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ADVERSE SELECTION, COMPETITION, AND LINEAR SELF-INSURANCE*

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A two-class insurance model is analysed. In addition to a competitive insurance market, the households can use a simple linear self-insurance technology. Using the recently proposed Coalition Proof Equilibrium with Private Information due to Kahn and Mookherjee (1995) the insurance market equilibrium is found to be either separating or pooling. There may be profits in equilibrium. The self-insurance option can, but does not necessarily, promote more efficient allocation of consumption; self-insurance may be dysfunctional, lowering welfare. The model is applied to a competitive private pension market where the households in addition can save in a bequeathable asset. (JEL D41, D82, D89)

1. Introduction

The research on competitive equilibria in insurance markets with adverse selection has revealed that, in many cases, consumers will not be offered full insurance. E.g. in the traditional Rothschild and Stiglitz (1976) model of an insurance market with two types of agents differing in the probability of suffering a loss, an agent with a low probability of a loss will, in a separating equilibrium, obtain a contract that does not even the net incomes in the loss- and the non-loss state. This can be given the interpretation of an endogenous quantity constraint; the contract obtained by the low-risk agents is actuarially fair in that it generates zero profits, but at an actuarially fair price the agent would, if he could choose the quantity freely, opt for full insurance.

If an agent cannot obtain full insurance in the insurance market, he may look for other ways to even his consumption. In particular, he may look for some method to self-insure.[1] A consistent analysis of adverse selection and self-insurance must, however, take the self-insurance option into account already when determining the outcome in the insurance market. Self-insurance from the point of view of the insurance firms constitute unobserved actions taken by the consumers. The unobservability means that the insurance contracts cannot be made contingent

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[1] *Examples of actions that can be interpreted as self-insurance may include choosing a less vulnerable crop to reduce the loss from e.g. hail, installing sprinkler systems to reduce the loss in case of fire, or choosing a portfolio to reduce the variance in consumption as e.g. in an application outlined in section 4.*

upon the action. However, we do not encounter the problems studied in the literature on simultaneous adverse selection and moral hazard (see e.g. Picard, 1987). The reason for this is that the unobserved action does not affect the profitability of a contract. Yet the insurance firms must be aware of the potential self-insurance activity since it will affect the consumer's preferences over contracts. Indeed, in that respect a self-insurance option has an effect that resembles that of a decrease in the level of risk aversion.

This paper investigates the effect of a simple linear self-insurance technology, available to all consumers, in a standard model of an insurance market with adverse selection. To do this the "Coalition Proof Equilibrium with Private Information" proposed by Kahn and Mookherjee (1995) is employed. The strenght of this concept lies in its consistency; in order to break an equilibrium a contract must itself be viable.[2] This has the effect that an equilibrium always exists. The main findings are; (i) the competitive insurance market equilibrium may separate or pool the two types, (ii) there may be profits in a separating equilibrium, (iii) in a pooling equilibrium the high-risk consumers engage in self-insurance, (iv) a self-insurance option can, but need not, improve the welfare of the consumers. The fourth observation indicates while self-insurance is functional on the individual level, it is need not be so in the equilibrium context.

We also provide an application. In a two period model of a private pension market, the option of "topping up" private pensions by saving in a bequeathable asset serves as a linear self-insurance technology.

The outline is as follows. Section 1 sets up the model and derives preferences over insurance contracts. Section 2 studies insurance market equilibria and considers the effect of the self-insurance option on the welfare of the consumers. Section 3 describes the application. Section 4 concludes.

[2] *The concept used is in this respect closely related to Riley's (1979) "reactive equilibrium". A comparison with other solution concepts, including the reactive equilibrium is provided at the end of section 3.*

2. The model

Consider the following standard setup. Each household has wealth m, but may suffer a loss l (of fixed size). This defines a *pre-insurance endowment* $\omega = (\omega_1, \omega_2)$, where $\omega_1 = m$ and $\omega_2 = m - l$. There are two types of households, $i = L, H$, differing in loss probabilities. A fraction θ_i of the households are of type i. The probability of a loss is p_i for type i with $p_L < p_H$.

Let $c = (c_1, c_2)$ dentote a *consumption plan* and denote type i's expected utility by $Eu_i\ (c)$,

$$(1) \qquad Eu_i\ (c) = (1 - p_i)\ u\ (c_1) + p_i u\ (c_2).$$

u is assumed to be strictly increasing, strictly concave and twice continuously differentiable. As is well-known, the preferences Eu_L and Eu_H satisfy a *single-crossing property* in the space of consumption plans; the level curve of Eu_L is, at any c, strictly steeper than that of Eu_H.

By signing a *contract* x with an insurance firm, a household changes its pre-insurance endowment ω to a *post-insurance endowment* $x = (x_1, x_2)$. Some definitions are useful.

Definition 1. Contract x is *actuarially fair* for type i if $\sigma_i\ (x) \equiv (1 - p_i)\ (\omega_1 - x_1) + p_i\ (\omega_2 - x_2) = 0$.

If x satisfies $\sigma_i\ (x) > 0$, type i is said to be *exploited* (at x), while if $\sigma_i\ (x) < 0$ type i is said to be *subsidized.*

Definition 2. A contract x is *actuarially fair* on average if $\bar{\sigma}\ (x) \equiv \sum_i \theta_i \sigma_i\ (x) = 0$.

The twist that we explore in this paper is that of a self-insurance option. Following Becker and Ehrlich (1972) we denote by self-insurance an activity that allows a household to increase its (net) income in the loss state by giving up income in the good state. A simple linear self-insurance technology is assumed; by giving up e units of income in the good state, income in the bad state is increase by λe units, where $\lambda > 0$. Given a contract x and a self-insurance decision e, planned consumption is $c = (x_1 - e, x_2 + \lambda e)$. All households have the same technology available.

