise terms, insurers in effect face the population average, while policyholders are uncertain about their own probabilities. This framework is, arguably, quite realistic, and we believe that allowing for uncertainty aversion provides insurance models with an extra degree of freedom without leading too far into "the wilderness of bounded rationality".

The analytics of our model are simple since we do not incorporate asymmetric information in the traditional sense of adverse selection and moral hazard. Nevertheless, it makes strongly a point that seems robust indeed: ambiguity in combination with uncertainty aversion leads to optimal contracts providing *more extensive insurance coverage* than would be the case in the absence of uncertainty aversion. This observation can reconcile the magnitude of observed deductibles with reasonable degrees of risk aversion.

The logic behind the over-insurance result is that an optimal insurance contract not only provides protection against risk, but also protection against ambiguity. This relates to the general point that a complete view of insurance needs to recognise that there is, in addition to trade in protection, trade in e.g. risk assessment and risk management. Mathematically, the over-insurance result is related to risk-averse parties' willingness to bet on an event on the basis of different beliefs.

There are several outstanding issues highlighted by our simple model. Firstly, the presence of uncertainty aversion raises a number of normative issues; in particular, what are the welfare implications of policyholders' being uncertainty averse. This issue is particularly pertinent regarding other types of insurance than have been considered here, such as health insurance. However, for normative work, more comprehensive models are desirable; moral hazard, adverse selection, and dynamics are important in practice, and a complete view must include them. Secondly, empirical work is too short in supply within the economics of uncertainty, as well as in information economics. It seems clear that the insurance market is one of the most promising laboratories for basic empirical research. In particular, we believe that market data from insurance would be a good source if one were to look for uncertainty aversion "out there"; we hope to explore this source in future research.

References

Andersson, F. (1997). "Adverse selection and bilateral asymmetric information." Manuscript, Department of Economics, Harvard University.

Anscombe, F.J., and R.J. Aumann (1963). "A definition of subjective probability." *The Annals of Mathematical Statistics* 34: 199–205.

Arnott, R., and J. Stiglitz (1991). Equilibrium in competitive insurance markets with moral hazard. NBER Working Paper, No 3588.

Brandenburger, A., E. Dekel, and J. Geanakoplos (1992). "Correlated equilibrium with generalized information structures." *Games and Economic Behavior* 4: 182–201.

Choquet, G. (1954). "Theory of capacities." *Ann. Stat. Fourier,* Grenoble, 5: 131–295.

Dionne, G., and S.E. Harrington (1992). In "An introduction to insurance economics." *Foundations of Insurance Economics.* Eds. G. Dionne, and S.E. Harrington. Dordrecht: Kluwer Academic Press.

Dow, J., and S. Werlang (1992). "Uncertainty aversion, risk aversion, and the optimal choice of a portfolio." *Econometrica* 60: 197–204.

Ellsberg, D. (1961). "Risk, ambiguity, and the Savage axioms." *Quarterly Journal of Economics* 75: 643–69.

Friedman, B. (1974). "Risk aversion and the consumer choice of health insurance option." *Review of Economics and Statistics* 56: 209–14.

Hendon, E., H.-J. Jacobsen, B. Sloth, and T. Tranaes (1994). "Expected utility with lower probabilities." *Journal of Risk and Uncertainty* 8: 197–216.

Hogarth, R., and H. Kunreuther (1989). "Risk, ambiguity, and insurance." *Journal of Risk and Uncertainty* 2: 5–35.

Karni, E., and D. Schmeidler (1991). "Utility theory with uncertainty." In *Handbook of Mathematical Economics.* Eds. W. Hildenbrand, and H. Sonnenschein. Vol IV. Amsterdam: North-Holland.

Keynes, J.M. (1921). *A Treatise on Probability.* London: MacMillan and Co.

Knight, F. (1921). *Risk, Uncertainty and Profit.* Boston: Houghton Mifflin.

Kreps, D.M. (1990). *A Course in Microeconomic Theory.* Princeton: Princeton University Press.

Kunreuther, H., R. Hogarth, and J. Meszaros (1993). "Insurer ambiguity and market failure. *Journal of Risk and Uncertainty* 7: 71–87.

Rothschild, M. and J. Stiglitz (1976). "Equilibrium in competitive insurance markets: an essay on the economics of imperfect information." *Quarterly Journal of Economics* 90: 629–49.

Savage, L.J. (1954). *The Foundations of Statistics.* New York: John Wiley.

Schmeidler, D. (1989). "Subjective probability and expected utility without additivity." *Econometrica* 57: 571–87.

Segal, U., and A. Spivak (1990). "First order versus second order risk aversion." *Journal of Economic Theory* 51: 111–25.

