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SOME DISTRIBUTIONAL PROPERTIES OF MONTHLY STOCK RETURNS IN SWEDEN 1919–1990*

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This paper examines the distributional properties of a newly constructed dataset of monthly returns on the Swedish stock market. The standard assumptions that stock returns are log-normally distributed, serially independent, non-seasonal and homoscedastic are all rejected by data. Swedish stock returns are more likely to belong to a peaked and fat-tailed distribution, with positive first-order autocorrelation, strong seasonality and changing volatility over time. These results are well in line with what has been reported from other national stock markets. Our major conclusion is that, given the failure of data to meet the usual distributional assumptions in finance, it may be worthwhile to pay more attention to modeling both the return generating process and the volatility generating process for the market index, instead of simply assuming a strict random walk model. (JEL G12, G14)

1. Introduction

The distribution of stock returns is an issue of fundamental importance in finance. Asset returns in finance are usually modeled as generated by a stochastic process with certain characteristics. Concepts such as return and risk in the mean-variance criterion and the efficient market hypothesis either depend on, or have strong implications for the assumptions of the underlying distribution of asset returns. Black and Scholes stock option pricing model assumes that logarithmic stock returns are normally distributed, and in practical application of the formula it is usually assumed they are serially independent. Furthermore stock return are often assumed to be approximately homoscedastic, i.e. the variance of the returns is constant over time.

The purpose of this paper is to examine the distributional properties of a newly constructed time-series for monthly returns on the Swedish stock market index. The study is organized in the following way. In the next section we review, very briefly, some of the theories where explicit or implicit assumption of stock return distributions are made. In Section 3 we give some stylized facts about our stock return data, and compare the empirical distribution with the normal distribution. In Section 4 we test the assumption of serial independence and non-seasonality. In Section 5 the stability of stock return volatility is examined and in Section 6 we summarize our results.

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2. *Common distributional assumptions in finance*

2.1 Stable normal distribution

There are several reasons for using the assumption of normality in finance. The most important one is that the normal distribution is fully described by only two parameters, the mean and the variance. This property is used as one of two possible arguments to motivate the mean-variance analysis [Markowitz (1959)], where an asset is fully described by its expected rate of return (mean) and its expected risk (variance).[1] In such a world investors either minimize expected variance for a given level of expected return or maximize expected return for a given level of expected variance. An important extension of the mean-variance analysis, the two fund separation theorem, is based on the same assumption.[2] The two fund separation theorem says that all efficient portfolios can be constructed as linear combinations of the market portfolio and a riskless asset. Thus, the only role for the individual investor is to choose the appropriate combination of the market portfolio and the risk-free asset, which matches his degree of risk aversion. Furthermore, in the practical implementation of the mean-variance analysis, the two-fund separation theorem and the CAPM, it is often assumed that asset return distributions as well as the covariance structure among individual assets are stable over time, with the implication that unconditional historical estimates of the distribution parameters (i.e. mean and variance) as well as of the covariance structure among individual securities, can straightforwardly be used in portfolio optimization.[3]

Another reason for using the assumption of normality is its computational convenience. The by far most well-known and practically implemented theorem in finance is Black and Scholes (1973) option pricing formula. This formula gives the price of an American call option (in case of no dividends) as a function of the time to expiration, the risk free rate of interest, the current stock price, the exercise price and the volatility of the logarithmic stock price. The last-mentioned is the only unknown parameter in the formula. The only assumption (apart from no transaction costs and no arbitrage possibilities) is that the price of the underlying stock follows a continuous log-normal distribution. The assumption of normality is by no means crucial in option pricing theory since the hard core of the theory is arbitrage relations, but it is mathematically convenient.[4]

Why would we expect asset returns in general, and monthly stock index returns in specific, to be normally distributed? The return on a stock index is a weighted sum of returns on individual stocks. Since the sum of normal variables is normally distributed, stock index returns would be normally distributed if returns on the individual stocks were normal. The argument for individual stock returns to be normally distributed comes from the assumption that the return over a specific time interval (for example one month) can be seen as the sum of independent and identically distributed returns over small trading intervals (say 15 minutes or so). Normality can be proven by applying the central limit theorem, which says that if (and only if) the variance of the random variables is finite then, in the limit, the sums of identically distributed random variables approach a normal distribution. However, if short interval returns do not have a finite variance and/or do not have stable distributions over time, then the central limit argument can not be applied. Thus, the assumption of normality is not only very often used, it is also based on a very sensitive theoretical reasoning. We will examine its empirical support in section 3.

