Finnish Economic Papers – Volume 8 – Number 2 – Spring 1995

AN IMPURE PUBLIC INPUT AS A DETERMINANT OF TRADE*

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This paper utilises an extended Ricardian model to examine the relationship between government spending on an impure public input and the pattern of trade. (JEL, F11, F41)

1. Introduction

A significant proportion of government budget is directed towards the provision of impure public inputs in most real economies. Examples of such inputs include roads, canals, bridges and harbours. Unlike a pure public input, the entire supply of an impure public good cannot be utilised by all firms simultaneously. In other words, an impure public input is congestible within industries and among firms across industries. For a given supply of an impure public input, the entry of an extra firm not only increases congestion within the relevant industry, but also increases the level of congestion in other industries. Congestion affects the overall usefulness of a public input to industries: an impure public input may not be equally congestible across industries; also, each industry is likely to contribute in different ways to the degree of congestion. Consequently, in the presence of an impure public input, the pattern of international trade is also influenced by (i) the level of congestion within the industries, and (ii) the level of inter-industry congestion.

Existing studies (e.g., McMillan, 1978, Manning and McMillan, 1979, Abe, 1990, Clarida and Findlay, 1992, and Anwar, 1992) have considered the relationship between government spending on a pure public input and trade pattern.[1] It has been shown that the differences in government spending on a pure public input alone can determine the pattern of trade in a two-country world. The relationship between government spending on impure public inputs and the pattern of international trade has not received much attention in the available literature.

The present study incorporates an impure public input into a Ricardian model of a closed economy where the private sector produces two final goods. The model is used to examine the relationship between the supply of an impure public input and the relative goods prices in a representative closed economy. The relationship between the supply of the impure public input and the relative goods prices in a representative closed economy is used to predict the pattern of trade in a two-country world. The two countries are assumed to be identical except for the differences in the supply of an impure public input. The paper shows that differences in the supply of an impure public input, in conjunction with congestion across industries and among firms within industries, alone can determine the pattern of international trade.

* *I am grateful to B. Copeland, D. Donaldson, A. Kotwal and the referees for invaluable comments and suggestions. Responsibility for any remaining errors or omissions is however solely mine.*

[1] *Pure public inputs are by definition non-congestible.*

The paper is organised as follows. An extended Ricardian model is developed in section two. In section three, the relationship between government spending on an impure public input and the pattern of trade is considered. The last section contains concluding remarks.

2. A Ricardian Model

Consider a representative closed economy that produces two final goods (X and Y) by means of labour and an impure public input (G).[2] The public input, which is produced by means of labour, is congestible across industries and among firms within each industry. The firms in each industry are identical. Consequently, congestion caused by each firm within an industry is identical. However, the public input may not be equally congestible across industries. In addition, each industry is likely to contribute in different ways to the degree of congestion.

Congestion is assumed to be positively related to the output of each industry. Accordingly, when an industry expands, the resulting congestion decreases the usefulness of the public input to all industries. In other words, congestion results in negative (Marshallian) production externalities. Each industry consists of a large number of identical firms. The production functions of the ith firm in industry X and Y are given below:[3]

$$X_i = h_x(G,X,Y)L_{xi};\ X = n_x X_i\ (i = 1, 2, \ldots n_x)$$

$$Y_i = h_y(G,X,Y)L_{yi};\ Y = n_y Y_i\ (i = 1, 2, \ldots n_y)$$

where X_i is the output of firm i in industry X; n_x is the number of firms in industry X; G is the supply of the impure public input; and L_{xi} is labour used in the production of X_i.

Each firm takes G, X, and Y as given. $h_x(.)$ and $h_y(.)$ measure the contribution of the public input to production. For a given X and Y, an increase in G reduces the congestion. In other words, $h_x(.)$ and $h_y(.)$ are positively related to the supply of congestible public input. However, for a given supply of public input, an increase in the output of either or both industries increases congestion. In other words both $h_x(.)$ and $h_y(.)$ are negatively related to X and Y. However, $h_x(.)$ and $h_y(.)$ are concave. The technology for public production is the following, where α is a positive constant and L_g is labour used in the production of G.

$$G = \alpha L_g$$

X is the numéraire and all markets are competitive. By using the production functions for the private and the public sectors, labour market clearing condition can be written as follows:

$$(1)\quad L_e = G/\alpha + [X/h_x(.)] + [Y/h_y(.)]$$

where $[X/h_x(.)]$, $[Y/h_y(.)]$, and G/α are labour used in the production of X, Y, and G respectively; L_e is the supply of labour which is fixed.