Two restrictions are imposed on the self-insurance technology. First, it cannot be reversed,

$e \geq 0$. Secondly, it is inferior to actuarial insurance in the following sense:

Assumption 1. Self-insurance is less efficient than actuarial insurance in transforming income in the good state into income in the bad state, $\lambda < (1 - p_i)/p_i,\ i = H, L.$[3]

It is assumed that no insurance firm can observe a household's self-insurance decision; the self-insurance decision can be made *after* an insurance contract has been signed.

Preferences over insurance contracts

To study the outcome in the insurance market we determine preferences over contracts. Given x, the utility achieved by type i is

$$(2)\qquad V^i(x) \equiv \max_{e_i \geq 0} \{Eu_i(c)\,|\,c_1 = x_1 - e_i,\ c_2 = x_2 + \lambda e_i\},\ i = L, H.$$

There are three observations to be made from this problem.

1. There is a set of contracts β_i such that type i self-insures at x if and only if x is in the interior of β_i. The boundary of β_i consists of the contracts where the indifference curves of Eu_i have slope $-\lambda$.
2. If a type i household self-insures it plans consumption on the boundary of β_i.
3. Since $p_H > p_L$, type H is more prone to self-insure.

The first two observations are illustrated in figure 1. Note that the same figure can be used to illustrate contracts as well as consumption plans. The figure depicts an arbitrary contract x, the locus of consumption plans attainable through self-insurance given x, and an indifference curve of Eu_i. Maximizing Eu_i given x and the self-insurance option, the household's chosen consumption plan is c^i, on the boundary of β_i.

To formalize these observations, let the solution to (2) be denoted $e_i^*(x)$, and define

$$(3)\qquad \beta_i \equiv \{x\,|\,(1 - p_i)\,u'(x_1) \leq \lambda p_i u'(x_2)\}.$$

[3] *Obviously, $p_L < p_H$ implies that if the inequality holds for i = H, it also holds for i = L.*

First we note some properties. From the definition of β_i we have

Lemma 1. Properties of β_i, $i = L, H$:
(1) *If $x \in \beta_i$, then $x_1 > x_2$.*
(2) $\beta_L \subset \beta_H$.

Part 1 follows from assumption 1 and part 2 follows from $p_L < p_H$. Stating observations 1 and 2 formally we now have:

Lemma 2. The solution to problem (2) satisfies:
(1) $e_i^*(x) > 0$ if and only if $x \in \text{Int}\,\beta_i$.
(2) *if* $x \in \beta_i$, then $(1 - p_i)\,u'(x_1 - e_i^*(x)) = \lambda p_i u'(x_2 + \lambda e_i^*(x))$

The proof is straightforward and is omitted.

Combined with part 2 of lemma 1, part 1 gives the third observation above.

V^i represents type i's preferences over contracts. To grasp the shape of these preferences we can again refer to figure 1. Type i considers all contracts along the segment x to c^i to be equally good – by self-insurance they all result in the consumption plan c^i. On the other hand, at some other contract $x' \notin \beta_i$ type i plans consumption x'. This reasoning shows an indifference curve of V^i coincides with the corresponding indifference curve of Eu_i at contracts not in β_i, but is flattened out in β_i by the self-insurance option. Rather than continuing to curve as the indifference curves for Eu_i, the indifference

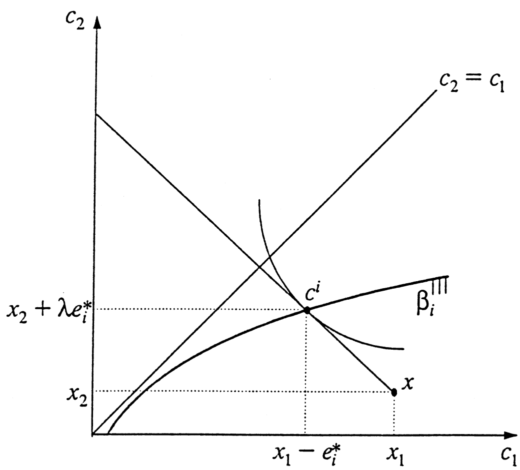

Figure 1: At contract x type i self-insures e_i^* and attains the consumption plan c^i where the slope of the indifference curve is $-\lambda$.

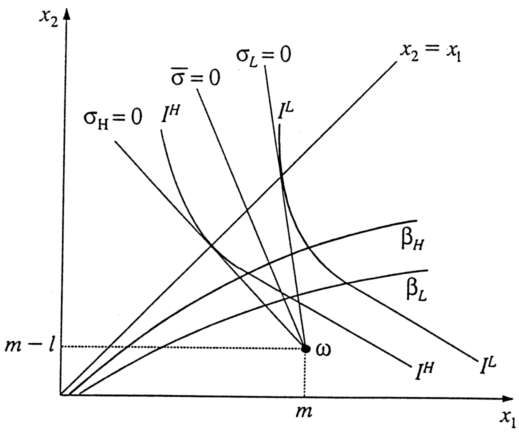

Figure 2: The sets of acturially fair contracts, some indifference curves for the preferences over insurance contracts, and the sets β_i, $i = L, H$.

curves for V^i become linear in β_i due to the linear self-insurance technology.

