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CO-INTEGRATION AND THE TERM STRUCTURE OF FINNISH SHORT-TERM INTEREST RATES*

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The term structure of Finnish HELIBOR interest rates is studied by modelling it as a co-integrated system. There are three co-integrating vectors among the six rates. They can be identified as the spreads between the two and one and three and one month rates, and a third vector tending to keep the yield curve linear. Co-integration analysis of partial systems suggests that it is only for the three shortest-term yields that the expectations hypothesis cannot be rejected. Recursive analysis reveals that the co-integration space has changed in time, which is not surprising given the changes in monetary policy regimes. (JEL C32, E43)

1. Introduction

Recent developments in econometric methods, especially in the theory of integrated time series and multivariate co-integrated systems, have brought up totally new ways of modelling the term structure of interest rates, i.e. the relationship between yields of securities that differ only with respect to their time to maturity. Several empirical studies have concluded that interest rates are integrated processes of the first order ($I(1)$ processes)[1]. The so called *expectations hypothesis*, according to which the term structure is determined by investors' expectations about future interest rates, has always played the central role in empirical work on the term structure, and as will be shown later, this hypothesis implies that the spread between the short-term and long-term interest rate should be stationary, i.e. they should co-integrate, if interest rates are $I(1)$ processes. Testing for these co-integration relations forms the basis for recent empirical research of the term structure of interest rates, which was pioneered by Campbell and Shiller (1987) in a bivariate setting, and further developed in a multivariate framework by Hall, Anderson and Granger (1992), Shea (1992), Zhang (1993), and Engsted and Tanggaard (1994).

In this paper we study the term structure of Finnish HELIBOR interest rates using the co-

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[1] *As Hall et al. (1992) have pointed out, this cannot be strictly true, since first-order integrated processes are unbounded, whereas nominal interest rates are bounded below by zero. Treating them as integrated processes in model building is appropriate, though, because their statistical properties are much closer to those of integrated processes than those of stationary processes. Although considering interest rates as I(1) processes is statistically convenient, it is dubious whether this feature can be given a meaningful economic interpretation. In addition to being bounded below by zero, interest rates are also rather stable in the sense that the variances of forecast errors cannot be considered to grow constantly with the forecast horizon. Moreover, if yields are I(1) processes, the prices of bonds (and stocks) should be I(2) processes, which poses further interpretational problems.*

integration methodology for modelling non-stationary data. Although they are all rather short-term rates and do not in any way represent the entire Finnish yield curve, the poor quality of data for longer-term rates due to thin markets, forces us to limit ourselves to the short end of the maturity spectrum. The goal is to try to find a sensible and parsimonious representation of the HELIBOR rates. Thus we not only attempt to test whether the expectations hypothesis holds, but having that hypothesis as a starting point or benchmark hypothesis, we aim at finding a suitable empirical model for the term structure. There are so far very few published studies of the term structure of HELIBOR rates. Rantala (1989), using daily data for the years 1987 and 1988 concluded that the one month HELIBOR rate follows an AR(1) process and a model based on the expectations hypothesis is rejected. Murto (1990), on the other hand, found with weekly data from 1987–1989 that the pure expectations hypothesis characterizes the market reasonably well, but that the possibility of time-varying term premium cannot be excluded.

In section 2 we derive the co-integration implications of the expectations hypothesis. Section 3 contains the empirical results. First, the complete yield curve will be dealt with, and then several subsets of the interest rates will be analyzed. Finally, recursive analyses will be conducted in order to find out whether the models are stable over time and to detect potential structural breaks in the system. Because of the limited sample size, methods based on recursive estimation are the only way to study the stability of the system, as the division of the data into subperiods is out of the question. Section 4 concludes the paper.

2. *The expectations hypothesis and co-integration*

For pure discount bonds the expectations hypothesis can be expressed by the following formula

$$(1) \quad R_{n,t} = \frac{1}{n}\sum_{i=0}^{n-1} E_t R_{1,t+i} + L_n,$$

where $R_{1,t}$ and $R_{n,t}$ are the 1-period and n-period interest rates, respectively, L_n is a term premium dependent only on the ratio of the maturities, and E_t denotes mathematical expectation conditional on public information at time t including current and lagged values of the two interest rates. Equation (1) states that the longer-term interest rate is an arithmetic average of the present and expected future 1-period interest rates over the life of the longer-term bond plus a constant premium term L_n. By rearranging equation (1) we obtain

$$(2) \quad R_{n,t} - R_{1,t} = \frac{1}{n}\sum_{i=1}^{n-1}\sum_{j=1}^{i} E_t \Delta R_{1,t+j} + L_n,$$

where $\Delta R_{1,t+j} = R_{1,t+j} - R_{1,t+j-1}$. Assuming that interest rates are $I(1)$ processes, their first differences are stationary $I(0)$ processes and because the term premium L_n is constant, it follows that as a sum of stationary components the right hand side of equation (2) must be stationary. For the term on the left-hand side of the equation to be stationary it is necessary that $R_{n,t}$ be co-integrated with $R_{1,t}$ such that their difference is stationary. In other words this means that the expectations hypothesis requires each yield to be co-integrated with the one period yield. Moreover, the type of co-integration should be such that, if there are p yield series, then each of the p–1, p-dimensional vectors in the set {(–1, 1, 0,..., 0)', (–1, 0 1, 0,...,0)',..., (–1, 0,..., 0, 1)'} is co-integrating for the vector consisting of the p yields.[2] It is also worth noting that it follows from this argument that *any* spread between two yields is co-integrating, since an arbitrary spread can always be expressed as a linear combination of two spreads including the one-

[2] *To take account of the constant term premium, the co-integrating vectors must be augmented with a constant term. Embedding the constant term in the co-integrating relations is natural since interest rates cannot be assumed to have a linear trend. Besides, under the expectations hypothesis, this leads to a model with a natural interpretation: although interest rates of different maturity may diverge in the short run, they will adjust when the spreads between them deviate from the equilibrium value given by the constant term.*

period yield, and linear combinations of stationary variables are stationary.

