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TAX EVASION AND GROWTH*

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In an overlapping generations model, in which savings and tax evasion are endogenous, tax evasion will have a negative effect on long-term growth if public services are productive inputs for private producers. It is shown in the paper that tax evasion reduces the endogenous growth rate. Moreover, the case of pure public consumption is considered. Growth is then exogenous at the steady state path. It is found out that the effect of a tax-enforcement parameter change on the long-run equilibrium heavily depends on the intertemporal elasticity of substitution. (JEL H26, O41)

1. *Introduction*

In all OECD countries penalties are imposed on the failure to comply to the rules of the tax system. Nevertheless, since control and search powers of tax authorities are strictly limited in most countries, the probability of detection is low (see OECD, 1990). Hence, deterrence by the penalty system is weak. Thus, tax evasion is widely practiced. In particular, a large part of capital income is concealed from the authorities, since either a withholding tax is not applied to capital income in many countries or the withholding tax rate is much less than the personal income tax rate. On the one hand, enforcement of the capital income tax is weak because the government of each country fears to loose its taxable base as the international capital mobility is high. On the other hand, taxation of capital income drives a wedge between the rate of time preference and the interest rate. Therefore, the economy uses less capital in the long run. Lowering the effective tax rate by tax evasion might counteract this negative influence of capital income taxation.

Despite the fact that income from capital is concealed from tax authorities in many countries, most of the literature on tax evasion ignores this subject. Usually it concentrates on income from labor and the black economy (see the standard reference Cowell, 1990). Nevertheless, some work has been done on the microeconomics of tax evasion and savings (see, e.g., Hagedorn, 1991). In general, microeconomic models of tax evasion are much more elaborated than models studying the macroeconomic impact of this individual behavior. Moreover, these macroeconomic models are typically non dynamic models which focus on the comparative statics of tax evasion in the short run (see Peacock and Shaw, 1982, Ricketts, 1984, and Lai and Chang, 1988).

The usual approach in the macroeconomic models mentioned above is to take the proportion by which tax is evaded as given. Neither income nor any fiscal parameters have an influence on the fraction of income people try to conceal. This is clearly not in line with the microeconomic view of noncompliance. Most

* *I have benefited from the comments of two anonymous referees from this journal.*

economists guess that a taxpayer is confronted by a problem of choice under risk. They treat the problem of a tax evader as identical to that of portfolio selection. Hence, risk averse individuals will change the fraction of the risky asset 'evaded tax' if the income or a fiscal parameter is altered.

The aim of this paper is to discuss the long run effects of tax evasion, in particular those of noncompliance with respect to capital income taxation. The standard neoclassical Solow growth model has been used before by Wrede (1993), chapter 6.2, to examine how tax evasion affects the capital-labor ratio (and therefore output) in an intertemporal equilibrium. In the Solow model marginal propensities to save and to evade are exogenous. It has been shown that an increase in the proportion of evaded tax increases the capital-labor ratio if the government pays for public consumption with taxes. If instead, the government invests a larger fraction of its balanced budget than the households save, the impact of tax evasion on the long-run output is negative. In contrast, debt finance of government spending will counteract any positive influence of tax evasion even if public investment is very low. While every dollar not received by the government increases the deficit and hence lowers investment one by one, the household will save only a fraction of any dollar evaded tax as long as the marginal propensity to save is less than one.

During the last decade a furious comeback of the growth theory could be observed following the seminal papers of Romer (1986, 1990) on the subject of endogenous growth. In contrast to the standard neoclassical growth model in which the steady state growth rate is exogenously given, the theory of endogenous growth is able to explain how individual preferences and hence savings might influence the long-term value of the growth rate (for a recent overview see Barro and Sala-i-Martin, 1995, and Verspagen, 1992). Many contributions to that theory dealt with the impact of taxation on the growth rate (see, e.g., Barro, 1990, Rebelo, 1991, Barro and Sala-i-Martin, 1992, Saint-Paul, 1992, Betten, Wenzel and Wrede, 1994). Especially, the model of Barro (1990) is of interest here since he constructs a growth model that includes publicly provided inputs of production. As a consequence, production has external effects if income is taxed. Higher output leads to greater tax revenue thereby increasing output of everybody since government purchases are an argument of the production function. Constant or increasing returns to capital and government purchases together may occur even if the production is subject to diminishing returns with respect to privately owned capital. Barro and Sala-i-Martin (1992) present a version of that model which is used here. Instead of the aggregate quantity of government purchases, they incorporated the government expenditures per capita in the production function.