Lemma 3. An indifference curve of V^i in the space of insurance contracts has slope

$$\text{(4)} \quad \left.\frac{dx_2}{dx_1}\right|_{V^i\ constant} = \begin{cases} -\dfrac{(1-p_i)\,u'(x_1)}{p_i u'(x_2)} & at\ x \notin \beta_i \\ -\lambda & at\ x \in \beta_i \end{cases}, \quad i = L, H$$

The upshot is that the derived preferences over contracts, V^L and V^H, do *not* exhibit the same strict single-crossing property as the primitive preferences over consumption plans, Eu_L and Eu_H. Instead, as can be verified from lemma 3, the indifference curve of V_L is steeper than that of V^H through any contract x, *except* at contracts in β_L, where both have slopes equal to $-\lambda$.

Figure 2 summerizes the main features obtained thus far. To make the problem interesting we assume that the pre-insurance endowment ω is in β_L which guarantees that, if left to their own devices, both types of households would engage in self-insurance.

3. Insurance market equilibria

In this section we use the derived preferences V^L and V^H to study the insurance market. Several solution concepts could be employed, notably the screening approach due to Rothschild and Stiglitz (1976), Wilson's (1977) "Anticipatory Equilibrium", Riley's (1979) "Reactive Equilibrium" and Hellwig's (1987) three-stage game.[4]

These well-known concepts have in common that they postulate dynamic games to represent the contracting process. An unattractive feature of this approach is that the predictions turn out to be quite sensitive to the imposed timing of play. Kahn and Mookherjee (1995) consider instead a simultaneous move game where firm offer contracts and households request contracts. To select among the set of Nash equilibria the authors suggest that coordinated deviations by coalitions should be allowed. This leads them to define a "Coalition Proof Equilibrium with Private Information" (CPEPI).

The CPEPI concept is intended to capture the essence of a decentralized preplay negotiation process. The decentralized nature of the negotiation process is reflected in an agreement between members of a coalition not being observed by non-members. Since this means that any player can choose not to honour an agreement (by deviating unilaterally) without the cheated party being able to react, there are essentially no commitment abilities.

A predicted outcome must be self-enforceable. There are three sides to this requirement. First, it must be that no individual player can gain by a unilateral deviation. Secondly, an equilibrium must not be threatened by coordinated deviations. Third, to break an equilibrium, a deviation must itself be stable – importantly, a deviation does not pose threat to a proposed equilibrium if a subset of the deviating players can gain by a deviating a second time.

The presence of private information requires careful modelling of self-enforceability.[5] The consumers are *indistinguishable* since their types are private information. The members of a deviating coalition must be confident of each

[4] *See e.g. Kreps (1990) for a short discussion of these concepts and the timing structures they impose.*

[5] *Here we present informally the definitions used in CPEPI concept. The formal definitions can be found in Kahn and Mookherjee, and are also reproduced in an appendix.*

others identities. More precisely, a deviation is *vulnerable to infiltration* if some non-member, indistinguishable from a true coalition member, (i) finds it to her advantage to enter the coalition and (ii) by doing so hurts some true member. In our case this simply means that a coalition involving a firm and type L consumers is at risk if the contract they agree to trade is attractive to type H and earns negative profits on the latter. A deviation is *credible* when not vulnerable to infiltration. An agreement (a coalition of players and a set of offers and requests) is *undermined* if there exists a credible deviation where the members of the deviating coalition (i) is a subset of the initial coalition, (ii) are made better off by deviating. An agreement is *self-enforcing* if it is not undermined by a deviation that is itself self-enforcing (an agreement involving only one player or type is always self-enforcing). A *CPEPI* is a self-enforcing agreement among all players.

We now describe the setting. The economy consists of large set of households and a (larger) set of risk neutral profit maximizing (potential) firms. Each firm selects a set of trade offer – a menu of contracts. Simultaneously, each household selects a firm and requests a contract. If the firm approached by a household offers the requested contract, trade occurs.

In looking for CPEPI, two alternative configurations play key roles. Let x^H be type H's most prefered (own) actuarially fair contract. Clearly, this is the (zero profit) full insurance contract $x^H = (m - p_H l,\, m - p_H l)$. Also, let X^0 be defined as the set of contracts that (i) at least break even on average, $\bar{\sigma}(x) \geq 0$, and (ii) are in β_L. Formally,

$$(5) \qquad X^0 \equiv \{x \,|\, \bar{\sigma}(x) \geq 0,\ x \in \beta_L\}.$$

The two configurations considered are:

Configuration C1:

$V^H(x^H) > V^H(x)$ for all $x \in X^0$.

Configuration C2:

$V^H(x^H) \leq V^H(x)$ for some $x \in X^0$.

Our first observation holds under both configurations.

Lemma 4. In any equilibrium a type H household obtains a level of utility no less than $V^H(x^H)$.

To see this, note that if a type H household obtained less utility, then an inactive firm and the type H household could form a coalition and agree to trade a full insurance contract (i.e. on the 45 degree line) slightly below x^H. This would give profits to the firm and increase the household's utility. The deviation would be credible since no other household could hurt the coalition members by infiltrating their coalition. The deviation would be self-enforcing since no coalition member could gain by a further (unilateral) deviation.

A separating equilibrium

We consider first configuration C1 and try to pin down the equilibrium contracts.

Lemma 5. When C1 applies, a type H household cannot be subsidized in equilibrium.