The fact that the term structure can be expressed as the aforementioned co-integrated system, can be given at least two intuitive interpretations based on different representations of a co-integrated system. First, the error correction representation relates the changes in each yield series to past equilibrium errors and past changes in all yields. In this case the yield spreads are the equilibrium errors that adjust the yields on bonds of different maturity when they diverge, so that in the long run different yields move together. As Campbell and Shiller (1988) have shown, the existence of an error correction model in this context does not necessarily reflect any kind of partial adjustment of one variable to another, but the co-integration relationship can also be interpreted in another way. Namely, co-integration can arise whenever agents are forecasting and have rational expectations. The yield spreads can be used to make more accurate forecasts of future short rates than would be possible using only the past observations of the short rate series, and the error correction model results from agents' forward-looking behaviour.

Co-integration can also be interpreted in terms of common trends underlying the interest rate series. More specifically Stock and Watson (1988) have shown that, with p $I(1)$ processes, if there are r co-integrating vectors, then the p variables can be expressed as a linear combination of $p-r$ $I(1)$ common trends and an $I(0)$ component. Thus, the expectations hypothesis implies that there should be a single non-stationary common trend underlying all yields. The hypothesis does not, of course, restrict the number of stationary common factors explaining the movements of interest rates.

As a starting point for the co-integration analysis we shall have an unrestricted p-dimensional VAR model

$$(3)\quad X_t = \Pi_1 X_{t-1} + \dots + \Pi_k X_{t-k} + \varepsilon_t, \quad t = 1,\dots,T$$

where the error terms $\varepsilon_1,\dots,\varepsilon_T$ are $IIN_p(\mathbf{0},\Lambda)$ and $X_{-k+1},\dots, X_0$ are fixed. It can be reparametrized as

$$(4)\quad \Delta X_t = \Gamma_1 \Delta X_{t-1} + \dots + \Gamma_{k-1}\Delta X_{t-k+1} + \Gamma_k X_{t-k} + \varepsilon_t$$

where $\Gamma_i = -(I-\Pi_1-\dots-\Pi_i)$, $i = 1,\dots,k$, and $\Pi = -\Gamma_k$. Under the assumption that $X_t \sim I(1)$ and thus ΔX_t is stationary, the components of X_t are co-integrated when the rank r of the impact matrix Π is greater than zero but less than p. In this case there are $p \times r$ matrices α and β such that $\Pi = \alpha\beta'$, and the columns of β are the co-integrating vectors having the property that the linear combinations $\beta_j'X_t$ ($j = 1,\dots, r$) are stationary. The space spanned by β is called the *co-integration space*, and the columns of α that give the weights with which each error-correction term enters each of the equations, span the *adjustment space*.

The maximum likelihood method developed by Johansen (1988, 1991) can be used to test for the number of co-integrating vectors, and conditional on the chosen number, the vectors can be estimated. Once the co-integration rank for a given set of interest rates has been determined and found to be in accordance with the theory, the restrictions imposed by the expectations hypothesis can be tested. In other words, testing the expectations hypothesis involves two stages. With p interest rate series, the first null hypothesis is that the co-integration rank is $p-1$, and then conditional on there being $p-1$ co-integrating vectors, the second null hypothesis is that the $p-1$ linearly independent spreads form a basis for the co-integration space. Thus, if the null hypothesis of $p-1$ co-integrating vectors is rejected, the expectations hypothesis is also rejected and no further testing is needed.[3]

[3] *Shea (1992), however, points out that a possible reason for finding too few co-integrating vectors is low power of the tests in detecting extra vectors. It is also known that power of these tests drops with the dimensionality of the system, and therefore it may be advisable to do the first test for several, gradually increasing sets of interest rates and, whatever the result, to do the second test assuming the co-integration rank to be p–1.*

3. Data and Empirical Results

3.1 Data

The data consist of monthly observations of HELIBOR (Helsinki Interbank Offered Rate) rates for maturities of 1, 2, 3, 6, 9 and 12 months as the average bid rate of the banks' certificates of deposit (CDs) as quoted by the five largest Finnish banks[4]. The CDs are issued by the banks and the Bank of Finland. The sample period ranges from January 1987 to May 1993, so that the total number of observations is 77.[5]

It is important to note that when studying the term structure we assume that the only difference between the bonds is their time to maturity. As most studies focus on the bonds issued by the government, this assumption holds. The banks' CDs are not, however, homogenous instruments, but because of the limited number of banks participating in the market, differences are likely to be immaterial (see Murto 1990). This is not, however, true for the whole sample period. Beginning in autumn 1990, other banks started requiring an extra premium for CDs issued by Skopbank, and this went on until Skopbank was taken over by the Bank of Finland in September 1991. To alleviate the effect of the premium, the average was calculated without the bid rate of Skopbank for the period 1990:9–1991:9.

The sample period included several incidents and changes in the institutional setting that were likely to affect the functioning of financial markets in Finland. By June 1991 the foreign exchange regulations were gradually abolished, and in July 1992 the system for regulating bank liquidity changed such that thereafter deposit and borrowing rates should be closer to market interest rates and also move in line with changes in market rates. In February 1993 the Bank of Finland introduced new guidelines for monetary policy in terms of an inflation target. It turned out, however, that the following three extraordinary events during the sample period needed to be modelled by dummy variables in most of the subsequent analyses. First, all rates declined remarkably in May 1990, probably due to the new cash reserve agreement between the banks and the Bank of Finland. Before the agreement the rate of interest payable to cash reserve deposits had been linked to average rates, so that it had not really followed changes in the market interest rates. The extra risk involved in this arrangement was now abolished by linking the cash reserve deposit rate to the three month HELIBOR rate. Second, there was a sharp increase in March 1991 in the one, two and three month rates. This was plausibly caused by heavy speculation on devaluation which was enforced by uncertainty concerning the formation of the new government after the parliamentary election. Third, severe speculation on devaluation broke out again in September 1991 causing enormous increase in all HELIBOR rates, although the shorter rates rose more sharply. The spreads diminished after the devaluation in November, but speculation went on in April 1992, when the spreads rapidly grew again. Finally the same development was experienced in September leading to the decision to float the markka, and thereafter in November all HELIBOR rates declined markedly and also the spreads became clearly narrower.