Because the continuous time model mostly used in the literature on endogenous growth is difficult to handle if tax evasion and hence uncertainty is included, we introduce a discrete time version of the Barro model. The two-period life-cycle model is used to examine tax evasion in the long run. Undoubtedly, this is not the first attempt to use the life-cycle model within the framework of endogenous growth theory (see King, 1992, in the context of the public debt analysis).

In addition, we use the OLG model in order to examine the relationship between tax evasion and the steady state capital-labor ratio if the government only provides consumption goods. This assumption leads to the standard neoclassical OLG growth model in which the growth rate in the steady state is determined by the growth rate of labor.

The paper is organized as follows. Section 2 investigates tax evasion in an overlapping generations model. Savings and tax evasion are determined by household utility maximization. We use the CRRA utility function in order to obtain clear-cut results. Private production is heavily influenced by publicly provided inputs. It is assumed that production is subject to constant returns to scale with respect to the private inputs capital and labor if the level of government purchases are given, but it is subject to increasing returns to scale when private and publicly provided inputs vary. The government levies a flat tax on capital income. Taxation generates an externality. The tax revenue depends on output and output depends on government purchases. A formula for the constant growth rate is derived in this section. Finally, we compare the results with the exogenous growth case. Conclusions are presented in the last section.

2. The Model

In order to examine savings and tax evasion behavior, we consider a two-period life-cycle model.

2.1 Consumption

The model is of the overlapping generations type in which people live for two periods. Individuals work only in the first period. They save part of their income to finance consumption in their retirement period.

A young person in period t supplies inelastically one unit of labor and earns a wage of w_t. He consumes c_t^1 in that period and saves the rest of his income, s_t. During the second period the individual consumes all his wealth, both saving and interest. r_{t+1} is the interest rate paid on savings held from period t to period $t+1$. Suppose that the government taxes the return to capital at the rate τ, where $0<\tau<1$. Since labor is not taxed and since the government budget is balanced at every moment, it is assumed that all government purchases are financed through taxation of capital income. The analysis is therefore partial since it ignores tax evasion from labor.

The taxpayer decides on the evaded fraction, e_{t+1}, where $0 \leq e_{t+1} \leq 1$, of tax on income in the retirement period, $s_t r_{t+1}\tau$. Moreover, there is a fixed probability, p, that tax evasion will be discovered and punished. The tax on income found to be concealed from the authority is subject to surcharge at a rate π, where $\pi>0$. If investigated by the tax authority the individual has to pay the regular tax on income, $s_t r_{t+1}\tau$. Additionally, he is obliged to pay the fine $\pi\tau e_{t+1} r_{t+1} s_t$. This type of penalty function, first analyzed by Yitzhaki (1974), is in common use in the literature on tax evasion. Denote individual consumption in the second period of life by c_{t+1}^{21} if the individual is caught and consumption in the second period of life by c_{t+1}^{22} if he is not caught. Hence, consumption is

(1a) $c_t^1 = w_t - s_t,$

(1b) $c_{t+1}^{22} = (1+r_{t+1}(1-\tau+\tau e_{t+1}))s_t,$

(1c) $c_{t+1}^{21} = (1+r_{t+1}(1-\tau-\pi\tau e_{t+1}))s_t,$

respectively. It is assumed that utility is additive separable. Furthermore, we work on the assumption that the individual's objective is to maximize the expected utility from consumption. Hence, each individual chooses savings s_t and the proportion e_{t+1} concealed from the authorities to maximize

(2) $u(c_t^1) + \delta[(1-p)u(c_{t+1}^{22}) + pu(c_{t+1}^{21})]$

where u(c) denotes a strictly concave utility function with $u'(c)>0$ and $u''(c)<0$ and δ denotes the discountfactor.[1,2] The first order conditions for an interior solution are