2.2 Serial independence and non-seasonality

One of the cornerstones in modern *finance* is the efficient market hypothesis (EMH) which states that prices in the capital markets

[1] *The other argument to motivate the mean-variance analysis is to assume that inventors have quadratic von Neumann-Morgenstern utility functions.*

[2] *Ross (1978) shows that two fund separation is also possible for a broader class of asset distributions than the normal.*

[3] *Today, however, most research is directed towards estimating conditional distribution parameters, following a lot of empirical evidence of non-stable asset return distributions. The evidence presented in this study supports this new direction of research.*

[4] *Merton (1973) and Cox and Ross (1976) have derived option pricing formulas for alternative assumptions of the stochastic process.*

fully reflect all available information [see e.g. Fama (1991) or Malkiel (1987)].[5] More formally the efficient market hypothesis can be stated [e.g. LeRoy (1976)]:

(1) $E(R_{it} \mid I_t) = \mu_{it}$,

where $E(R_{it} \mid I_t)$ is the conditional expected rate of return on asset i in period t, given the information available at time t, and μ_{it} is the required equilibrium rate of return on an asset of i:s riskiness. It is difficult to determine reasonable boundaries for the required rate of return but it is usually assumed to be at least non-negative.[6] If the flow of relevant information is uncorrelated over time and if changes in the required rate of return is also uncorrelated over time then (1) implies that asset returns should be serially independent. Formally it can be expressed:

(2) $E[(R_{i,t}-\mu_{i,t})(R_{i,t-k}-\mu_{i,t-k})] = 0$ for all $k > 0$

This means that the return in period t should be independent of the return in any previous period $t-k$, or, in other words, that the return in the current period t has no predictive power for the return in any future period $t + k$. This can be tested by estimating the equation:

(3) $$R_t = b_o + \sum_{j=1}^{n} b_j R_{t-j} + e_t$$

If (2) holds then all b_j should be equal to zero, and b_o equal to μ, the mean rate of return. The assumption of serial independence is used in all sorts of financial modeling both theoretical and empirical. For example, the intensive debate of whether or not speculative bubbles exists in the capital markets is closely related to the autocorrelation structure of asset returns.[7]

Another implication of (1) is that asset returns should not contain any seasonal patterns since the calendar is certainly a subset of the information set, I_t, at any time t. This implies that if we estimate the equation:

(4) $$r_t = \sum_{j=1}^{12} b_j D_j + e_t$$

where D_j are monthly dummies, then the estimated coefficients b_j should all be approximately equal. We will examine this issue further in Section 4.

3. *Examining the assumption of normality*

In this section we examine the validity of the normality assumption of monthly stock returns on the Swedish stock market.

3.1 Data

Our data is the monthly nominal return, including dividends, on the Swedish market portfolio 1919: 1–1990: 12. The return on the market portfolio has been calculated using the *Affärsvärldens generalindex*, adjusted with respect to dividens, and applying the definitions of Ibbotson and Sinquefield (1989).[8] This study focuses on nominal returns rather than on real (nominal returns minus inflation/deflation) or excess returns (nominal returns minus the return on short-term money market instruments). The reasons for this choice is simple. The relevance of all statistical analysis depends strongly on the accuracy of data, i.e. to what extent the data describes what it is meant to describe. We believe that our stock return data have a fairly high accuracy while inflation data, no matter from which country it comes, are only weak proxies of the true process they are meant to describe, in particular when it comes to monthly data. Unfortunately, reliable money market rates are not available for Sweden before the 1980s.

5 The capital market efficiency concept is sometimes divided into informational efficiency and market efficiency. Informational efficiency means that there is no information that can be used to predict deviations from expected returns, i.e. abnormal returns. Market efficiency means that there is no information that, net of all collection- and transaction-costs, can be used to earn excess returns. See e.g. Malkamäki (1989).

6 However, this is not necessary for all assets. Assets that are negatively correlated with the market portfolio, like put options, have typically a negative expected return due to their insurance characteristics since investors are risk averse.

7 See LeRoy (1989).

8 See Frennberg & Hansson (1992a) for further details of the dataset.

Table 1. Sample mean, Standard Deviation, Skewness, Excess Kurtosis, Studentized Range and X^2-test statistic for normal distribution, of monthly market portfolio returns 1919–1990.