Shavell, S. (1979). "On moral hazard and insurance." *Quarterly Journal of Economics* 93: 541–62.
Stiglitz, J.E. (1977). "Monopoly, non-linear pricing and imperfect information: the insurance market." *Review of Economic Studies* 44: 407–30.
Stuart, C. (1983). "Pareto optimal deductibles in property-liability insurance: The case of homeowner insurance in Sweden." *Scandinavian Actuarial Journal* 1983: 227–38.
Viscusi, K. (1993). "The risky business of insurance pricing." *Journal of Risk and Uncertainty* 7: 117–39.
Viscusi, K., W. Magat, and J. Huber (1991). "Communication of ambiguous risk information." *Theory and Decision* 31: 159–73.

Appendix

Proof Proposition 3. The distortion is due to the policyholder perceiving the success probability to be $\hat{q}+r(e)$ rather than $Q + r(e)$. The monopoly problem is

$$\max \quad p-(1-Q-r(e))(L-D)$$

(A.1) $$\text{s.t.} \quad (\hat{q}+r(e))\cdot v(w-p)+(1-\hat{q}-r(e))\cdot v(w-p-D)-e \geq \underline{u} \qquad (\lambda)$$
$$r'(e)\cdot(v(w-p)-v(w-p-D))-1=0 \qquad (\mu)$$

and the first-order conditions are

(A.2) $$1-\lambda\cdot[(\hat{q}+r(e))v'(w-p)+(1-\hat{q}-r(e))v'(w-p-D)]+\mu\cdot r'(e)\cdot[v'(w-p-D)-v'(w-p)]=0,$$

(A.3) $$1-Q-r(e)-\lambda\cdot(1-\hat{q}-r(e))v'(w-p-D)+\mu\cdot r'(e)v'(w-p-D)=0,$$

(A.4) $$r'(e)\cdot(L-D)+\mu\cdot r''(e)\cdot((v(w-p)-v(w-p-D)))=0,$$

where the "λ-term vanishes from (A.4) due to the policyholder's first-order condition. Subtracting (A.3) from (A.2) we get

(A.5) $$Q+r(e)-\lambda\cdot(\hat{q}+r(e))v'(w-p)-\mu\cdot r'(e)\cdot v'(w-p)=0,$$

and dividing (A.5) by (A.3) we get,

(A.6) $$\frac{Q+r(e)}{1-Q-r(e)}=\frac{\lambda\cdot(\hat{q}+r(e))v'(w-p)+\mu\cdot r'(e)\cdot v'(w-p)}{\lambda\cdot(1-\hat{q}-r(e))v'(w-p-D)-\mu\cdot r'(e)v'(w-p-D)}$$

or,

(A.7) $$1=\frac{(1-Q-r(e))(\hat{q}+r(e))v'(w-p)+\mu/\lambda\cdot(1-Q-r(e))\cdot r'(e)\cdot v'(w-p)}{(Q+r(e))\cdot(1-\hat{q}-r(e))v'(w-p-D)-\mu/\lambda\cdot(Q+r(e))\cdot r'(e)\cdot v'(w-p-D).}$$

The difference compared to a case without uncertainty aversion is precisely presence of $\hat{q}$ which would be Q in the absence of uncertainty aversion; this difference distorts $v'(w-p-D)$ downwards, and hence distorts D downwards.

Q.E.D.

Proof of statement concerning differing levels of uncertainty aversion

We start by noting that *if* the principal could observe the type of agents (i.e., their degree of uncertainty aversion and hence their values of $\hat{q}$), it follows immediately from Proposition 1 that she would offer contracts to each type along the lines of that proposition. Since she does not, we must consider incentive compatibility. Consider 2 types $i = 1, 2$, with $\hat{q}^1 < \hat{q}^2$ and the proportion of type-2 agents μ; with obvious notation$\hat{q}^i \leq Q^i$ (the natural case is $Q^1 = Q^2$ if the difference between agents is uncertainty aversion, but the analysis is the same for this slightly more general case). We will characterise the optimal contracts, for the case where $s(F)$ is negative enough for the solution not to be at the kink. Let us introduce the following notation, and let primes denote ordinary derivatives evaluated at the points indicated by the indices (note that throughout, superscripts refer to type),

$$v_1^1 = v(w - p^1 - D^1), \quad v_2^1 = v(w - p^1),$$
$$v_1^2 = v(w - p^2 - D^2), \quad v_2^2 = v(w - p^2).$$