3.2 Co-integration analysis of the complete system[6]

As a first step the underlying unrestricted VAR model was estimated, and the lag length

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[4] The six interest rates will be denoted R_1, R_2, R_3, R_6, R_9 and R_{12}, respectively.

[5] Weekly and daily data from that period are also available, but monthly data are preferable. The use of more frequently sampled data introduces the problem of numerous extraordinary observations or outliers that would have to be modelled by dummy variables in order to satisfy the assumptions of the statistical methods. Besides, in light of some simulation studies (see e.g. Shiller and Perron 1985, and Hakkio and Rush 1991) the effect of sampling the data more frequently on the power of Dickey-Fuller type tests is negligible, and the same is likely to apply to maximum likelihood methods as well. In fact, Eitrheim (1991) has shown with Monte Carlo simulation experiments that with constant sample size, the power of the trace test increases with decreasing sampling frequency. Most of the tests presented in this section were also performed with weekly data, and the results are qualitatively the same.

[6] Computations were mostly performed using the software package Cointegration Analysis of Time Series

Table 1. Results of co-integration analysis.

H_0	$r = 0$	$r \leq 1$	$r \leq 2$	$r \leq 3$	$r \leq 4$	$r \leq 5$
Eigenvalues	0.569	0.436	0.343	0.203	0.166	0.006
Trace test	168.696	105.542	62.550	31.088	14.062	0.441
95% fractiles	119.537	89.027	62.168	40.856	23.360	10.814
90% fractiles	113.637	84.029	58.058	37.419	20.873	8.793
λ_{max} test	63.153	42.992	31.462	17.026	13.621	0.441
95% fractiles	47.533	40.574	33.190	25.948	18.482	10.898
90% fractiles	43.905	37.117	29.966	23.178	16.120	8.816

The critical values are the quantiles of the asymptotic distributions given in Osterwald-Lenum (1992, 467, Table 1*) corrected for sample size. The corrections were computed using response surface estimates in Cheung and Lai (1993a, 318, Table 1).

was chosen to be the minimum that still guaranteed non-autocorrelatedness and normality of the residuals. The three dummy variables mentioned above, were also included in the vector autoregression. It is conceivable that the dummies may effect the distributions of the test statistics to be used in the following analysis, although they have no influence asymptotically. It is virtually impossible to take account of this effect, though. According to the diagnostic tests the residuals were non-autocorrelated and normally distributed.

The results of co-integration analysis are presented in Table 1. It has been discovered that Johansen's likelihood ratio tests easily lead to too high a co-integration rank in finite samples, and therefore the critical values were computed using the response surface estimates in Cheung and Lai (1993a). At the 5% level the critical values thus obtained give the same result as the asymptotic values given in Osterwald-Lenum (1992) as far as the trace test is concerned. According to the maximal eigenvalue test the co-integration rank would be two if the corrected fractiles are used and three if the asymptotic fractiles are used. At the 10% level both the trace test and maximal eigenvalue test agree, no matter which of the two distributions is used. As the trace test has also been found to be more robust to possible misspecifications, we conclude that the rank of the co-integration space is three. This means that the expectations hypothesis cannot hold for all HELIBOR rates, or in other words, there are three non-stationary common stochastic trends driving the system of interest rates, not a single one as the expectations hypothesis implies.[7]

Because it is possible that the tests have chosen too low a rank for the co-integration space, let us for a moment assume that the co-integration rank is indeed five, as it would be under the expectations hypothesis. Given this assumption, we can test whether the co-integration space is spanned by the five spreads, i.e. by vectors (asterisk denotes that there are no restrictions on the constant term) (1, –1, 0, 0, 0, 0, *), (1, 0, –1, 0, 0, 0, *), (1, 0, 0, –1, 0, 0, *), (1, 0, 0, 0, –1, 0, *) and (1, 0, 0, 0, 0, –1, *). The null hypothesis can thus be expressed in matrix notation as (for details of this and subsequent likelihood ratio tests on the co-integration space and adjustment space, see Johansen and Juselius 1992):

$$\beta = H\varphi = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \varphi,$$

(CATS) by Katarina Juselius and Henrik Hansen. Routines for recursive analysis were written and run in RATS 3.11.

[7] *We also tested for the stationarity of each of the interest rate series, and according to both Johansen's stationarity tests and standard unit root tests they are I(1) processes. Theoretically this finding is difficult to justify (see footnote 1), but it indicates that the properties of the series resemble more those of I(1) than I(0) processes.*

where φ is a (6 × 5) matrix, and the likelihood ratio statistic follows asymptotically the χ^2 (5) distribution. By expanding this expression it can easily be seen that the hypothesis simply implies that the first six elements of each co-integrating vector sum to zero. The value of the test statistic, 7.29 clearly indicates that the restrictions cannot be rejected (p-value = 0.20).

According to the expectations hypothesis (see equation (2)) the spread between the long and short rate should forecast changes in the short rate over the life of the long rate, and furthermore the relationship should be positive. In other words, increases in the spread should lead to rises in the short rate. This feature can be tested by examining the coefficients of the alpha matrix consisting of the factor loadings that measure the weights of the co-integrating vectors in each of the VAR equations. If the expectations hypothesis holds, the spreads span the co-integration space and the factor loadings should thus be significant in the equations of the short-term rates and close to zero in the equations of the long-term rates. This can be tested by testing weak exogeneity of each of the rates with respect to the long-run parameters, i.e. under the null hypothesis none of the co-integrating vectors enters the respective equation. In matrix notation this can be expressed as

$$\alpha = A\psi,$$

where A and ψ are (6 × 5) and (5 × 5) matrices, respectively, and one of the rows in A consists, in turn, of zeros. The results of these tests are shown in Table 2. In the rightmost column we test simultaneously both weak exogeneity and the zero-sum restriction implied by the expectations hypothesis (test statistics follow asymptotically χ^2 (10)), while the statistics in the left column come from a test of weak exogeneity only and follow the χ^2 distribution with 5 degrees of freedom. At the 5% level the critical values for the R_1 two tests are 18.31 and 11.07, respectively, so that weak exogeneity with the zero-sum restriction is rejected for R_1, R_2 and R_3, and weak exogeneity only for these as well as R_6.