Period	Number of Observations	Mean	Standard Deviation	Skewness	Excess Kurtosis	Studentized Range	X^2
1919–1990	864	0.75	4.46	<u>–0.79</u>	<u>5.63</u>	<u>11.06</u>	<u>45.68</u>
Excl. outliers*	853	0.85	3.92	–0.05	<u>0.45</u>	6.57	<u>51.31</u>
1919–1939	252	0.05	4.85	<u>–0.91</u>	<u>7.66</u>	<u>10.19</u>	26.53
Excl. outliers*	247	0.16	4.01	0.03	0.38	6.10	<u>31.94</u>
1940–1959	240	1.05	3.04	<u>–0.61</u>	<u>2.32</u>	<u>7.44</u>	21.00
Excl. outliers*	239	1.11	2.89	–0.20	0.52	6.12	21.15
1960–1979	240	0.60	3.88	–0.13	0.02	5.14	18.51
Excl. outliers*	240	0.60	3.88	–0.13	0.02	5.14	18.51
1980–1990	132	1.85	6.28	<u>–1.05</u>	<u>3.31</u>	<u>6.59</u>	16.12
Excl. outliers*	127	2.17	5.02	–0.24	–0.00	5.01	20.75

Underlined values indicate a rejection of H_0, Normal distribution, at the 95 percent level.
*) Outliers are somewhat arbitrarily defined as observations more than three standard deviations away from the mean (calculated over the total sample). The observations are (in chronological order): 1921: 7 (+17.7), 1922: 2 (–13.5), 1922: 5 (+15.6), 1931: 9 (–13.5), 1932: 3 (–31.6), 1940: 4 (–13.9), 1983: 1 (+14.2), 1983: 2 (+17.1), 1987: 10 (–23.0), 1987: 11 (–15.3) and 1990: 9 (–24.3).

3.2 Methodology

The standard test statistics for the assumption of normality are the coefficients of skewness and kurtosis, which are the expected value of the third- and the fourth-order moment respectively. The coefficient of skewness is zero for any symmetric distribution as the normal distribution. A negative coefficient of skewness indicates a distribution that is skewed to the left of its mean, and a positive coefficient of skewness indicates a distribution skewed to the right of its mean. The coefficient of kurtosis measures the degree of peakedness of the distribution. For a normal distribution the expected value of the coefficient of kurtosis is three. The excess kurtosis measures the departure from normal distribution kurtosis. A value below zero indicates a more peaked distribution than the normal distribution, and a value above zero indicates a more flat distribution than the normal distribution. It should be noted that both the coefficient of skewness and the coefficient of kurtosis are very sensitive to outliers, i.e. they are more sensitive to asymmetry in the tails than to asymmetry close to the mean.[9] To deal with this problem we will first calculate the coefficients of skewness and kurtosis including all observations; we then exclude all observations more than three standard deviations away from the mean, and recalculate the third and fourth moments. Taken together the skewness and the kurtosis coefficients are fairly sensitive to detect any departure from normality.

The X^2-test is another method of testing the departure of an empirical distribution from a theoretical one. For this purpose the sample and subsamples have been divided into 10 as well as 20 equiprobable classes. In the case of 20 classes, each class represents five percent of the probability mass of the standardized normal distribution, and the first and the twentieth class represent all observations more than 1.65 standard deviations away from the mean. This test is therefore not sensitive to extreme outliers, it is instead more sensitive to observations around the population mean. The X^2-test is less sensitive in small samples but in large samples it performs well, especially when it comes to detect departures in the center of the population.

As a last method for testing the normality assumption we use the Studentized range statistics which has particularly good properties against symmetric short- or long-tailed distributions, but it is completely insensitive to asymmetry (Shapiro *et al.* 1968).

3.3 Empirical results

All our samples have a positive excess kurtosis, most of them significant at the 95 % level, which indicates a flat and fat-tailed distribution (see Table 1). However, to judge from

[9] *See Shapiro, Wilk and Chen (1968) for a comparative study of various tests for normality.*