Labour is fully mobile within the country and the public sector is determined (either by law or a custom) to pay the private sector wages.[4] Zero profit conditions for industry X and Y are the following:

$$(2)\quad w/[h_x(X,Y,G)] = 1$$

$$(3)\quad w/[h_y(X,Y,G)] = p$$

The autarky price ratio (p) in the economy is determined by the interaction of supply and demand. On the demand side, preferences are assumed to be homothetic.[5] The relative demand for the two goods therefore depends on the rel-

[2] *In order to reduce the mathematical complexity of the model, the present study considers only one primary factor. Abe (1990) has considered two factors of production, i.e, capital and labour. However, in order to derive his result, Abe assumes that the public and the private sectors are equally capital intensive. This assumption effectively reduces Abe's model to one primary factor.*

[3] *The production structure of the present study is similar to Melvin (1969) and Chang (1981). However, these studies, which do not include public inputs, exclusively deal with the implications for trade theory of variable returns to scale. In the present study, variable returns are attributed to the congestibility of the public input.*

[4] *I am indebted to a referee for clarifying this point.*

[5] *This assumption is widely used in the literature on international trade including the Heckscher-Ohlin model.*

ative price ratio alone. Market equilibrium for the final goods can be written as

(4) $Y/X = \phi(p)$

where $\phi(p)$ is relative demand of the final goods and $\partial\phi(p)/\partial p$ is negative.

The cost of public input production is financed by means of a proportional income tax as follows, where t is the tax rate

(5) $t(wL_c) = w(G/\alpha)$

The right hand side of (5) is total cost of public input whereas the left hand side is government tax revenue.

As far as the supply of public input is concerned, the government is a Stackelberg leader with respect to the private agents. In other words, the private sector of the economy takes the supply of public input as given.[6] The equilibrium of the private sector can be characterised by (1) to (4). In four equations there are four endogenous variables; w, p, X, and Y.[7]

3. *Pattern of International Trade*

The relationship between the supply of an impure public input and the pattern of trade can be examined by differentiating equilibrium conditions (1) to (4) with respect to G as follows:[8]

(6) $$(\partial p/\partial G)(G/p) = [\gamma(\alpha_{xg} - \alpha_{yg}) + \alpha\ \{(\mu_{xx} - \mu_{yy}) + (\lambda_{xy} - \lambda_{yx})\}]\ /\Delta$$

where $\gamma = [L_y(1 - \mu_{yy} - \lambda_{yx}) + L_x(1 - \mu_{xx} - \lambda_{xy})] > 0$; $\sigma = \alpha_{xg}L_x + \alpha_{yg}L_y - L_g$; λ_{yx} and μ_{yy} are the elasticity of $h_y(.)$ with respect to X and Y respectively; μ_{xx} and λ_{xy} are the elasticity of $h_x(.)$ with respect to X and Y respectively; α_{xg} and α_{yg} are the elasticity of $h_x(.)$ and $h_y(.)$ with respect to G respectively; Δ is positive provided that the equilibrium is stable.[9]

The sign of (6) which, which describes the impact on the autarky price ratio of a country when its supply of the impure public input increases by a small amount, depends on: (1) σ; (2) the relative size of congestion within each industry $(\mu_{xx} - \mu_{yy})$; (3) the relative size of inter-industry congestion $(\lambda_{xy} - \lambda_{yx})$; and (4) the relative benefits of the public input $(\alpha_{xg} - \alpha_{yg})$.