Although the necessary condition for the expectations hypothesis, that there should be five co-integrating vectors, is clearly rejected, the data seem to some extent to be in conformity with it. This is demonstrated by the fact that the hypothesis on the spreads spanning the co-integration space under the assumption of five co-integrating vectors cannot be rejected. Furthermore, the longer-term rates seem to »drive» the system as the co-integrating vectors (spreads) cannot be excluded from the equations for the 1, 2, 3 and maybe even 6 month yields. This evidence is thus somewhat inconclusive. The system has too many common trends, and since several of the longer-term rates are weakly exogenous w.r.t. alpha and beta, it seems that they together drive the system. To find out about the dynamics of the interest rate series we shall next take a closer look at the model with three co-integrating vectors and try to identify the co-integration space by testing structural hypotheses on it.

Table 2. Testing for weak exogeneity of the factor loadings w.r.t. the long-run parameters.

Variable	$\alpha = A\psi$	$\beta = H\varphi$, $\alpha = A\psi$
R_1	30.54 (0.00)	36.87 (0.00)
R_2	28.60 (0.00)	34.50 (0.00)
R_3	23.10 (0.00)	28.12 (0.00)
R_6	12.47 (0.03)	17.87 (0.06)
R_9	7.81 (0.17)	13.42 (0.20)
R_{12}	6.11 (0.30)	12.03 (0.28)

The figures in parentheses are significance levels.

As an initial stage, significance measures of each element of the three co-integrating vectors and of the factor loadings are computed. These measures are not necessarily very reliable, and therefore this analysis is to be considered just as a first step in identifying the structure of the model (see Juselius 1993). Significance measures for the variables in the system can be obtained by testing the hypothesis $\beta_{ij} = 0$ for $i = 1, ..., p$ and $j = 1, ..., r$, where p is the number of series and r is the number of co-integrating vectors. For each i the hypothesis implies that variable X_i does not enter the co-integration space, and if it cannot be rejected, that variable can be excluded from the long-run relations. By doing this test for all possible values of r, and for $\hat{r} = 2, ..., r$, by subtracting the value for $\hat{r}-1$ from the value for $\hat{r}$ for each variable, we get statistics for the significance of each beta-coefficient in each co-integrating vector. Analogously, by testing the hypothesis $\alpha_{ij} = 0$ for $i = 1, ..., p$ and $j = 1, ..., r$ with different values of r and doing the same subtraction operation, we get marginal significance statistics for all the factor loadings. In this case the hypothesis implies that the process X_i does not contain any information about the long-run relations, i.e. it is a test for weak exogeneity of X_i w.r.t. beta. In both cases the significance statistics follow asymptotically the χ^2 (1) distribution. The 'overall' significance statistics computed for each variable (Q_β) and for each equation (Q_α) for the chosen co-integration rank, r, are asymptotically χ^2 (r) distributed.

The beta and alpha matrices are given in Table 3, along with the above mentioned marginal significance statistics (in parentheses for each coefficient) and the overall significance statistics Q_β and Q_α. The only variable that can be excluded from the co-integration space is the twelve month yield for which Q_β value is below the 5% critical value; at the 10% level it cannot be excluded, but it is clearly the 'least significant' of the six interest rates in the co-integrating system. It is difficult to give any clear interpretation for the co-integrating vectors. In β_1 R_1 and R_2 are the most significant, and the coefficients suggest that it might be the spread between these rates. Similarly, the second beta vector could be interpreted as the spread between R_3 and R_6. As for the third co-integrating vector, all variables except the twelve month rate seem to be significant, but for the linear combination given by the coefficients one cannot easily find any intuitive characterization. According to the weak exogeneity tests Q_α, the equations for ΔR_6, ΔR_9 and ΔR_{12} do not contain information about the long-run relations, i.e. they can be considered the driving trends in the system of equations. The marginal significance statistics indicate, however, that the third co-integration relation enters the equation for ΔR_6. In addition, the first and third relation seem to enter the equations of the three shortest-term yields, whereas the second relation is not significant in any equation. The conclusion is thus that presumably only R_{12} can be omitted from the co-integration space and the two longest-term rates, R_9 and R_{12}, can be considered weakly exogenous for the long-run parameters and therefore the driving trends in the system.

Table 3. Testing for significance of β and α.

β-matrix

Variable	β_1	β_2	β_3	Q_β
R_1	1.000	1.000	1.000	
	(16.38)	(0.07)	(13.81)	30.26
R_2	–1.985	25.552	–1.758	
	(12.05)	(4.89)	(14.11)	31.05
R_3	0.006	–51.751	0.465	
	(0.00)	(11.10)	(11.58)	22.69
R_6	1.872	58.441	1.201	
	(5.87)	(9.83)	(12.01)	27.71
R_9	–0.867	–52.623	–2.278	
	(1.07)	(4.93)	(5.56)	11.56
R_{12}	–0.025	19.692	1.369	
	(0.00)	(2.28)	(4.58)	6.86
Constant	–0.052	–2.625	–0.060	

α-matrix

Equation	α_1	α_2	α_3	Q_α
ΔR_1	2.409	–0.038	–4.065	
	(4.11)	(0.31)	(12.46)	16.87
ΔR_2	2.035	–0.038	–2.917	
	(4.76)	(0.56)	(11.49)	16.81
ΔR_3	1.460	–0.016	–2.246	
	(3.60)	(0.14)	(8.60)	12.34
ΔR_6	0.528	–0.023	–1.330	
	(0.71)	(0.44)	(4.07)	5.22
ΔR_9	0.422	–0.009	–0.871	
	(0.57)	(0.10)	(2.06)	2.73
ΔR_{12}	0.380	–0.003	–0.748	
	(0.51)	(0.01)	(1.69)	2.21

Figures in parentheses follow asymptotically the χ^2 (1) distribution and Q_β and Q_α follow asymptotically the χ^2 (3) distribution. The critical values at the 5% level are 3.84 and 7.82, respectively.