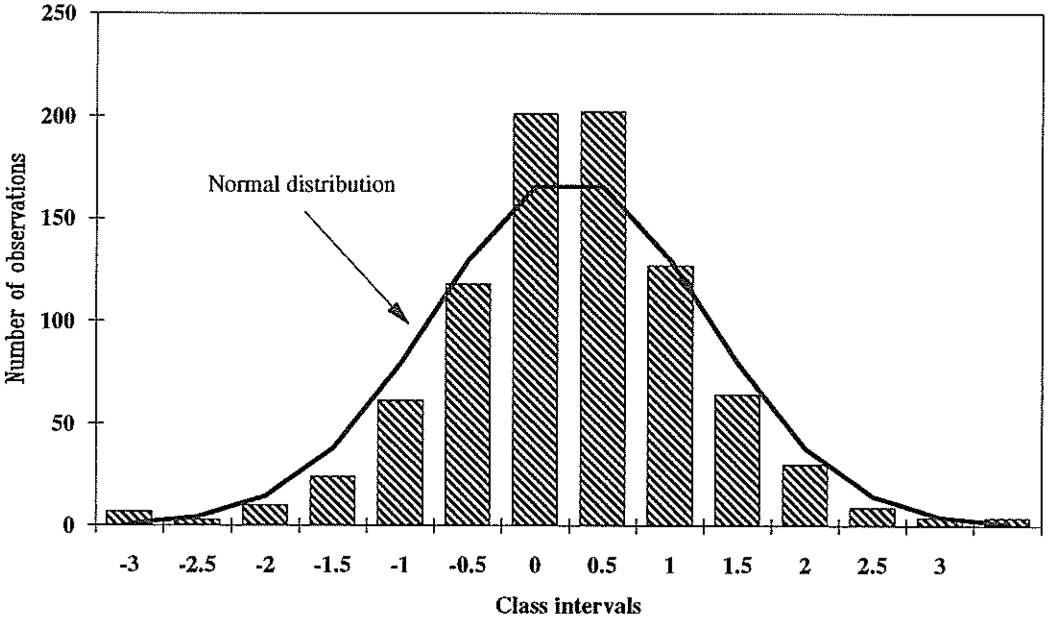

Figure 1. Distribution of standardized monthly stock returns 1919: 1–1990: 12.

the histogram in Figure 1 our empirical distribution is more peaked than flat compared to the normal distribution. The explanation is to be found in the existence of long (rather than fat) tails.

For the whole sample period all test statistics are clear-cut. They all reject the normal distribution. This is both due to too many extreme observations (more than three standard deviations from the mean) and too many observations close to the mean. The extreme observations are well detected by the kurtosis, skewness and Studentized range statistics, while the concentration to the mean is detected by the equiprobable chi-squared test statistic. The latter test is not sensitive to outliers as can be seen in Figure 2.

It has been proposed that the rejection of normal distribution for stock returns could be caused by contaminated distributions, i.e. changing mean and variance over time. This hypothesis is to some extent supported by our subsamples. The X^2-test does not reject the normal distribution for any of our subsamples. However, due to the extreme observations the tests of skewness, kurtosis and of Studentized range reject normality for all subperiods except one. For example in the 1919–39 subsample there is one observation 6.5 standard deviations from the mean: the probability of finding such an observation, in a sample of our size, taken from a normal distribution, is only one in 6.5 millions.

When we exclude all observations more than three standard deviations away from the mean of the total sample, the remaining sample shows no skewness (asymmetry) and only a small positive kurtosis. On a subperiod basis all samples pass the normality tests. Thus, with the important exception of a considerably higher probability density in the »tails», i.e. the probability of observing extreme (in relation to the mean) observations and with the reservation that the expected return and the volatility may change over time, Swedish stock returns, on a monthly basis, seem to have been fairly close to a normal distribution.

Looking at the extreme outliers the distribution is heavily skewed to the left which means that extreme negative returns are more likely than extreme positive returns. The presence of extreme outliers, in particular negative ones, should not be disregarded. They are probably very important for the way investors perceive