To see this, note that if a type H household was subsidized in equilibrium it would have to be pooled with some type L households, else the firm trading with the type H household could increase its profits by dropping the contract from its menu. Assume then that the type H households are pooled with the type L households at some contract x that gives non-negative profits, $\bar{\sigma}(x) \geq 0$[6]. Since C1 applies and lemma 4 holds, x cannot be in β_L. The pooling arrangement can then be "cream-skimmed" – pick a contract x' close to x such that type L strictly prefers x' to x, but such that type H is indifferent between the two. Since x is not in β_L the indifference curves through x strictly intersect so it is clearly possible to find such an x'. Let a type L household and an inactive firm deviate to trade x'. This deviation is not vulnerable to infiltration (by construction) and is self-enforcing (no one can gain from a second deviation). Hence this deviation undermines the

[6] *Multiple pools can be ruled out; if there were multiple pooling contracts, self-enforcability would require that both types of consumers be indifferent between all contracts. This would require that all contracts be in β_L (because indifference curves that intersect outside β_L never intersect again). But then all pooling contracts would satisfy $V^H(x) \geq V^H(x^H)$ and $x \in \beta_L$. By C1 all pooling contracts would then satisfy $\bar{\sigma}(x) < 0$, which clearly cannot happen since at least one pool has to contain a proportionate number of type H households.*

pooling arrangement – the households cannot be pooled.

Next we can pin down more exactly which contracts can be traded in equilibrium.

Lemma 6. If C1 applies and an equilibrium exists, then (i) every type H household must obtain the contract x^H, and (ii) every type L household must obtain a contract that solves problem P_L,

Problem P_L:

$$\max_x \{V^L(x) \mid \sigma_L(x) \geq 0 \text{ and } V^H(x) \leq V^H(x^H)\}.$$

Assuming a CPEPI exist, the first part follows from lemma 4 and lemma 5. Given part (i) type L clearly cannot be subsidized. A contract obtained by a type L household must not attract type H; if it did, type H would deviate and request it (violating self-enforcability). Thus any contract obtained by type L must be feasible in P_L. Figure 3 shows a case where configuration C1 applies and where P_L does *not* have a unique solution. All contracts on the segment c^L to x^L solve P_L; by self-insurance, they all result in type L planning consumption c^L. This is, however, not the only possibility. Formally defining x^L as the intersection of the $\sigma_L = 0$ locus with the indifference curve of V^H through x^H, two possibilities arise. Either x^L is in β_L as in figure 3 or x^L is above β_L. In the first case P_L has a continuum of solutions. In the second case x^L is the unique solution to P_L.

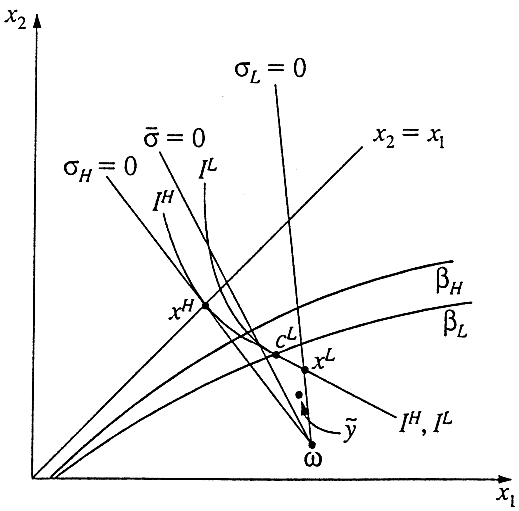

Figure 3: A separating CPEPI. Type H obtains contract x^H and does not self-insure. Type L's final consumption plan is c^L.

Focusing on the case displayed in figure 3 (the other case is similar), assume that a type L household obtains a contract that does not solve P_L. It may then obtain a contract like $\tilde{y}$ that is off the segment c^L to x^L. An inactive firm and the L household can then deviate to trade a contract closer to the segment that the household prefers to $\tilde{y}$ and that is profitable. This deviation is credible (it does not attract type H) and self-enforcing (no one could gain from further unilateral deviation). Thus the agreement to trade $\tilde{y}$ would not be self-enforcing.

Even when problem P_L does not have a unique solution, it uniquely identifies a consumption plan for type L. Notably, if x^L is in β_L, then at any contract that solves P_L type L plans consumption c^L as in figure 3. If x^L is not in β_L, x^L is the unique solution to P_L and type L does not engage in self-insurance, i.e. type L plans consumption x^L. Hence problem P_L always uniquely identifies a consumption plan for type L. This consumption plan is henceforth denoted c^L.

If problem P_L has a continuum of solutions as in figure 3, all of them except x^L are below the $\sigma_L = 0$ locus. Can type L, in equilibrium, sign a contract that generates profits? The next proposition suggest this.

Proposition 7. When C1 applies there exists a CPEPI where firm 1 offers the contracts x^H and c^L, all households of type H request x^H from firm 1, all households of type L request c^L from firm 1, and all other firms offer nothing.

Consider what type of deviation could potentially break the equilibrium. Clearly, no unilateral deviation can do so – all players are choosing a best reponse. A firm cannot gain from entering a deviating coalition with type H only since any contract that type H prefers to x^H yields negative profits (on type H). Neither can a firm enter a deviating coalition with type L households only. To see this, suppose first that x^L is in β_L as in figure 3. Any contract preferred by type L to c^L also attracts type H and makes negative profits on type H. A deviation by a firm and type L households would therefore be vulnerable to infiltration. Suppose then that x^L is not in β_L. Then (since the indifference curves intersect at x^L) there do exist contracts that attract type L but not type H. However, all such contracts make negative profits on type L.