Next the stationarity of different linear combinations of the rates will be examined under the assumption of there being three co-integrating vectors. In other words, we shall test whether a given vector lies in the space spanned by the three co-integrating vectors, the columns of β. First of all, the presence of the spreads in the co-integration space is an interesting hypothesis because it may help us see, which rates in particular are responsible for the rejection of the expectations hypothe-

Table 4. Testing structural hypotheses on the co-integration space.

Hypothesis	df	LR statistic	
$(1, -1, 0, 0, 0, 0, *)' \in sp(\beta)$	3	6.11	(0.11)
$(1, 0, -1, 0, 0, 0, *)' \in sp(\beta)$	3	6.25	(0.10)
$(1, 0, 0, -1, 0, 0, *)' \in sp(\beta)$	3	8.08	(0.04)
$(1, 0, 0, 0, -1, 0, *)' \in sp(\beta)$	3	10.15	(0.02)
$(1, 0, 0, 0, 0, -1, *)' \in sp(\beta)$	3	11.86	(0.01)
$\beta_1 = (1, -1, 0, 0, 0, 0, *)$, $\beta_2 = (1, 0, -1, 0, 0, 0, *)$ and			
$H^1 : \beta_3 = (1, 0, 0, -1, 0, 0, *)$	9	39.16	(0.00)
$H^2 : \beta_3 = (0, 0, 0, a, b, 0, *)$	8	42.76	(0.00)
$H^3 : \beta_3 = (0, 0, 0, 0, a, b, *)$	8	40.32	(0.00)
$H^4 : \beta_3 = (0, 0, 0, a, b, c, *)$	7	36.16	(0.00)
$H^5 : \beta_3 = (a, 0, 0, b, c, d, *)$	6	7.70	(0.26)
$H^6 : \beta_3 = (a, 0, 0, b, c, 0, *)$	7	9.34	(0.23)
$H^7 : \beta_3 = (a, 0, 0, 0, b, c, *)$	7	31.90	(0.00)
$H^8 : \beta_3 = (a, 0, 0, b, 0, 0, *)$	8	30.77	(0.00)
$H^9 : \beta_3 = (a, 0, 0, 0, b, 0, *)$	8	40.23	(0.00)
$H^{10} : \beta_3 = (a, 0, 0, 0, 0, b, *)$	8	41.35	(0.00)
$H^{11} : \beta_3 = (1, 0, 0, -2, 1, 0, *)$	9	10.22	(0.33)

Figures in parentheses are significance levels. Asterisk * is used to denote that there are no restrictions on the constant term.

sis. The results of these tests are presented in Table 4, and each likelihood ratio (LR) statistic follows asymptotically the χ^2 (df) distribution. The stationarity of the spreads between R_1 and R_2, and R_1 and R_3 cannot be rejected, and also stationarity of the spread between R_1 and R_6 is rather close to being accepted, while the stationarity of other spreads is clearly rejected at the 5% level.

Based on the results of the stationarity tests for the spreads it seems natural to test whether the first three spreads span the co-integration space. This can be accomplished by estimating the co-integrating vectors under the subsequent restrictions and forming the corresponding LR test statistic. This hypothesis is clearly rejected with the value of the LR statistic being 39.16 while the 5% critical value is 16.92. In light of the stationarity tests of the spreads it is conceivable that the rejection is due to forcing the spread between R_1 and R_6 to lie in the co-integration space. Therefore we shall moderate this hypothesis somewhat and proceed by testing more general hypotheses concerning the presence of the longer-term rates in the third co-integrating vector. Throughout it will be assumed that the first two spreads lie in the co-integration space.

As can be seen from Table 4, hypotheses that only two or three of R_6, R_9 and R_{12} enter the third vector are strongly rejected. It seems that at least one of the shorter-term yields has to enter the vector too, and next we run the same tests with the exception that R_1 is also allowed to enter the third vector. The hypothesis that R_1, R_6, R_9 and R_{12} enter the third vector (H^5) cannot be rejected since the value of the test statistic 7.70 is well below the 5% critical value of 12.59. The third vector can further be simplified by excluding R_{12}; for H^6: $\beta_3 = (a, 0, 0, b, c, 0, *)$ the value of LR tests statistic becomes 9.34 with 5% critical value of 14.07. The difference between the two χ^2 statistics, 1.64, is also insignificant at any reasonable level, so that leaving R_{12} out of the co-integrating space can be considered a harmless simplification. From Table 3 it is clear that any further simplifications are out of the question.

The scaled estimate of the third co-integrating vector in the restricted model is (1.000, 0.000, 0.000, –1.891, 0.885, 0.000, 0.116), which suggests testing the hypothesis $\beta_3 = (1, 0, 0, -2, 1, 0, *)$. If this hypothesis is accepted, the third co-integration relation can be considered as the difference between R_6 and the arithmetic average of R_1 and R_9 (plus the constant equilibrium value). Testing this additional hypothesis yields the value of the LR statistic of 10.22, so that the hypothesis cannot be rejected (the critical value from χ^2 (9) distribution at the 5% level is 16.92). The constant terms also deviate significantly from zero; the corresponding LR statistic under the null of zero equilibrium values obtains value 28.12 and the critical value from χ^2 (12) at the 5% level is 21.03.

Estimates of the co-integration and adjustment spaces are given in Table 5. As the alpha matrix indicates, the two yield spreads enter the equations of the interest rates with negative signs as expected, i.e. if e.g. the spread between R_1 and R_2 is greater than the threshold value 0.009, then this implies negative future changes in the shorter-term yields. The situation is similar with the spread between R_1 and R_3, but the threshold value is 0.057. It is the third co-integrating vector that poses more serious problems in terms of interpretation. Basically it says that if R_6 is greater than the average of R_1 and R_9 (plus the threshold value 0.019), then the third co-integration relation obtains a positive value. The rightmost column of α-matrix, on the other hand, tells that this relation enters the equations for the three shortest-term yields with a positive and the equations for the three longest-term yields with a negative coefficient. These two facts allow us infer that the third co-integrating vector deals with the curvature of the yield curve in the following way: when $R_6 - 0.019 > (R_1 + R_9)/2$ (the yield curve is »concave»), then R_1, R_2 and R_3 tend to rise, while R_6, R_9 and R_{12} tend to lower, thus restoring the equilibrium value of 0.019. When $R_6 - 0.019 < (R_1 + R_9)/2$ (the yield curve is »convex»), the implied changes are the opposite. What the third vector does, is that it tends to keep the yield curve linear.