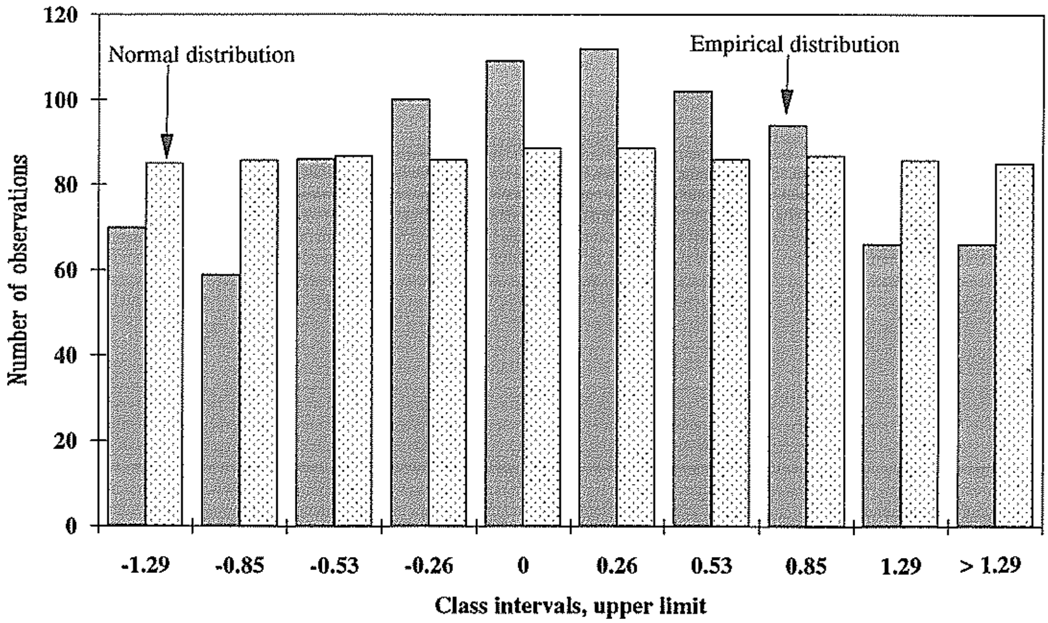

Figure 2. Distribution of standardized monthly stock returns 1919: 1–1990: 12. Each class represents ten per cent of the Normal distribution probability mass.

the riskiness of holding common stocks.[10] The high frequency of outliers may also seriously affect the valuation of options.

To summarize, monthly Swedish stock index returns are not perfectly normally distributed. Compared to the normal distribution they are more likely to be very far from the mean (outliers) as well as very close to the mean: we have a peaked distribution with very long tails. Furthermore, the tails are skewed to the left which means that extreme returns are more likely to be negative than positive.

4. *Testing the assumptions of serial independence and non-seasonality*

In this section we examine the autocorrelation structure and the seasonal properties of monthly Swedish stock index returns 1919–1990 in subsection 4.1 and 4.2.[11] In subsection 4.3 we look at the possibility of predicting stock returns by estimating an AR(1) model with seasonal dummies for the stock returns.

4.1 Serial independence (no autocorrelation)

The usual way to test for serial independence in time series analysis is to examine the autocorrelation structure of the series under investigation. Serial independence implies zero autocorrelation while the opposite is not necessarily true, i.e. zero autocorrelation is a necessary but not a sufficient condition for independence. In Table 2 the twelve first autocorrelation coefficients are given for ordinary calculated returns as well as absolute returns. The latter is of interest if volatility is changing over time and especially if it is positively au-

[10] *It is often argued that various measures of downside risk are more relevant than the usual variance or standard deviation measure. The former measures focus on the deviation from some prespecified target rate of return and are thus more sensitive to the probability of large negative returns than the latter measures (See e.g. Harlow and Rao (1989)).*

[11] *The autocorrelation structure of Swedish stock returns for return intervals longer than one month is examined in Frennberg and Hansson (1992b).*

Table 2. Autocorrelation at lag 1 to 12.

Period	Autocorrelation at Lag												Box-Pierce
	1	2	3	4	5	6	7	8	9	10	11	12	Q(12)
1919–1990	.18	–.01	.03	.03	.05	.11	.02	.02	.09	.08	.09	.05	62.9
$\mid R_t \mid$	.22	.14	.23	.17	.12	.19	.12	.11	.19	.14	.11	.11	267.3
1919–1939	.20	–.06	.01	–.04	.01	.13	.03	.10	.20	.12	.10	.02	33.4
$\mid R_t \mid$	.18	.14	.18	.16	.16	.15	.18	.17	.20	.19	.11	.04	88.4
1940–1959	.14	–.03	.04	.04	.10	.14	.02	.02	.01	–.04	.10	.21	26.1
$\mid R_t \mid$	.15	.12	.14	.04	–.00	.15	.00	.12	.07	.11	.04	.12	31.7
1960–1979	.06	–.10	.10	.03	–.10	.08	–.06	–.02	.02	.08	.02	.02	11.5
$\mid R_t \mid$	.16	.10	.22	.13	.05	.25	.04	–.01	.18	.04	–.04	.16	56.6
1980–1990	.26	.06	–.06	.06	.14	.04	–.01	–.14	.02	.03	.11	–.01	15.7
$\mid R_t \mid$	.18	–.01	.14	.06	–.07	.02	–.08	–.10	.07	–.05	.02	–.03	11.9