The problem faced by the principal is now

$$\max_{p^1, p^2, D^1, D^2} \quad \pi = (1 - \mu)[p^1 - (1 - Q^1)(L - D^1)] + \mu[p^2 - (1 - Q^2)(L - D^2)]$$

$$\text{(A.8)} \quad \begin{aligned} \text{s.t.} \quad & (1 - \hat{q}^1)[v_1^1 + s] + \hat{q}^1 v_2^1 \geq \underline{u}^1 & (\alpha^1) \\ & (1 - \hat{q}^2)[v_1^2 + s] + \hat{q}^2 v_2^2 \geq \underline{u}^2 & (\alpha^2) \\ & (1 - \hat{q}^1)[v_1^1 + s] + \hat{q}^1 v_2^1 \geq (1 - \hat{q}^1)[v_1^2 + s] + \hat{q}^1 v_2^2 & (\lambda^1) \\ & (1 - \hat{q}^2)[v_1^2 + s] + \hat{q}^2 v_2^2 \geq (1 - \hat{q}^2[v_1^1 + s] + \hat{q}^2 v_2^1 & (\lambda^2) \end{aligned}$$

where the first two constraints are the participation constraints just like above, while the last two constraints are the *incentive compatibility constraints*, guaranteeing that the principal's intention to separate the two categories of agents works as intended. The first-order conditions are,

$$\frac{\partial L}{\partial p^1} = 1 - \mu - (\alpha^1 + \lambda^1)[(1 - \hat{q}^1)v'^1_1 + \hat{q}^1 v'^1_2] + \lambda^2[(1 - \hat{q}^2)v'^1_1 + \hat{q}^2 v'^1_2] = 0,$$

$$\frac{\partial L}{\partial D^1} = (1 - \mu)(1 - Q^1) - (\alpha^1 + \lambda^1)(1 - \hat{q}^1)v'^1_1 + \lambda^2(1 - \hat{q}^2)v'^1_1 = 0,$$

$$\frac{\partial L}{\partial p^2} = \mu - (\alpha^2 + \lambda^2)[(1 - \hat{q}^2)v'^2_1 + \hat{q}^2 v'^2_2] + \lambda^1[(1 - \hat{q}^1)v'^2_1 + \hat{q}^1 v'^2_2] = 0,$$

$$\frac{\partial L}{\partial D^2} = \mu(1 - Q^2) - (\alpha^2 + \lambda^2)(1 - \hat{q}^2)v'^2_1 + \lambda^1(1 - \hat{q}^1)v'^2_1 = 0.$$

Manipulations according to $\partial_{p^i} L - \partial_{D^i} L / (1 - Qi) = 0$ gives for the two categories respectively,

$$\text{(A.9)} \quad (\alpha^1 + \lambda^1)\left[\left\{\frac{1 - \hat{q}^1}{1 - Q^1} - (1 - \hat{q}^1)\right\}v'^1_1 - \hat{q}^1 v'^1_2\right] + \lambda^2\left[\left\{(1 - \hat{q}^2) - \frac{1 - \hat{q}^2}{1 - Q^1}\right\} v'^1_1 + \hat{q}^2 v'^1_2\right] = 0,$$

$$\text{(A.10)} \quad (\alpha^2 + \lambda^2)\left[\left\{\frac{1 - \hat{q}^2}{1 - Q^2} - (1 - \hat{q}^2)\right\}v'^2_1 - \hat{q}^2 v'^2_2\right] + \lambda^1\left[\left\{1 - \hat{q}^1 - \frac{1 - \hat{q}^1}{1 - Q^2}\right\} v'^2_1 + q^{\wedge 1} v'^2_2\right] = 0;$$

or, factoring out $1/(1 - q^i)$,

$$\text{(A.11)} \quad (\alpha^1 + \lambda^1)\ [\{Q^1(1 - \hat{q}^1)\}v'^1_1 - (1 - Q^1)\hat{q}^1 v'^1_2] + \lambda^2[\{-Q^1(1 - \hat{q}^2)\}v'^1_1 + (1 - Q^1)\hat{q}^2 v'^1_2] = 0,$$

$$\text{(A.12)} \quad (\alpha^2 + \lambda^2)\ [\{Q^2(1 - \hat{q}^2)\}v'^2_1 - (1 - Q^2)\hat{q}^2 v'^2_2] + \lambda^1[\{-Q^2(1 - \hat{q}^1)\}v'^2_1 + (1 - Q^2)\hat{q}^1 v'^2_2] = 0.$$

In the absence of incentive compatibility constraints, the expressions in the first brackets are zero, and it is easy to verify that, as a consequence, at that solution, the last term of (A.11) is positive, while the last term of (A.12) is negative. This implies that if $\lambda^2 > 0$, then D^1 is distorted further downward by incentive compatibility, while if $\lambda^1 > 0$, then D^2 is distorted upwards.