The only remaining possibility is that a firm forms a deviating coalition with some households of both types. Any such deviation would have to include all type H households, else it would be vulnerable to infiltration.[7] Moreover, the households would have to be pooled in the deviation, else the firm would deviate a second time by dropping the contract requested by type H. It may be that no profitable pooling contracts exist. But even if there do exist pooling contracts that could attract both types and still earn profits (as is the case in figure 3), these can be cream-skimmed. To see this, suppose that some firm, all type H households, and, say, all type L households deviate to trade a pooling contract x' which attracts both types and satisfies $\bar{\sigma}(x') > 0$. In a second deviation, the firm can then withdraw x' and offer a nearby contract, x'', preferred by type L to x' but such that type H is indifferent. Such an x'' exists since x' cannot be in β_L.[8] Type L simultaneously switch to requesting x''. This second deviation undermines the deviation to x' – it is credible since x'' doesn't attract type H and it is self-enforcing since no one can gain from a further deviation. The agreement to trade x' is therefore not self-enforcing.

Thus there may be equilibria with profits. In a situation like that depicted in figure 3, there is some leeway along the segment c^L to x^L. Households only engage in self-insurance as a means to a (consumption) end. Rather than offering a contract at which type L self-insures, a firm can always offer the consumption plan c^L directly.

One would expect that if positive profits are made by some firm (say, by firm 1 as in proposition 7), then a competing firm would try to outbid the incumbent. This turns out to be difficult since it cannot credibly pool the households. The best it can do is to offer type L a contract somewhere along the segment c^L to x^L, and hope that some type L households (and *only* type L households) accept this offer despite being indifferent between it and firm 1's original offer.

Note that if a type L household signs a contract other than c^L (say, x^L in figure 3), then the household is no better off, but the aggregate profits are lower. This means that if Pareto dominated outcomes are ruled out (see definition A6 in the Appendix), then type L must sign c^L; no self-insurance will then be used.

A pooling equilibrium

Configuration C2 will now be examined. For simplicity it is assumed that the inequality in C2 is strict for some $x \in X^0$. This avoids a knife-edge case.

Lemma 4 established that type H cannot obtain a utility level less than $V^H(x^H)$. To obtain a higher level of utility type H has to be subsidized. Moreover, if subsidized, type H must be pooled with type L since unprofitable contracts can be dropped. Finally pooling arrangements at contracts not in β_L are at risk from cream-skimming. However under C2, there exist contracts in β_L that type H prefers to x^H and that break when the households are pooled. This raises the question if pooling can be sustained. First we can show that pooling is the only possible outcome.

Lemma 8. When C2 holds, every type H households must, in any equilibrium, be pooled with some type L households.

To see this, assume that some type H household is not pooled. This household then cannot be subsidised (since unprofitable contracts can be dropped) and must obtain contract x^H. Self-enforceability then requires that all type H households obtain the utility level $V^H(x^H)$ and that contracts obtained by type L households do not attract type H. For the same reasons as is

[7] *This follows since any contract that attracts type H away from* x^H *generates negative profits on type H.*

[8] *For type H to join the deviating coalition they must prefer x' to* x^H*. By C1, x' then cannot be in* X^0*. But for the firm to join the deviation, x' must be profitable,* $\bar{\sigma}(x') > 0$*. From the definition of* X^0 *it then follows that x' cannot be in* β_L*. Since x' is not in* β_L *the indifference curves intersect at x'.*

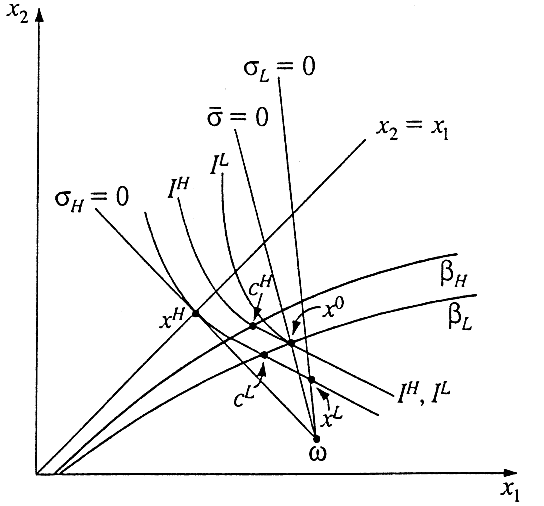

Figure 4: A pooling CPEPI where both types obtain the contract x^0. Type H engages in self-insurance and attains the consumption plan c^H. Type L does not engage in self-insurance.

in lemma 6 every type L household then obtains a contract that solves problem P_L and plans consumption c^L. Figure 4 illustrates a case where configuration C2 applies.

Now pick a contract x that satisfies $\bar{\sigma}(x) > 0$, $V^H(x) > V^H(x^H)$ and is on the boundary of β_L (i.e. somewhere on the segment from c^L to x^0 in figure 4). Since C2 holds with strict inequality for some contracts, such a contract exists. Let an inactive firm and all consumers form a coalition and let them trade x. This is an undermining agreement; all coalition members are made better off and the deviation is not vulnerable to infiltration since it includes all households. Moreover it is self-enforcing. To see this note first that, by construction, it is not possible to find a contract that attracts type L from x but does not also attract type H (and makes losses on type H). A second deviation would therefore have to include (all) type H for it not to be vulnerable to infiltration. Moreover, it would then have to involve pooling. But any new pooling contract that attracts both types *and* earn more profits cannot be in β_L (it must be to the north-west of x), and, by arguments now familiar, pooling contracts not in β_L are not viable. Thus no self-enforcing agreements can undermine the agreement to trade x. The assumption that some type H household is not pooled thus leads to a contradiction.

Which pooling contract can be sustained? Pooling at contracts not in β_L is not viable. The obvious candidate is then the contract most preferred by all households in X^0. Formally, let x^0 be the intersection of the $\bar{\sigma}(x) = 0$ locus and the boundary of β_L (as illustrated in figure 4).

Lemma 9. When C2 holds, all households must, in any equilibrium, obtain contract x^0.