(3a) $[(1-p)u'(c_{t+1}^{22}) - \pi pu'(c_{t+1}^{21})]\delta\tau r_{t+1}s_t = 0,$

(3b) $-u'(c_t^1) + \delta[(1-p)u'(c_{t+1}^{22})$
$+pu'(c_{t+1}^{21})][1+r_{t+1}(1-\tau)] = 0,$

where the first condition has been used to obtain the second condition. Both conditions are well known. Firstly, the marginal rate of substitution between consumption in the two outcomes in the second period is equal to the price ratio, the penalty rate. Secondly, the expected marginal rate of substitution – the trade off between consumption in the first period and in the second period – equals the return net of tax, $1+r_{t+1}(1-\tau)$.

In order to obtain clear-cut results, we assume CRRA utility,

(4) $u(c) = \begin{cases} c^{1-\varepsilon}/(1-\varepsilon), & if\ \varepsilon \neq 1, \varepsilon>0, \\ \ln c, & if\ \varepsilon = 1. \end{cases}$

[1] *The basic portfolio model has been applied to tax evasion analysis by Allingham and Sandmo (1972). For further references see Cowell (1990), chapter 4, Hagedorn (1991), chapter 1, and Wrede (1993), chapter 3.*

[2] *For a discussion of capital income tax evasion see Hagedorn (1991). The Hagedorn two-period microeconomic model is similar to that of Andersen (1977) who studied the relationship between labor supply and tax evasion.*

The constant relative risk aversion is denoted by ε, which also indicates the inverse of the intertemporal elasticity of substitution.

With constant relative risk aversion the optimal evaded fraction and optimal savings are

$$\text{(5a)}\quad e_{t+1} = \frac{(1-p)^{1/\varepsilon} - (p\pi)^{1/\varepsilon}}{\pi(1-p)^{1/\varepsilon} + (p\pi)^{1/\varepsilon}} \; \frac{1+r_{t+1}(1-\tau)}{r_{t+1}\tau},$$

$$\text{(5b)}\quad s_t = \frac{w_t}{1 + \dfrac{\left[\left(1+r_{t+1}(1-\tau)\right)(1+\pi)\right]^{\frac{\varepsilon-1}{\varepsilon}}}{\delta^{1/\varepsilon}\left[\pi^{\frac{\varepsilon-1}{\varepsilon}}(1-p)^{1/\varepsilon} + p^{1/\varepsilon}\right]}}.$$

If the inverse of the intertemporal elasticity of substitution is less than (larger than) one, the interest elasticity of savings is positive (negative). Both cases can be justified from an empirical point of view.[3]

The expected rate of return to a unit of evaded tax, $1-p(1+\pi)$, has to be positive to exclude truthful declaration. Nevertheless, the penalty system has to be severe to prevent the taxpayer from blatant dishonesty. Due to the constant relative risk aversion the optimal evaded fraction of tax just depends on the rate of return, the tax rate, the probability of investigation, and the penalty rate. Tax evasion is independent of savings. A decrease in the probability, the penalty rate, the tax rate, or the rate of return will increase the proportion of taxable income that is being concealed.[4] If wages increase, savings will increase as well. A rise in the rate of time preference will leave the evaded fraction of tax unchanged. In contrast, savings will rise if the rate of time preference decreases. The effects of the penalty rate as well as of the probability of detection on the saving decision depend on the degree of relative risk aversion as the following proposition states.

Proposition 1. An increase of one of the tax-enforcement parameters (p, π and τ) has a negative (positive) effect on savings if $\varepsilon < 1 (\varepsilon > 1)$.

The proof is given in the appendix.

The impact of a higher tax rate is due to the fact that the interest elasticity of savings is negative (positive) if the intertemporal elasticity of substitution is below (above) unity. A change of the penalty system has a similar effect as a rise of the nominal tax rate. The individuals change their savings in the same direction when they are told to pay more taxes as when they are forced to do so.

Moreover, if utility is assumed to be logarithmic, savings are independent of the rate of return and the tax-enforcement system, since the formulas become

$$\text{(6)}\quad e_{t+1} = \frac{1-p(1+\pi)}{\pi\tau}\,\frac{1+r_{t+1}(1-\tau)}{r_{t+1}}$$

$$\text{and}\quad s_t = \frac{\delta}{1+\delta} w_t.$$

The individual saves a fixed proportion of his wage in the working period depending on the discountfactor.