The sign of σ can be determined by differentiating (1) with respect to G. Since the supply of labour is fixed, the derivative of the left hand side of (1) with respect to G is zero.[10]

$$\begin{aligned} 0 = dL_c/dG = [1/\alpha] &+ \{[1/h_x(.)^2]\,[h_x(.) - Xh_{xx}(.)] \\ &- [1/h_y(.)^2]\,[Yh_{yx}(.)]\}\,(dX/dG) \\ &+ \{[1/h_y(.)^2][h_y(.) - Yh_{yy}(.)] \\ &- [1/h_x(.)^2]\,[Xh_{xy}(.)]\}\,(dY/dG) - [X/h_x(.)^2]\,[h_{xg}(.)] \\ &- [Y/h_y(.)^2]\,[h_{yg}(.)] \end{aligned}$$

The above can be written as

$$\begin{aligned} 0 = &[L_x(1 - \mu_{xx}) - L_y\lambda_{yx}]\,(dX/dG)\,(G/X) \\ &+ [L_y(1 - \mu_{yy}) - L_x\lambda_{xy}]\,(dY/dG)\,(G/Y) - \sigma \end{aligned}$$

G enters as an input in the production of X and Y. The marginal productivity of G in X and Y is assumed to be positive, i.e., both (dX/dG) and (dY/dG) are positive. Also notice that the coefficients of both (dX/dG) and (dY/dG) are positive. Clearly the above condition will be satisfied only if σ is positive.

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6 *The present study attempts to examine the problem of what happens to the pattern of trade when the supply of public input is marginally increased. It is clear that after a marginal increase from the non-optimal initial position, the supply of public input does not yet necessarily maximise social welfare. The optimal supply of public input can be determined by maximising the relevant indirect utility function.*

7 *In the present case, labour is the only non-produced input and the technology exhibits constant returns to scale at the firm level. However, due to the presence of congestion, the equilibrium prices are not independent of tastes.*

8 *An increase in the supply of a public input can also be interpreted as an improvement in production technology. Markusen (1983), and Markusen and Svensson (1985) have shown that differences in production technologies alone can also determine the pattern of trade.*

9 *Stability condition is discussed in the appendix. Chang (1981) and Okamoto (1985) have used a similar stability condition.*

10 *A similar procedure is used in intermediate micro theory to derive the Cournot and Engel aggregation conditions.*

If the public input is equally congestible within each industry (i.e., $\mu_{xx} = \mu_{yy}$) and congestion caused by each industry is symmetric (i.e., $\lambda_{xy} = \lambda_{yx}$), then the relative size of the direct benefits determines the sign of (6). The following proposition follows from (6)

Proposition 1. When two countries have (i) identical homothetic preferences, technology, labour supply, (ii) $\mu_{xx} = \mu_{yy}$, and $\lambda_{xy} = \lambda_{yx}$; then the country that produces more public input exports (imports) the output of that industry which derives more (less) benefits from its supply.[11]

On the other hand, when the impure public input is equally productive in both industries (i.e., $\alpha_{xg} = \alpha_{yg}$) and congestion caused by each industry in the other is symmetric (i.e., $\lambda_{xy} = \lambda_{yx}$), then the following proposition follows from (6).

Proposition 2. When two countries have (i) identical homothetic preferences, technology, labour supply, (ii) $\alpha_{xg} = \alpha_{yg}$, and $\lambda_{xy} = \lambda_{yx}$; then the country that produces more public input exports (imports) the output of that industry in which the public input is relatively more (less) congestible.

An increase in the supply of the impure public input increases the output of both industries. Suppose that the public input is relatively more congestible in X industry as compared to Y in the sense that external diseconomy works relatively more intensively in X industry. Hence if G increases, then the relative cost of X industry would decline, since the same percentage decrease in the production levels would reduce the congestion by a greater amount in X industry. This effect will bring an increase in the relative supply of good X for any relative price of private goods, which, in turn, would decrease the relative price of good X in the autarky equilibrium. Therefore, the country that produces more public input will export the output of the industry where the public input is relatively more congestible.

When the impure public input is equally productive in both industries (i.e., $\alpha_{xg} = \alpha_{yg}$) and it is also equally congestible within each industry (i.e., $\mu_{xx} = \mu_{yy}$), then the following proposition follows from equation (6).