To see this, assume that the households are pooled at some contract x' other than x^0 that (i) is in β_L, and (ii) generates non-negative profits $\bar{\sigma}(x') \geq 0$.[9] But x^0 is clearly the unique solution to the problem of maximizing $V^i(x)$ by choice of $x \in X^0$ for both types, $i = L, H$. Therefore if the types are pooled at x' there are contracts close to x^0 that are preferred by both types to x' and earn positive profits. In particular, pick a contract x'' on the boundary of β_L (slightly) below x^0 and select a coalition of an inactive firm and all households and let them deviate and trade x''. For the same reasons as in the proof or lemma 8, the agreement to trade x'' is self-enforcing and undermines the pooling arrangement at x'.

We can again demonstrate the existence of an equilibrium.

Proposition 10. When C2 holds, there exists a CPEPI where firm 1 offers x^0, all consumers request x^0 from firm 1, and all other firms offer nothing.

A complete proof is similar to that for proposition 7 and is only sketched here. Since all players choose a best response in the described strategy vector no unilateral deviation can break the proposed equilibrium. No firm can gain from deviating with type H only – any contract that attracts type H from x^0 would be unprofitable (on type H). Neither can a firm deviate with type L only since any contract that attracts type L from x^0 also attracts type H and makes

[9] *To keep the argument simple all households are assumed to be pooled at one contract. A slightly extended argument covers the case with many pools (some pool must contain at least the average proportion of type H).*

losses on the latter, making any such deviation vulnerable to infiltration. A deviating coalition would thus have to involve pooling arrangement – all type H would have to be included and be subsidized. However, any pooling contract that is profitable on average and is preferred by both types to x^0 is not in β_L (as can be seen from figure 4.) and would fail to be self-enforcing. Hence no self-enforcing agreements can undermine the pooling agreement described in the proposition, so this pooling agreement is indeed self-enforcing.

From figure 4 we see that, in a pooling equilibrium, type H engages in self-insurance and plans consumption at c^H, while type L does not self-insure and thus plans consumption at x^0. There are no profits in a pooling equilibrium.

Separation, pooling and welfare

Which type of equilibrium obtains depends foremost on the distribution of types. For fixed preferences u, efficiency of the self-insurance technology, λ, and probabilities p_L and p_H, changing the distribution of types changes the location of the $\bar{\sigma}(x) = 0$ locus. Consider e.g. figure 3. In this figure C1 holds and the types are separated. Increasing the fraction of type L households rotates the $\bar{\sigma}(x) = 0$ locus closer to the $\sigma_L(x) = 0$ locus. At some critical value θ_L^{crit}, the locus goes through c^L. At larger values for θ_L C2 holds and pooling occurs.

An interesting comparative statics exercise that can be carried out is with respect to λ. It turns out that the more efficient is self-insurance, the more likely is pooling. This follows from two observations. First, the larger is λ, the larger is the region β_L (and β_H) and hence the larger is X^0. Secondly, the set of contracts that type H prefers to x^H clearly grows when λ increases. Pooling occurs when the intersection of the latter set with X^0 is non-empty. Since both sets grow as λ grows, pooling becomes more likely. Conversely, when λ is small, separation is likely to occur. Indeed, if λ is so small that x^L is not in β_L, then separation occurs for every distribution of types. When λ is so small that x^L is not even in β_H, then the self-insurance option is completely immaterial. Hence we can conclude:

Proposition 11. There exists a critical proportion $\theta_L^{crit} \in (0, 1]$ such that the two types are separated (pooled) in the insurance market if and only if $\theta_L < \theta_L^{crit}$ ($\theta_L \geq \theta_L^{crit}$). θ_L^{crit} is weakly decreasing in λ, the efficiency of the self-insurance technology.

A second (and related) question is how the efficiency of the self-insurance technology affects the welfare of the consumers. A full investigation of this issue is beyond the scope of this paper but two observations can readily be made.

Claim 12. As long as the insurance market equilibrium is separating, an increase in the efficiency of the self-insurance technology, λ, does not affect type H and will, if anything, lower the welfare for type L.

Claim 13. If a more efficient self-insurance technology induces pooling rather than separation in the insurance market, this can lead to all households being better off.

To see the first observation formally recall that the type H households obtain contract x^H which does not depend on λ (and they do not self-insure). Recall also the final consumption plan for type L, c^L, is itself a solution to P_L (defined in lemma 6). When λ increases, the set of contracts that type H prefers to x^H expands, whereby the feasible set in P_L shrinks. It follows that type L cannot be better off. The dominating effect of an increase in λ is thus to aggravate the self-selection problem.

For the second observation, it is well-known that subsidising type H may lead to Pareto improvements. By subsidising type H, type H can be made better off than at x^H, which expands the set of contracts that can be given to type L without attracting type H. On the other hand, subsidies to type H also requires that the type L households consume less than their expected wealths. If initially the self-selection contraint is sufficiently "tight" and there are sufficiently few type H households, then subsidies to type H can lead to a Pareto improvement. Though pooling is a crude way in which type H can be subsidised it may still allows both types to be made better off compared to a situation where they are separated.

The first observation is particularly striking. It is rational for an individual consumer to use self-insurance to improve the partial insurance that can be obtained in the competitive market. Self-insurance would hence serve a purpose and be welfare improving *if the market contracts were unaffected by the self-insurance option.* This will, however, not be the case. Indeed, the first observation shows that, when we require equilibrium, the opposite applies; the more efficient is the self-insurance technology, the worse off are the consumers. In separating equilibria, self-insurance thus turns out to be completely dysfunctional, completely undermining its own expediency. The second observation indicates that the self-insurance technology can actually be welfare improving; but then it is because it opens up for subsidisation of the high-risk type, and, through that, a relaxation of the self-selection constraint. This welfare improving effect is indeed very remote from the individual justification for using self-insurance.