2.2 *Production*

In each period t the representative firm buys labor from the young and capital from the old. Additionally, the government provides public services as a productive input for producers. Barro (1990) and Barro and Sala-i-Martin (1992) constructed several different versions of growth models that include public services. We consider only one version of this type of model. Public services are assumed to be rival and excludable (e.g., education and health). Each producer gets property rights to a specified quantity of public services. Moreover, it is assumed that government purchases per head (of the young generation), g_t, are the quantity allocated to each firm. The firm operates under the production function

$$\text{(7)}\quad Y_t = F(K_t, L_t, g_t)$$

where Y_t indicates the output, K_t the employment of capital, and L_t the employment of labor. It is assumed that F is concave and homogeneous of degree one in K_t and L_t. Therefore, a competitive equilibrium can be established in

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[3] *Hall (1988) showed that there is no strong evidence for a positive elasticity of intertemporal substitution. For a survey of the empirical studies and an analytical explanation see Gylfason (1993).*

[4] *These results are well known from the basic analysis of tax evasion in the framework of the Allingham-Sandmo model (see, e.g., Wrede, 1993, pp. 40–53).*

each period. Since F is increasing in publicly provided services it follows that F exhibits increasing returns to scale. Hence, following Romer (1986),

$$(8)\quad \lambda F(K_t, L_t, g_t) = F(\lambda K_t, \lambda L_t, g_t) < F(\lambda K_t, \lambda L_t, \lambda g_t).$$

By the assumption that F is homogeneous of degree one in K_t and L_t, the production function in per capita terms is

$$(9)\quad y_t=f(k_t,g_t)$$

where y_t and k_t indicate output and capital per capita, respectively.

Taking government purchases as given, the profit maximizing choice of capital and labor equalizes the marginal product of each factor and its payment

$$(10a)\quad r_t = \frac{\partial f(k_t, g_t)}{\partial k_t},$$

$$(10b)\quad w_t = f(k_t, g_t) - r_t k_t.$$

The firm ends up with zero profits.

Once again we specify the functional form. If technology is of the Cobb-Douglas type, the production function is[5]

$$(11)\quad y_t=f(k_t,g_t)=Ak_t^{\alpha}g_t^{1-\alpha}$$

where α denotes the constant elasticity of production with respect to capital and A a parameter of efficiency. Factor payments in equilibrium are

$$(12a)\quad r_t = \alpha \frac{f(k_t, g_t)}{k_t},$$

$$(12b)\quad w_t = (1-\alpha) f(k_t, g_t).$$

2.3 Government budget constraint and public services

In order to examine the impact of tax evasion on the growth rate, we have to analyze the government budget constraint. Since the government runs a balanced budget, the quantity of publicly provided services is equal to the taxes paid in that period. By assumption, taxes are levied on capital income only. But – at least partially – taxes are evaded. Although the sum of taxes and fines paid by each household is uncertain, tax revenue equals the expected tax payments by each individual times the number of persons if the number of individuals is sufficiently large.

The expected tax revenue per capita (of the young generation) in period t is equal to the per capita tax payments of the old times the ratio of population. Assuming a constant growth rate of population, n, this ratio is $1/(1+n)$. Hence, the government expenditures per capita (of the young generation) are

$$(13)\quad g_t = \frac{s_{t-1} r_t \tau [1 - e_t(1 - p(1+\pi))]}{1+n}.$$

2.4 Equilibrium and growth

The capital per capita available in period $t+1$ is equal to the amount of savings of the preceding generation of workers, i.e.

$$(14)\quad k_{t+1} = \frac{s_t}{1+n}.$$

Using this asset market equilibrium condition and the definition of private consumption and public expenditures, we have the per-capita-GNP use

$$(15)\quad y_t = c_t^1 + \frac{E[c_t^2]}{1+n} + g_t + (1+n)k_{t+1} - k_t.$$

where $E[c_t^2]$ indicates the expected value of consumption of a member of the old generation. Output is of the same size as the sum of consumption of both generations, government expenditures, and savings of the young minus dissavings of the old.