First row in each time-period refers to ordinary monthly stock returns while the second row of autocorrelation coefficients refers to absolute monthly returns. Underlined coefficients are significantly different from zero at the 95 percent level.

tocorrelated as have been suggested by many authors.[12]

Starting with the ordinary returns we can see that there is a significant tendency towards positive first-order autocorrelation. In fact, using the standard Portmanteau test the hypothesis of zero first-order autocorrelation is rejected for all but one subperiod.[13] The Box-Pierce statistic for higher order autocorrelation rejects serial independence at the 95 percent level for the whole period as well as for two subperiods. The evidence of positive first-order autocorrelation could be an indication of that the Swedish stock market is not fully informational efficient, i.e. that abnormal returns on the market portfolio could be predicted. However, it could also be consistent with the »non-synchronous-trading-hypothesis». This hypothesis says that if a stock index, which is calculated as the weighted sum of a number of individual stock prices, contain stock prices that are not collected at the same time (generally because of low trading volume), then this index would have a positive first-order autocorrelation even if all individual stocks where truly serially independent.[14] However, since the stock index we use, *Affärsvärldens generalindex,* is based on bid-prices and not the latest transaction price, the risk of positive autocorrelation induced by non-synchronous trading is minimal as long as bid-prices are updated to reflect current information. A third possible explanation to the serial dependence in stock returns is that the required rate of return vary over time in a predictable way.

The significant autocorrelation of the absolute returns, i.e. non-linear dependence, (Table 2) confirms the rejection of H_0 that stock returns are not serially independent. It is also an indication of changing variance [see Akgiray (1989 p. 62)]. The issue of changing variance will be examined in section 5.

4.2 Seasonality

The seasonal pattern of stock market returns has been examined in a number of studies. The well-known January effect has now been documented for a large number of stock markets.[15]

[12] *The phenomenon is called conditional heteroscedasticity and was modeled by Engle (1982) (so called ARCH models). However, it was already noted by Mandelbrot (1963) that large changes in stock prices tended to be followed by large changes and small changes by small changes.*

[13] *This test assumes a homoscedastic null distribution which makes r_k distributed N(0,1/N). If the underlying null distribution is heteroscedastic, which is strongly indicated, the test will have a tendency to reject H_0 too often. Diebold (1987) provides a heteroscedasticity robust estimation method of the standard deivation of r_k. However, using Diebolds method does not change our results in any significant way.*

[14] *See Sholes and Williams (1977) or Lo and MacKinlay (1988) for further details.*

[15] *Gultekin and Gultekin (1983) investigated 17 national stock markets for the period 1959–1979 and found the January-effect to be present in most of them. For a review of capital market return regularities see e.g. Dimson (1988)*

Table 3. Mean and standard deviation of month-to-month stock market return, and test of equality of mean returns, 1919–1990.

Period	Jan	Feb	Mar	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec	*F*	*K-W*
1919–1990														
Mean	3.05	0.57	0.56	1.05	1.04	0.71	2.68	–0.25	–1.13	–0.39	–0.24	1.41	5.75	72.05
SD	4.57	4.73	5.33	4.32	4.46	3.04	3.74	3.94	4.52	4.96	4.69	3.00		
1919–1939														
Mean	2.00	–0.79	–1.13	1.18	0.45	0.73	2.12	–0.70	–2.25	–0.90	–1.55	1.49	1.99	23.19
SD	3.77	5.16	8.28	4.54	5.58	3.03	5.06	4.54	4.03	3.62	3.71	3.22		
1940–1959														
Mean	2.84	0.59	0.47	0.56	1.94	0.39	2.95	1.23	0.49	–0.30	0.13	1.27	2.49	29.96
SD	3.22	3.14	3.20	3.83	2.96	2.11	2.18	2.29	2.92	3.38	3.41	2.03		
1960–1979														
Mean	3.96	0.44	1.37	0.92	0.37	0.29	2.04	–1.17	–1.13	–0.59	0.09	0.57	2.89	29.91
SD	4.74	3.74	2.64	4.71	3.30	3.38	3.65	3.84	2.64	4.32	4.08	2.73		
1980–1990														
Mean	3.77	3.36	2.50	1.91	1.74	2.03	4.43	–0.42	–1.92	0.79	0.97	3.07	0.90	9.85
SD	7.27	6.93	4.52	4.44	6.13	3.83	2.91	5.14	8.63	9.42	8.33	4.06		
Bonds														
1919–1990														
Mean	1.01	0.69	0.40	0.11	0.47	0.65	–0.28	0.53	0.38	0.22	0.40	0.46	1.61	15.19
SD	2.27	2.40	1.55	2.16	1.70	1.48	4.11	1.35	1.73	2.05	1.48	1.79		