These results are closely akin to the results obtained by Arnott and Stiglitz (1991) in a moral hazard setting (i.e. in a setting where an individual can affect the probability of a loss by a choice of effort). Starting from the observation that the market response to moral hazard is to offer only partial insurance, the authors note that there is an incentive for coalitions of friend or family member to engage in mutual assistance. Though perfectly rational on the individual level, the authors show that mutual assistance can lead to everyone being worse off – the mutual assistance turns out to be dysfunctional.

Thus in this paper as well is the paper by Arnott and Stiglitz it is found that non-market insurance can be harmful despite being individually rational; mutual assistance and self-insurance as institutions can thus arise and persist, despite being welfare damaging.

Before proceeding we will indicate a certain robustness of our results by making a comparison with a solution concept that seems to have gained in popularity; namely Riley's (1979) "reactive equilibrium". The equilibrium outcomes with the CPEPI concept are in fact also equilibrium outcomes with the reactive equilibrium. Roughly, the definition of a reactive equilibrium says that to break a proposed equilibrium, it must be possible for a firm to add a contract to the existing menu that will be strictly profitable, and that will not become strictly unprofitable if other firms can react by adding more contracts to their menues.

Consider first configuration C1 and assume that an "incumbent" firm is trading c^L and x^H (the CPEPI contracts) and to be specific assume that the incumbent is making profits as in figure 3. What can a rival firm (a *defector*) do? Attracting only type H would bring losses. Attracting only type L is not possible. It cannot attract both types without pooling them since a second firm (a *reactor*) could attract away the profitable type L, leaving the defector negative profits. This leaves pooling. But this would mean pooling at a contract such as x' that is not in β_L (since C1 holds) and the defector would be at risk from a reactor attracting type L from the pool. Thus there is no way a defector can outbid the incumbent. Similarly assume that C2 holds and that an incumbent is trading x^0 (the unique CPEPI contract) with all households. Any contract that attracts type L from x^0 also attracts type H. Thus a defector would have to attract both types and try to pool them. But the profitable pooling contracts that attract the consumers away from x^0 are not in β_L, which would leave the pooling arrangement at risk from cream-skimming by a reactor.

The CPEPI and the reactive equilibrium have in common that they both emphasize that for a deviation to break a proposed equilibrium, it must itself be viable. While in the CPEPI framework a firm cannot commit not to cream-skim its own arrangements, in the reactive equilibrium it is the fear of having ones menu cream-skimmed by a rival firm combined with an inability to withdraw the menu that is the driving force. In both cases, the cream-skimming argument is forceful. The specification of the reactive equilibrium has, however, been criticized for being incomplete. Attempts to formulate extensive-form games underlying the reactive equilibrium concept have led to the insight that multiple equilibria generally exist; in particular it has be shown that implicit collusion can be sustained as a perfect Nash equilibrium.[10]

4. An application – Private pensions and bequeathable savings

In this section we will show how an option to save in a bequeathable asset can be interpreted as a linear self-insurance technology. The application draws on Eichenbaum and Peled (1987) who show that some consumers may hold mixed portfolios of private pensions and a bequeathable asset even though they have no bequest motives.

Consider the following setup. There are two periods. Each consumer is sure to be alive in the first period and is alive in the second period with some probability. There are two types, $i = L, H$, where the probability for type i of being alive in the second period is p_i with $p_L < p_H$.

The consumers plan consumption for both periods and maximize

$$U_i(c) \equiv u(c_1) + p_i u(c_2),\ i = L, H \tag{6}$$

For simplicity we assume that there is no other discounting of utilities and zero rate of interest on savings. The consumer have no bequest motives. Each consumer is endowed with w units of income in the first period and nothing in the second.

Following Yaari (1965) the consumers are assumed to have two investment alternatives available – private pensions and a bequeathable asset (which we can refer to as "cash"). A *private pension contract* is written as $x = (x_1, x_2)$. Contract x provides the consumer with income x_2 in the second period *if* the consumer is alive. In return the consumer has to give up $w – x_1$ of first period income, i.e. x_1 is what the consumer has left in the first period after signing x. By saving in cash a consumer can also carry wealth over one to one from the first- to the second period, i.e. by saving e in cash in the first period the consumer has e more units of income in the second period ($\lambda = 1$). If the consumer dies at the end of the first period, the cash savings are bequeathed. The insurance companies cannot observe the consumers' cash savings.

Slightly redefining the sets of actuarially fair contract we have

Definition 3. A private pension contract x is *actuarially fair* for type i of $\sigma_i(x) \equiv (x_1 - w) + p_i x_2 = 0$.

This redefinition takes into account that first component of the contracts will always be traded since the consumer is sure to be alive in this period. As before

Definition 4. An private pension contract x i *actuarially fair on average* if $\bar{\sigma}(x) \equiv \sum_i \theta_i \sigma_i(x) = 0$.

The consumers' preferences over contracts are obtained from solving the problem of choosing e optimally conditional on x,

$$V^i(x) \equiv \max_{e_i \geq 0} \{U_i(c) \,|\, c_1 = x_1 - e_i,\ c_2 = x_2 + e_i\}. \tag{7}$$

The restriction that $e_i \geq 0$ states that the consumer is not allowed to borrow in cash. The rationale for this is that a consumer is not allowed to die indebted.