Tax revenue in the asset market equilibrium may be written as (using (11), (12a), (13), and (14))

$$(16)\quad g_t = \{A\alpha\tau[1 - e_t(1 - p(1+\pi))]\}^{1/\alpha} k_t.$$

[5] *A lot of the work on endogenous growth concentrates on the Cobb-Douglas technology (see, e.g., Barro, 1990, Saint-Paul, 1992, and Barro and Sala-i-Martin, 1992).*

Hence, the production function is given by

$$(17)\quad f(k_t) = A^{\frac{1}{\alpha}}\left\{\alpha\tau\left[1-e_t(1-p(1+\pi))\right]\right\}^{\frac{1-\alpha}{\alpha}} k_t .$$

Though each producer faces diminishing returns with respect to the private inputs, externalities generate a constant marginal product of capital in equilibrium.

Equation (17) implies that the value of output and the marginal product of capital are determined through the fraction of evaded tax. On the other hand, the fraction of evaded tax depends on the rate of return, i.e. the marginal product of capital. We can solve this equation system for the competitive equilibrium. In the appendix a sufficient condition and a necessary condition for a time independent equilibrium evasion ratio are derived. Moreover, it is shown that in the asset market equilibrium the proportion by which the tax is evaded is only determined by the tax rate, τ, the parameters of the enforcement mechanism, π and p, the elasticity of production with respect to capital, α, and the efficiency parameter, A.

From equations (12a) and (17) we know that evasion increases if the rate of return decreases. Moreover, uniqueness of the (e^*,r^*)-equilibrium values is not guaranteed.

Now, let us consider the dynamic behavior of the economy. Since the marginal product of capital is fixed in the asset market equilibrium, the model generates a special case of the linear Ak-growth model. This basic model of endogenous growth has been thoroughly analyzed by Rebelo (1991). Proposition 2 derives the formula for the growth rate.

Proposition 2. The growth rate of the capital-labor ratio is

$$(18)\quad \hat{k} = \frac{(1-\alpha)A^{1/\alpha}\left\{\alpha\tau\left[1-e(1-p(1+\pi))\right]\right\}^{\frac{1-\alpha}{\alpha}}}{(1+n)\left\{1+\frac{\left[\left(1+(\alpha A)^{1/\alpha}\left\{\tau\left[1-e(1-p(1+\pi))\right]\right\}^{\frac{1-\alpha}{\alpha}}(1-\tau)\right)(1+\pi)\right]^{\frac{\varepsilon-1}{\varepsilon}}}{\delta^{1/\varepsilon}\left[\pi^{\frac{\varepsilon-1}{\varepsilon}}(1-p)^{1/\varepsilon}+p^{1/\varepsilon}\right]}\right\}} - 1,$$

where e is implicitly defined by equations (5a), (12a), and (17).

Proof. Using equations (5b), (12b), (14), and (17), k_{t+1} can be written as a function of k_t. If e_t is a constant, (18) follows.

QED

Firstly, note that the growth rate of output per capita equals the growth rate of the capital-labor ratio since production is linear in capital.

Secondly, the growth rate (18) becomes

$$(19)\quad \hat{k} = \left\{\frac{\delta}{1+\delta}\frac{1-\alpha}{1+n}A^{\frac{1}{\alpha}}\left\{\alpha\tau\left[1-e(1-p(1+\pi))\right]\right\}^{\frac{1-\alpha}{\alpha}}\right\} - 1$$

if utility is logarithmic.

Thirdly, note that the growth rate increases when the discountfactor increases. Only a sufficiently low rate of time preference generates positive growth. If individuals conceal a large fraction of capital income from the tax authorities, the rate of time preference has to be very low in order to attain positive growth. If the utility function is logarithmic, using (19) this can be easily seen, since the growth rate is positive when

$$(20)\quad \delta > \frac{1+n}{(1-\alpha)A^{\frac{1}{\alpha}}\left\{\alpha\tau\left[1-e(1-p(1+\pi))\right]\right\}^{\frac{1-\alpha}{\alpha}} - (1+n)}.$$

Otherwise the capital-labor ratio and the output fall.