The *F*-value is the one-way analysis of variance test-statistic for equal means, and *K-W* is the non-parametric Kruskal-Wallis test-statistic for the same hypothesis, *i e* equal means. Underlined values indicates a rejection of H_0 at the 95 percent level.

Judging from Table 3 there is a strong seasonal pattern in Swedish stock returns. The hypothesis of equal means for all months is rejected by the analysis of variance test as well as by the non-parametric Kruskal-Wallis test. Since the conclusion is the same for both the parametric F-test and the non-parametric Kruskal-Wallis test the exclusion of outliers will not change the results in any dramatic way.

Specifically we have two »good» months, January and July, four »bad» months, August, September, October and November, and six »normal» months. The mean returns of the »good» months are significantly higher than the mean returns of the »bad» months, using the F-test with a pooled variance as well as by the non-parametric Kruskal-Wallis test. The pattern is also fairly stable over time. For the whole sample, the mean return in September is particularly low, in fact, even significantly below zero.

For the Swedish market we seem to have not only the internationally well known January-effect, but also a July-effect and a September effect as well. The September effect is an international phenomenon that is rarely mentioned. In Gultekin and Gultekin's study (1983, Table 2) 16 out of 17 stock markets had a negative mean return in September (the exception Austria was close to zero). This effect challenges the efficient market hypothesis since the expected return of a market portfolio should always be positive. It should however be noted that the evidence of *ex post* seasonality does not prove the existence of trading strategies with abnormal returns.

The low mean returns of the period August to November calls for further examination. In Table 4 we have contrasted the August-November return against the pattern for the two remaining four-months periods: December-March and April-July. The results confirm the pattern of the monthly returns. The hypothesis of equal means is once again strongly rejected. The mean return of the »bad» period is significantly below the two »good» periods which holds both for the F-test and the Kruskal-Wallis statistic. The pattern is the same for all subperiods, though not always statistically significant. From a theoretical point of view the most disturbing characteristic of the »bad» period is its negative, or at least close to zero, mean nominal return. How can we explain that a risky asset systematically yields a negative or close to zero return for a third of the year?

Table 4. Mean and standard deviation of stock market returns december-to-march, april-to-juli and august-to-november, and test of equality of mean returns, 1919–1990.

Time Period	December to March	April to July	August to November	*F*-test	Kruskal-Wallis-test
1919–1990 (obs)	71	72	72		
Mean	5.76	5.48	–2.01	<u>13.79</u>	<u>21.28</u>
SD	10.12	8.10	11.66		
1919–1939 (obs)	20	21	21		
Mean	2.28	4.48	–5.41	<u>4.49</u>	<u>7.11</u>
SD	12.63	8.91	11.84		
1940–1959 (obs)	20	20	20		
Mean	4.78	5.84	1.54	2.57	3.26
SD	5.32	6.60	6.71		
1960–1979 (obs)	20	20	20		
Mean	6.51	3.62	–2.79	<u>8.36</u>	<u>12.77</u>
SD	7.48	7.35	7.26		
1980–1990 (obs)	11	11	11		
Mean	12.54	10.11	–0.58	2.26	1.92
SD	13.25	9.42	21.14		

The F-value is the one-way analysis of variance test-statistic for equal means, and *K-W* is the non-parametric Kruskal-Wallis test-statistic for the same hypothesis, *i e* equal means. Underlined values indicates a rejection of H_0 at the 95 percent level.

Implementing the obvious corresponding »excess profit» trading strategy: buying stocks at the end of November and selling them and putting the proceeds into risk-free bills at the end of July would have yielded an annual (geometric) mean return of 13.8 percent which should be compared with only 9.5 percent for the pure buy and hold strategy (transaction costs excluded). The risk, measured in terms of standard deviation, would at the same time have been reduced by over 20 percent.