Table 5. The restricted co-integration space.

β-matrix

Variable	β_1	β_2	β_3
R_1	1.000	1.000	–0.500
R_2	–1.000	0.000	0.000
R_3	0.000	–1.000	0.000
R_6	0.000	0.000	1.000
R_9	0.000	0.000	–0.500
R_{12}	0.000	0.000	0.000
Constant	0.009	0.057	–0.019

α-matrix

Equation	α_1	α_2	α_3
ΔR_1	–1.089	–0.315	0.351
ΔR_2	0.053	–0.811	0.229
ΔR_3	–0.610	–0.040	0.420
ΔR_6	–0.824	–0.775	–1.288
ΔR_9	–0.616	–0.383	–0.692
ΔR_{12}	–0.610	–0.035	–0.251

3.3 *Co-integration Analysis of Partial Systems*[8]

Because the expectations hypothesis that we have as a benchmark should hold for any set of yields, it seems natural to examine the term structure also in subsets consisting of 2, 3, 4 and 5 interest rates. On the basis of the analysis of the previous section it is conceivable that the shorter-term yields fulfill the requirements of the expectations hypothesis while the movements of at least the 9 and 12 month yields cannot be explained by it. Partial analysis may give convincing evidence on this observation. Furthermore, analyzing partial systems is useful because it helps in assessing the reliability of the tests. In the previous section we could not reject the restrictions implied by the expectations hypothesis *assuming* that there are five co-integrating vectors among the six interest rates. To what extent our finding »too few» co-integrating vectors is caused by low power of the tests, can be evaluated through partial analysis.

Under expectations hypothesis, there should always be p–1 co-integrating vectors among p interest rates; i.e. one co-integrating vector between two rates, two among three rates etc. Furthermore, the co-integration space should be spanned by the spreads. Following Shea (1992), test statistic for this zero-sum restriction was in each case computed assuming that the co-integration rank is p–1, regardless the results of the trace and maximal eigenvalue tests. Several partial systems consisting of 2, 3, 4 or 5 interest rates and each including R_1 were considered.[9] In all cases lag length of 2 is sufficient as far as autocorrelation and normality are concerned; ARCH effects may, however, be a problem in some systems, especially for the longer-term rates. The expectations hypothesis cannot be rejected in any system consisting of only two rates. Of the three-variable systems only those that include neither R_9 nor R_{12} have co-integration rank of two. Of the four-variable systems only the one consisting of R_1, R_2, R_3 and R_6 has three co-integrating

[8] *The term 'partial system' is here used somewhat loosely. We do not refer to a system that is partially modelled in that only some of the variables appear as endogenous (see Johansen 1992). Instead models including various subsets of the interest rates are considered in isolation, i.e. in each case the rest of the rates are excluded from the model altogether.*

[9] *For brevity the detailed results are not given here, but they are available from the author.*

vectors, and none of the five-variable systems has a co-integration rank of four. The system excluding only R_{12} has three co-integrating vectors and the system excluding only R_9 is very close to having a co-integration rank of three; the rest of the five-variable systems have a co-integration rank of at most two. The zero-sum restriction cannot be rejected at the 5% level in all except two systems; of the subsets having p–1 co-integrating vectors it is, however, rejected for systems consisting of R_1, R_3 and R_6, and R_1, R_2, R_3 and R_6. The conclusion from this partial analysis is thus that the expectations hypothesis holds for at least R_1, R_2 and R_3, and is close to being accepted for the subset including R_6 in addition. Moreover, it is the relation between R_3 and R_6 that does not seem to be in accordance with expectations hypothesis, since the zero-sum restriction is always rejected in subset containing both R_3 and R_6. In general, the twelve month yield does not seem to be co-integrating because the co-integration rank always remains the same once R_{12} is added to the system. R_9, on the other hand, may be co-integrating because augmenting the system with it seems to increase the co-integration rank in systems consisting of R_1, R_3 and R_9, and R_1, R_2, R_3 and R_9. The results thus confirm the conclusions obtained in the previous section.

3.4 Recursive Analysis

So far we have implicitly assumed that the parameters of the estimated models are constant in time. A standard method for evaluating the constancy of the parameters is to consider the long-run parameters as given and test for the constancy of the short-run and adjustment parameters in an error-correction model. Hansen and Johansen (1992) have, however, suggested recursive estimation as a method for examining the constancy of all the parameters simultaneously or of the long-run parameters while keeping the short-run dynamics fixed. Recursive analysis is especially useful for detecting non-constancy, when there is no prior knowledge of structural breaks or time dependencies in the parameters. The analysis typically proceeds in three steps: First, the co-integration rank tests can be done recursively. Second, based on the chosen rank we can test whether the parameters of the co-integration space have been constant through time, or whether the same restrictions have been fulfilled at each point in time. Finally, it may sometimes even be interesting to see how the values of certain single parameters have developed through time. Hansen and Johansen also point out that the slope of the recursive trace tests computed for different hypothetical values of the co-integration rank, $\hat{r}$, can be used as an auxiliary tool in determining the true rank r in the whole sample. This is based on the result that the trace test as a function of time is upward sloping for $\hat{r} < r$ while the test statistics are approximately constant for $\hat{r} \geq r$.

For recursive analysis Hansen and Johansen introduce two slightly different approaches, one based on the »Z-representation» and the other on the »R-representation» of the model. The idea in the Z-representation is to re-estimate all parameters at each point in time, while in the R-representation the short-run parameters are first estimated from the whole sample, and then the long-run parameters are re-estimated recursively given the fixed short-run dynamics. The difference between these two approaches can best be illuminated by the recursive trace test in both cases. In the Z-representation we answer the question of which co-integration rank would have been chosen, had we only had observations from 1 to t, where $t = T_0, ..., T$, whereas with the R-representation the relevant question is the constancy of the co-integration rank given the full sample estimates of the short-run dynamics.