Fourthly, consider the impact of tax evasion on the growth rate. Analyzing the derivative of (18) with respect to e allows to state the following.

Proposition 3. An increase in the proportion by which tax is evaded decreases the rate of growth.

An increase in the evaded fraction leads to less tax revenue. Hence, government expenditures decrease. Thus, since the government provides productive inputs for private producers, output and wages fall. If utility is logarithmic, savings are also reduced. Less capital is used in the next period. Tax evasion has only a negative effect on the growth rate. The reason is that, on the one hand, savings are unaltered if wages are unchanged. On the other hand, wages of the next generation are reduced. This effect is channeled through the publicly provided services. If the interest elasticity of saving is positive, growth is further discouraged by tax evasion, since an increase in the evaded fraction decreases the interest rate. In contrast, tax evasion has partly a positive effect on the growth rate under conditions of a negative interest elasticity of savings. It can be shown, however, that this positive effect is smaller than the negative effect.

Let us consider next the logarithmic case more carefully. The growth rate is maximal if no evasion occurs. Nevertheless, if the expected rate of return to evasion is positive, tax evasion cannot be stamped out. Although increasing growth does not imply in general a pareto-improvement, a higher growth rate through less tax evasion improves welfare of each individual when the intertemporal elasticity of substitution is equal to unity. Not only the growth rate, but also consumption in the retirement period and therefore welfare increase if the evaded fraction is lowered. Substituting (6) into (1) yields

$$\text{(21a)}\quad c_t^1 = \frac{1}{1+\delta} w_t,$$

$$\text{(21b)}\quad c_{t+1}^{22} = \left(1+r_{t+1}(1-\tau)\right)\frac{(1-p)(1+\pi)}{\pi}\frac{\delta}{1+\delta} w_t,$$

$$\text{(21c)}\quad c_{t+1}^{21} = \left(1+r_{t+1}(1-\tau)\right)p(1+\pi)\frac{\delta}{1+\delta} w_t.$$

Consumption in the second period of life and therefore utility is larger, the higher the rate of return, i.e., the lower the proportion of evaded tax.

Remark 1. A reduction in the proportion by which tax is evaded increases the utility of each individual when the intertemporal elasticity of substitution is equal to unity.

2.5 *Public consumption and exogenous growth*

Now we consider the case in which public expenditures are not provided as productive inputs to the producers. Hence, growth is no longer endogenous. As before, the production function is log-linear and homogeneous. For the sake of concretness the Cobb-Douglas function is used (in per capita terms: $y_t = Ak_t^{\alpha}$).[6]

Suppose first that the utility function is logarithmic. Using (5b) and (14) routine analysis yields

$$\text{(22)}\quad k_{t+1} = \frac{\delta}{1+\delta}\frac{1-\alpha}{1+n} Ak_t^{\alpha}.$$

The slope of the (k_t, k_{t+1})-curve is positive and diminishing. The value of the stable steady state is

$$\text{(23)}\quad k = \left(\frac{\delta}{1+\delta}\frac{1-\alpha}{1+n}A\right)^{\frac{1}{1-\alpha}}.$$

As savings are not influenced by evasion and government purchases do not affect production, the penalty rate, the probability of detection, and the tax rate are of no relevance for the steady state value of capital intensity. As is well-known, the growth rate is exogenously given by the growth rate of the population.

Now consider the more general case of the CRRA utility function. Using (5.b) and substituting into (14) yields

$$\text{(24)}\quad k_{t+1} = \frac{(1-\alpha)Ak_t^{\alpha}}{(1+n)\left\{1+\frac{\left[\left(1+\alpha Ak_{t+1}^{\alpha-1}(1-\tau)\right)(1+\pi)\right]^{\frac{\varepsilon-1}{\varepsilon}}}{\delta^{1/\varepsilon}\left[\pi^{\frac{\varepsilon-1}{\varepsilon}}(1-p)^{1/\varepsilon}+p^{1/\varepsilon}\right]}\right\}}.$$

In general, the effects of changes of the tax-enforcement parameters on the steady state capital-labor ratio depend on the degree of relative risk aversion. The following proposition states some results in comparative statics.