We can now verify that this example fits the general model in the previous sections. First, the preferences U_i, $i = L, H$ behave just like the preferences Eu_i. They satisfy the same single-crossing property, and a consumer would insure fully if he choose from the set of (own) actuarially fair contracts. Second, the cash saving technology is naturally linear and cannot be reversed, $e_i \geq 0$. Third, the cash saving option will necessarily have a return that is less than that implied by actuarial insurance. This follows since insurance is not paid out if the consumer dies, which happens with some positive probability.

Figure 1 can then be interpreted as illustrating a consumer's cash saving decision at some contract x that offers poor income insurance. Figure 2 still applies except that the initial endowment is now on the horizontal axis. Note however that an initial endowment such as that in figure 2 could come about through a mandatory uniform public pension scheme, which,

[10] *On this, see Engers and Fernandez (1987) and Kreps (1990) p. 650.*

when budget-balanced, moves the initial endowment along the $\bar{\sigma}(x) = 0$ locus.[11]

The conclusions from the general model thus carry over to the application:

- There may be a separating equilibrium in the competitive private pension market. In such an equilibrium there may be positive profits (or type *L* may save in cash).
- There may be a pooling equilibrium in the competitive private pension market. In a pooling equilibrium type *H* saves in cash.

Claim 12 from section 3 can be used to compare a situation with a cash savings option with a situation where no such option exists. The latter corresponds to a situation with a completely inefficient self-insurance technology, $\lambda = 0$.

- If, with a cash saving option, the two types are separated in the private pension market, then eliminating the cash saving option will not affect type *H* but will typically make type *L* better off.

This reflects the observation that a self-insurance option, though individually rational to use, will may well be harmful when its equilibrium impact is taken into consideration.

5. Conclusions

In this paper we have considered the effects of a simple linear self-insurance option in a standard two-class model of an insurance market with adverse selection. It is well-known that in markets troubled by adverse selection, some consumers may not obtain full insurance. This suggests that they might resort to self-insurance. However, a consistent analysis must account for the self-insurance option when determining the outcome in the insurance market.

Using the recently proposed concept of a "Coalition Proof Equilibrium with Private Information" due to Kahn and Mookherjee (1995) we have shown that, with a self-insurance option, the competitive insurance market outcome can be either separating or pooling. In the former case, there may be positive profits in equilibrium. In the latter case, pooling can be sustained since the self-insurance technology generates a set of contracts where the consumers can be pooled without the arrangement being exposed to cream-skimming.

A self-insurance technology also has some unexpected effects on welfare. E.g. a long as the insurance market equilibrium in separating, a more efficient self-insurance technology does not make any consumer better off and typically makes some consumers worse off. We noted that this result is closely akin to that obtained by Arnott and Stiglitz (1991) who consider mutual assistance between family members when the competitive insurance market only offers partial insurance as a response to moral hazard. In both cases, non-market insurance has an identifiable function on the individual level, but can turn out to be welfare damaging when its equilibrium repercussions are accounted for.

An application was also noted. The option of supplementing private pensions with savings in some bequeathable asset can be interpreted as a linear self-insurance technology. Consequently there may be a separating equilibrium in a competitive private pension market, possibly with profits, or there may be a pooling equilibrium where some consumers choose mixed portfolios of private pensions and bequeathable assets.

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[11] This is used e.g. in Eckstein, Eichenbaum and Peled (1985) to show that a mandatory pension system can be Pareto improving when the private pension market is separating. A uniform pension system subsidises type H. This helps to relax the incentive constraint in the private pension market and can be Pareto improving if the fraction of type H consumers is small.

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Appendix

Here we restate the definitions from Kahn and Mookherjee (1995). The set of players (the firms and the consumer) is *P*. Player *i* has a strategy set S_i. A *strategy vector* is $s \in \text{x}_{i \in P} S_i$. Player *i*'s payoff is $u_i(s)$. The consumers have private information which makes then *indistinguishable*. A *coalition* C is a subset of *P*, and an *agreement* is a pair (*C*, *s*).

Definition A.1. An agreement (*C*, *s*) is a *deviation* from a strategy vector *v* if $s_{-C} = v_{-C}$.

Definition A.2. A deviation (*C*, *s*) from a strategy vector *v* is *vulnerable to infiltration* if there exists players τ, ϕ, and η and a stratety α_τ for player τ such that:

1. $\tau \notin C$, but τ is indistinguishable from $\phi \in C$,
2. $\eta \in C$, $\eta \neq \phi$,
3. $u_\tau(\alpha_\tau, s_{-\tau}) > u_\tau(v)$,
4. $u_\eta(\alpha_\tau, s_{-\tau}) < u_\eta(s)$.

Definition A.3. A deviation (*C*, *s*) from a strategy vector *v* is *credible* if it is not vulnerable to infiltration.

Definition A.4. An agreement (*C*, *s*) *undermines* the agreement (*D*, *v*) if

1. $C \subset D$,
2. $s_i = v_i$ for all $i \notin C$,
3. $u_i(s) > u_i(v)$ for all $i \in C$,
4. (*C*, *s*) is a credible deviation from the strategy vector *v*.

Definition A.5. If the set *C* contains a single type, then for any strategy vector *s*, the agreement (*C*, *s*) is *self-enforcing.* If the set *C* contains more than one type, then the agreement (*C*, *s*) is *self-enforcing* if for no proper subset *D* of *C*, there is a self-enforcing agreement (*D*, *v*) which undermines (*C*, *s*).

Definition A.6. A strategy vector *s* is said to be a *Coalition Proof Equilibrium with Private Information* (CPEPI) if

1. the agreement (*P*, *s*) is self-enforcing,
2. there does not exist any other self-enforcing agreement (*P*, *v*) which Pareto dominates (*P*, *s*).