We start the recursive analysis by examining the graphs of recursive trace statistics for the complete system in Figure 1. The statistics have been computed from both the Z- and R-representation, and they have been scaled by the critical values at the 5% level, which means that values greater than unity imply rejection of the null hypothesis. The critical values have again been corrected for sample size using the response surface estimates in Cheung and Lai (1993a). This implies that the critical values are not constant in time but they get smaller the longer the sampling period is, tending to the asymptotic values of Osterwald-Lenum (1992). Thus the effect of this correc-

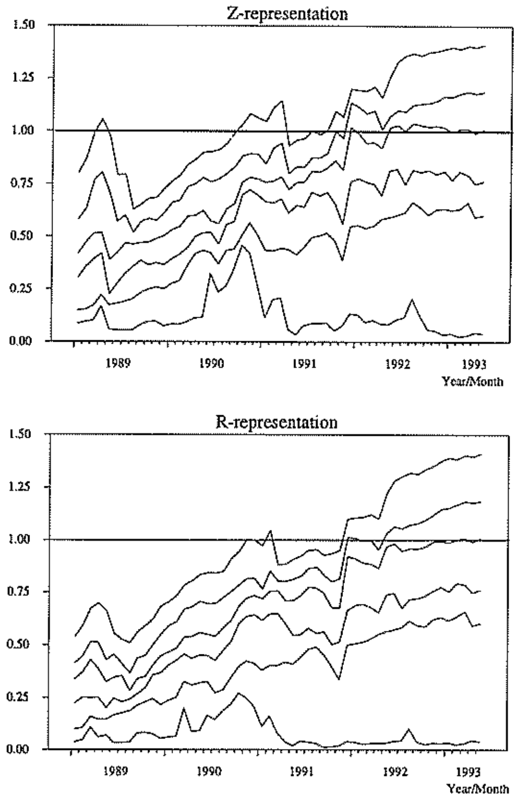

Figure 1. Recursive trace tests for the complete system.

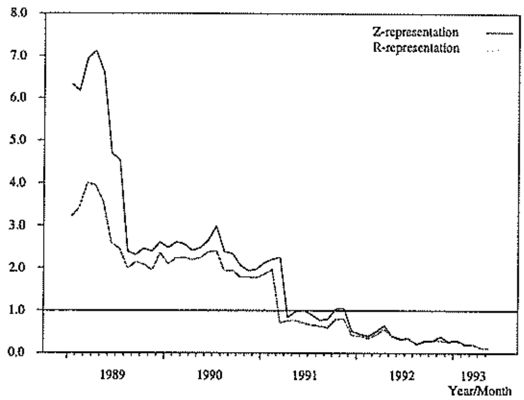

Figure 2. LR test for the constancy of the co-integration space.

tion is at its greatest in the first few observations, when the sample is still very small. With very few observations the estimates are not likely to be very reliable, so that we have computed all recursive statistics in this section from the beginning of 1989, thus leaving the first two years out of the graphs. The uppermost graph in each figure is the scaled recursive trace statistic for the hypothesis $r = 0$, for the one below that the null hypothesis is $r \leq 1$ etc. Figure 1 shows that the co-integration rank has not been stable over the sample period. The differences between the Z- and R-representations seem to be almost negligible, although the effect of short-run dynamics is less pronounced in the R-representation, making the graphs somewhat smoother. There is a large jump in the values of all test statistics in November 1991, which probably has to do with the contemporary devaluation of the Markka. This suggests a large structural break, since in December 1991 the co-integration rank jumped from one to two according to both graphs. The general conclusion is that the co-integration rank has not been stable. In addition, we get some confirmation for the hypothesis that there indeed are three co-integrating vectors because toward the end of the sample period the two topmost graphs seem to be upward sloping, while the rest tend to be constant[10].

It is also interesting to examine the stability of the co-integration space. This can be accomplished by testing if the co-integration space is spanned by some known vectors at each point in time. Since the estimate from the entire sample has the smallest sample variation, it is natural to take that as the known vectors. Assuming that there are three co-integrating vectors, the likelihood ratio test statistics are computed for that kind of a constancy hypothesis, and the values of the statistics divided by the critical value at the 5% level are plotted for both the Z- and R-representation in Figure 2. The differences between the two graphs are minor, and both indicate that the parameters have not been constant over time. The hypothesis of the co-integration space being spanned by the beta vectors in Table 3 is

[10] *The fact that the critical values here are not constant in time increases the slopes of the graphs. The differences between these and those computed with constant critical values are not remarkable, though.*

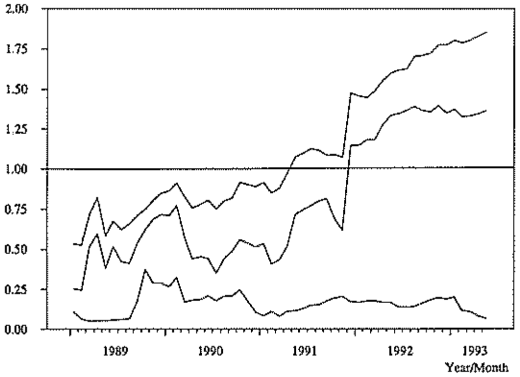

Figure 3. The scaled recursive trace statistics for the system consisting of R_1, R_2 and R_3.

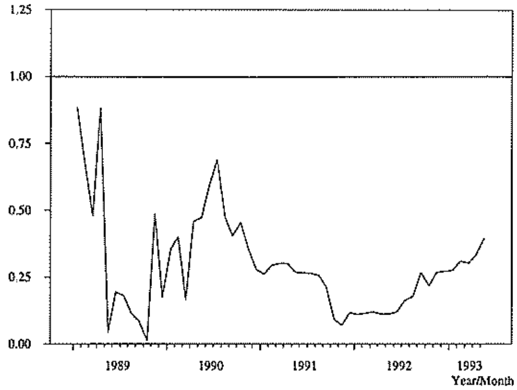

Figure 4. LR test statistic for the zero-sum restriction for the system consisting of R_1, R_2 and R_3.

accepted only after February or March 1991, with the exception of autumn 1991 for the Z-representation. As far as changes in institutional conditions are concerned, the concurrent abolition of nearly all remaining foreign exchange regulations seems a potential explanation for the remarkable drop in the value of LR statistic in the beginning of 1991. It must be borne in mind, however, that the test is conditional on there being three co-integrating vectors in the entire sample. We also tested whether the co-integration space has been spanned by the restricted vectors in Table 5 at each point in time. Naturally, the result is virtually the same as in Figure 2, and so the figure is not presented.