Proposition 3. When two countries have (i) identical homothetic preferences, technology, labour supply, (ii) $\alpha_{xg} = \alpha_{yg}$ and $\mu_{xx} = \mu_{yy}$; then the country that produces more public input exports (imports) the output of that industry which causes less (more) congestion in the other industry.

An increase in the supply of the public increases the output of both industries which increases congestion. Suppose that X industry causes more congestion in Y in the sense that external diseconomy works relatively more intensively in Y industry. Hence if G increases, then the relative cost of Y industry would decline, since the same percentage decrease in the production levels would reduce the congestion by a greater amount in Y industry. This effect will bring an increase in the relative supply of good Y for any relative price of private goods, which, in turn, would decrease the relative price of good Y in the autarky equilibrium. Therefore, the country that produces more public input will export the output of the industry which creates less congestion in the other industry.

Proposition one suggests that if congestion faced by both industries is symmetrical then an increase in the supply of the public input can decrease the average cost of production and hence increase exports. Propositions two and three exclusively consider the role of congestion. These propositions are based on the assumption that the public input is equally productive in both industries. Propositions two and three imply that an increase in the supply of the public input may not increase exports.

4. Concluding Remarks

Abe (1990) has shown that if two countries have identical tastes, technology and identical factor supplies then differences in the supply of a pure public input alone can determine the pattern of international trade. If two countries have identical tastes, technology, factor supplies and the pure (i.e., non-congestible) public input is equally productive in private goods industries

[11] *Abe (1990) has derived a similar result in the case of a non-congestible (pure) public input.*

then Abe's model cannot be used to predict the pattern of trade.

This paper attempts to extend Abe's result by incorporating an impure (i.e., congestible) public in a Ricardian model. The model consists of two-goods, one-primary factor, one public input, incorporating the Marshallian externalities which cause the congestion of the public input. It is shown that if two countries have identical tastes, technology, factor supplies and the public input is equally productive in both industries then differences in the supply of the public input can explain the pattern of trade as long as the public input is not equally congestible within and across industries.

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Appendix

Equations (1) to (4) can also be used to derive the Routh-Hurwitz stability conditions. The postulated dynamic adjustment process is described by means of the following equations, where the left hand side is the time derivative of the relevant variable.

$$dw/dt = \theta_w\{[X/h_x(.)] + [Y/h_y(.)] + G/\alpha - L_e\}$$
$$dX/dt = \theta_x\{1 - w/[h_x(X,Y,G)\}$$
$$dY/dt = \theta_y\{p - w/[h_y(X,Y,G)]\}$$
$$dp/dt = \theta_p\{\phi(p) - Y/X\}$$

where the relevant speeds of adjustment (θ_w, θ_x, θ_y, and θ_p) are assumed to be positive constants.

The economic meanings of the above equations are obvious, therefore the interpretation is not included. The relevant Jacobian matrix, denoted by J is the following:

$$J = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

where

$a_{11} = 0$, $a_{12} = L_x(1 - \mu_{xx}) - L_y\lambda_{yx}$,
$a_{13} = L_y(1 - \mu_{yy}) - L_x\lambda_{xy}$, $a_{14} = 0$, $a_{21} = -1$,
$a_{22} = \mu_{xx}$, $a_{23} = \lambda_{xy}$, $a_{24} = 0$, $a_{31} = -1$,
$a_{32} = \lambda_{yx}$, $a_{33} = \mu_{yy}$, $a_{34} = 1$, $a_{41} = 0$, $a_{42} = 1$,
$a_{43} = -1$, $a_{44} = [p/\phi(p)]\,[\partial\phi(p)/\partial p]$

One of the Routh-Hurwitz stability conditions requires $(-1)^4\,|J|$ to be positive, where »| J |» stands for the determinant of J. In the present case the determinant of the Jacobian matrix is, except for a positive scalar, identical to Δ. Accordingly, the determinant condition is satisfied if Δ is positive. The stability of the equilibrium ensures that production takes place on the concave part of the production possibilities curve both before and after a change in government spending on the impure public input.