Proposition 4. Under conditions of stability the steady state capital labor ratio, k*, changes according to:

$$sign\left(\frac{dk^*}{d\tau}\right) = sign\left(\frac{dk^*}{dp}\right) = sign\left(\frac{dk^*}{d\pi}\right) = sign(\varepsilon-1).$$

The proof is given in the appendix.

If the government levies a higher tax, the net interest rate decreases. The individuals reduce (increase) their savings if the interest elasticity of savings is positive (negative). Hence, the economy produces with less (more) capital in the long-run equilibrium.

As an increase of one of the enforcement parameters changes savings in the same direction as the tax rate, the effects of these parameters on the long-run equilibrium are similar.

[6] *This assumption is not essential. The following results hold if the production function is linear-homogeneous and fulfills e.g. the Inada-conditions.*

3. *Concluding remarks*

Consider once again the question raised at the outset. How will tax evasion affect the economy in the long run? The life-cycle model with a constant marginal product of capital leads to the following conclusion if the government pays for expenditures with taxes: when the government spending contributes to the economy´s productive capacity, tax evasion will lower the accumulation of capital. This result holds for a logarithmic utility function as well as for the more general CRRA utility function.

In contrast, if the taxes are used for public consumption and if therefore the growth rate is exogenously determined by the growth rate of labor, the impact of tax evasion is ambiguous. If the intertemporal elasticity of substitution equals one, tax evasion has no impact on the long-run equilibrium. Under conditions of negative or positive interest elasticity of savings the role of tax evasion is quite different. It has been shown that an increase of one of the tax-enforcement parameters shifts the long-run capital-labor ratio upwards (downwards), if the intertemporal elasticity of substitution is lower(higher) than one.

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Appendix

Proof of proposition 1:

Define $\Psi = \delta^{1/\varepsilon}\left[\pi^{\frac{\varepsilon-1}{\varepsilon}}(1-p)^{1/\varepsilon} + p^{1/\varepsilon}\right]$. Then

$$sign\left(\frac{\partial s_t}{\partial p}\right) = sign\left(\frac{\partial \Psi}{\partial p}\right)$$

$$= sign\left(\frac{1}{\varepsilon}\delta^{1/\varepsilon}\pi^{\frac{\varepsilon-1}{\varepsilon}}\left((p\pi)^{\frac{1-\varepsilon}{\varepsilon}} - (1-p)^{\frac{1-\varepsilon}{\varepsilon}}\right)\right).$$

Since an interior solution is characterized by $1-p>p\pi$, the sign is negative (positive) if $\varepsilon<1(\varepsilon>1)$. Moreover,

$$sign\left(\frac{\partial s_t}{\partial \pi}\right) = -sign\left(\frac{\partial\left((1+\pi)^{\frac{\varepsilon-1}{\varepsilon}}/\Psi\right)}{\partial \pi}\right)$$

$$= -sign\left(\frac{\delta^{1/\varepsilon}(\varepsilon-1)}{\Psi^2(1+\pi)^{1/\varepsilon}\pi^{1/\varepsilon}\varepsilon}\left((p\pi)^{1/\varepsilon}-(1-p)^{1/\varepsilon}\right)\right).$$

As above, the sign is negative (positive) if $\varepsilon < 1 (\varepsilon > 1)$. This is also the case if the tax rate is changed:

$$sign\left(\frac{\partial s_t}{\partial \tau}\right) = -sign\left(\frac{\partial\left(\left[1+r_{t+1}(1-\tau)\right]^{\frac{\varepsilon-1}{\varepsilon}}\right)}{\partial \tau}\right)$$

$$= sign\left(\frac{\varepsilon-1}{\varepsilon}r_{t+1}\left[1+r_{t+1}(1-\tau)\right]^{-\frac{1}{\varepsilon}}\right).$$

QED

Existence of an equilibrium:

In the asset market equilibrium e*, $0 < e^* < 1$, is determined by τ, π, p, α, and A, and it is independent of the capital-labor ratio and therefore constant in time if

$$\frac{(1-p)^{1/\varepsilon}-(p\pi)^{1/\varepsilon}}{(1-p)^{1/\varepsilon}(\tau(1+\pi)-1)+(p\pi)^{1/\varepsilon}} < (\alpha A)^{1/\alpha}\{\tau p(1+\pi)\}^{\frac{1-\alpha}{\alpha}}$$

and if the necessary condition for the existence of an equilibrium is fulfilled: $\tau(1+\pi)>1-[p\pi/(1-p)]^{1/\varepsilon}$.