It is also illuminating to study the behaviour over time of the test statistics for the partial systems. Of special interest is the system consisting of the three shortest-term yields for which the expectations hypothesis cannot be rejected. Figure 3 depicts the recursive trace statistics, and Figure 4 shows the time path of the LR test statistic for the null hypothesis that the spreads span the co-integration space (scaled by their critical values at the 5% level) for the system including R_1, R_2 and R_3. All statistics have been computed from the R-representation. Since December 1991 there seems to have been two co-integrating vectors and the zero-sum restriction implied by the expectations hypothesis cannot be rejected in the whole sample period. Also the topmost graph in the figure of the trace statistics is strongly positively sloped while the other two graphs are almost constant, thus confirming our finding a co-integration rank of two.

All in all, the conclusion that can be drawn from the above recursive analysis, is quite clear. The co-integration rank has not been stable over the sample period. According to the results concerning the complete system and partial models, there is a break in December 1991, and since then there are at least two co-integrating vectors among the six interest rates. The zero-sum restriction implied by the expectations hypothesis cannot be rejected at any point for the three-variable system, and the restricted co-integration space for the entire system is accepted at all points since December 1991. Thus it seems that the expectations hypothesis holds at the short end of the maturity spectrum and the vectors of Table 4 span the co-integration space for the complete system since December 1991, when, after the devaluation, there appears to be a structural break in the process generating the term structure.

4. *Conclusion*

The purpose of this study has been to empirically examine the term structure of Finnish HELIBOR interest rates with monthly data from the period 1987:1–1993:5. As a benchmark hypothesis we had the classical expectations hypothesis. The methodology was based on studying the time series properties of the interest rates using methods developed in the co-integration literature.

The expectations hypothesis implies that if interest rates are $I(1)$ processes, then each pair of them should be co-integrated with a stationary spread. Pairwise modelling is, however, somewhat restrictive, and recent developments in the theory of co-integrated time series enable us to model the entire yield curve simultaneously. For any p interest rates the implication of the expectations hypothesis is then that there should be $p-1$ co-integrating vectors, and the spreads should span the co-integration space.

With the multivariate approach the results are not very encouraging for the expectations hypothesis. Among the six interest rates there seemed to be only three co-integrating vectors. In subsequent tests these could be identified as the spreads between the two and one month and three and one month yields, and a third vector that tends to keep the yield curve linear. The twelve month rate was not co-integrating and the nine month rate seemed to be the »driving trend» of the system. Several sub models consisting of only a part of the yield curve were also considered, and the conclusion was that the expectations hypothesis can be accepted for the three shortest-term yields but not for larger subsets. The rather exceptional behaviour of the longest-term rate could probably to some extent be explained by clearly smaller trading volumes, especially at the beginning of the sample period, but this issue lies beyond the scope of this paper. It has, however, turned out to be difficult to explain the developments of long-term yields in other countries as well (see e.g. Campbell and Shiller 1991).

In order to examine the stability of the model over time, recursive analysis was conducted. It revealed that the number of co-integrating vectors has not been constant. The parameters of the co-integration space have not been stable either, but it seems that as long as the co-integration rank has been three, the aforementioned three vectors have spanned the co-integration space. For the three shortest-term rates there was a single structural break in December 1991, and since then the restrictions implied by the expectations hypothesis cannot be rejected for the short end of the yield curve.

It must be borne in mind when interpreting the results that the sample period has been very exceptional in the financial markets. Not only have there been several fundamental changes in the institutional circumstances, but also heavy speculation and several realignments in the foreign exchange markets presumably affected the determination of interest rates. Although we attempted to take account of these factors by using dummy variables, their presence was bound to disturb the econometric analysis. Another problem concerns the limited information set that was used. With explicit knowledge of the institutional arrangements in the market, it would probably have been possible to find an intuitive economic rationale for the econometric results whose interpretation now remains rather technical in nature. Considering the issue of market microstructure by studying the actual transactions data might result in a more satisfactory description of the working of the market.

The results obtained with US data very much resemble ours. Hall et al. (1992), Shea (1992) and Zhang (1993) all discovered that the expectations hypothesis holds for the shortest-term yields, whereas it could be rejected for the entire yield curve. Shifts in monetary policy also seemed to have an effect on the determination of US interest rates, and that showed up as changes in the co-integration space; Hall et al. concluded that for the periods when the Federal Reserve had a clear interest rate target, the expectations hypothesis could not be rejected. Later, however, Engsted and Tanggaard (1994) found out that the shift in monetary policy only had effected the short end of the yield curve, but at the longer end the co-integrating relations had been robust to policy changes. Such shifts in monetary policy might also here be an explanation for the instablity results obtained in the recursive analysis.

The result that interest rates are $I(1)$ processes can be considered a good approximation and modelling device, but as was already pointed out in the Inroduction, this notion cannot easily be given a meaningful economic interpretation, because accepting it would lead us to such assumptions concerning other economic variables that do not coincide with observed reality. Fortunately the framework of integrated time series can be generalized. Already Shea (1991, 1992) brought up the issue that interest rates indeed may be so called fractionally integrated processes. Recently Cheung and Lai (1993b) have introduced the generalized concept of fractional co-integration which might prove useful also in modelling the term structure.

The present study can be extended in several ways. First, the model could be augmented with variables such as inflation, exchange rate and foreign interest rates, that are likely to affect the term structure of Finnish interest rates. This would not only give a more general framework for analysis but also facilitate the simultaneous test of e.g. the expectations hypothesis and uncovered interest rate parity. Second, throughout it has been assumed that the term premia are constant. Their potential non-constancy is one possible explanation for the rejection of the expectations hypothesis, and this aspect also deserves further research.

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