Proof: Using equations (5a), (12a), and (17), yields

$$\frac{1}{\overline{B}e_t\tau-(1-\tau)} - \overline{A}\{1-e_t(1-p(1+\pi))\}^{\frac{1-\alpha}{\alpha}} = 0$$

where

$$\overline{A} = (\alpha A)^{\frac{1}{\alpha}}\tau^{\frac{1-\alpha}{\alpha}} \quad \text{and} \quad \overline{B} = \frac{\pi(1-p)^{1/\varepsilon}+(p\pi)^{1/\varepsilon}}{(1-p)^{1/\varepsilon}-(p\pi)^{1/\varepsilon}}.$$

Hence, optimal tax evasion is independent of capital per capita. Since we concentrate on interior solutions, we have to verify that there exists a solution of this equation in the open interval (0,1). Therefore, we define:

$$\mathfrak{I}(e) = \frac{1}{\overline{B}e\tau-(1-\tau)} - \overline{A}\{1-e(1-p(1+\pi))\}^{\frac{1-\alpha}{\alpha}}.$$

Note that

$$\mathfrak{I}(0) = -\frac{1}{1-\tau} - \overline{A} < 0 \quad \text{and} \quad \lim_{e\to\frac{1-\tau}{\overline{B}\tau}-0}\left(\mathfrak{I}(e)\right) = -\infty$$

$$\text{and} \quad \lim_{e\to\frac{1-\tau}{\overline{B}\tau}+0}\left(\mathfrak{I}(e)\right) = \infty.$$

A necessary condition for $\mathfrak{I}(e)=0$ is $\bar{B}e\tau > 1-\tau$. Since $\bar{B}e\tau-(1-\tau)$ is an increasing function of e, at least at $e=1$ this inequality has to be fulfilled. Therefore, $\tau(1+\pi)>1-[p\pi/(1-p)]^{1/\varepsilon}$ is necessary.

If the condition given above is fulfilled it follows that $\mathfrak{I}(e)<0$. Hence, at least one solution of $\mathfrak{I}(1)=0$ exists.

Since the signs of $\partial\mathfrak{I}(e)/\partial e$ and $\partial^2\mathfrak{I}(e)/\partial e^2$ are ambiguous, more than one solution might exist.

QED

Proof of proposition 4:

Using the implicit function theorem it can be shown that the slope of the (k_t,k_{t+1})-curve is positive, since

$$\frac{dk_{t+1}}{dk_t} = \frac{\Psi(1-\alpha)\alpha A k_t^{\alpha-1}}{(1+n)\left\{\Psi+\Theta^{\frac{\varepsilon-1}{\varepsilon}}-(1-\alpha)\alpha A k_{t+1}^{\alpha-1}(1-\tau)(1+\pi)\frac{\varepsilon-1}{\varepsilon}\Theta^{-\frac{1}{\varepsilon}}\right\}}$$

where

$$\Theta = \left(1+\alpha A k_{t+1}^{\alpha-1}(1-\tau)\right)(1+\pi).$$

Stability of the steady state requires the slope to be less than one.

The equation

$$\Phi(k,\tau,p,\pi,n) = (1+n)k - s(k,k,\tau,p,\pi) \equiv 0$$

where

$$s(k,k,\tau,\pi,p) = \frac{(1-\alpha)Ak^\alpha}{1+\frac{\left[\left(1+\alpha A k^{\alpha-1}(1-\tau)\right)(1+\pi)\right]^{\frac{\varepsilon-1}{\varepsilon}}}{\Psi}}$$

implicitly defines the steady state. Stability ensures that $\partial\Phi/\partial k>0$. Using the implicit function theorem it can be obtained that the proposition holds, since

$$sign\left(\frac{dk^*}{d\tau}\right) = sign\left(\frac{ds(k,k)}{d\tau}\right).$$

The sign of the term on the right-hand side follows from proposition 1. This relationship holds also for the other parameters.

QED