We do not have access to an international stock return database which include dividends, which is absolutely necessary when examining seasonalities due to the strong seasonality in the dividend payment process in most countries. However, we can use the results from other studies to get an approximate picture of the international evidence for the »autumn-effect». Using Table II in Corhay, Hawawini and Michel (1987), who examined the seasonal pattern of beta-coefficients on four stock markets for the period 1970–1983, we find a slightly modified autumn effect in all four markets. The average return is negative for all countries for the two-month period September-October and negative in all markets, except the NYSE, for the three-month period September-November. Extending to our four-month period, August-November, the average return is still below the risk-free rate of return in all four markets, i.e. the riskpremium is negative. The Gultekin and Gultekin (1983) study covered stock return data from 17 countries for the period 1959: 1–1979: 12, but unfortunately their data did not include dividend payments.[16] With the important reservation of the exclusion of dividends, we find that 10 out 17 countries had negative mean returns for the four-month period August-November. Assuming a 0.5 percent monthly risk-free rate of return, 16 countries had a negative risk premium for the same four-month period (the exception was the U.S.). Thus, we can conclude that with exception of the largest stock market in the world, the NYSE, there seems to be an international case for the »autumn-effect».

Clearly, the market portfolio can not have a negative *ex ante* expected return, in particular for a period of four months. We are however dealing with *ex post* data. The task is to find a reasonable explanation for the poor *ex post* performance of stocks in the autumn. Using the standard dividend discount model:

[16] *The effect of excluding dividends on the Swedish data is that the returns in March, April, May and June are significantly underestimated, since about 80 percent of total dividends on the Swedish stock market are paid in these months.*

$$(5)\quad P_t = E_t\left[\sum_{k=1}^{\infty} DIV_{t+k}(1+r)^{-k}\right]$$

There are basically two possibilities for falling stock prices: either an upward shift in the required rate of return due to a higher risk-free rate of return (i.e. premium for waiting) and/or a higher riskpremium, or a downward shift in the expectations of future dividends. The latter seems implausible since it would imply some kind of seasonality in investors expectations. We will briefly examine the former possibility, seasonality in the required rate of return, below.

If the seasonality in stock returns are due to seasonalities in the risk-free rate of return, then we would expect a similar seasonal pattern in long-term bond returns since changes in the premium for waiting affects stock and bond prices in the same way. This hypothesis gets only little support in our data.[17] The average monthly return on long-term government bonds in the four-month period August-November is 0.38 % which is just marginally below the overall mean return of 0.42 % (see bottom of Table 3). The only seasonality in the bond market seems to be a slightly higher return in January and a slightly lower return in July. Thus, we are left with either a rikspremium seasonality or a »negative news» seasonality to explain the poor performance of stocks in the autumn.

4.3 Predictability of stock returns

The presence of predictable components in monthly stock index returns could either be a sign of informational inefficiency or a sign of predictable variations in the required rate of return. The seasonal pattern, in particular the negative mean returns in some months, points towards the former case. Regardless of which interpretation of the predictable component in stock returns is the right one, we want to investigate the magnitude of the predictability. To do this we have estimated the following seasonal AR(1)-model for stock returns implied by the findings in section 4.1 and 4.2:

$$(6)\quad R_t = b_0 R_{t-1} + \sum_{i=1}^{12} b_i D_i + e_t$$

where D_i are monthly dummy variables. The results are reported in Table 5. The model explains around 9 % of the *in-sample* variance (somewhat lower in the subperiods). However, the interesting question is whether the seasonality and the first-order autocorrelation have any out-of-sample forecasting power. To examine this we have regressed the actual return in month t, $R_{act,t}$, against the one-step ahead forecasts generated by (6) at the end of month t–1, $R_{frc,t}$, in the following way:

$$(7)\quad R_{act,t} = \alpha + \beta R_{frc,t} + u_t$$

If actual returns were truly unpredictable we would expect α to be equal to the mean rate of return, μ, and β to be equal to zero. On the other hand, if returns were predictable and $R_{frc,t}$ were an *unbiased* estimator of $R_{act,t}$ we would expect α to be zero and β to equal to one. The results in Table 5 clearly reject the hypothesis of unpredictable returns since β is always significantly different from zero. The forecast has an explanatory power of 6.3 % for the period 1940–1990. However, the results also reject the hypothesis that $R_{frc,t}$ is an unbiased estimator of $R_{act,t}$, but the rejection is concentrated to the period 1940–1959.

To sum up: Swedish stock returns do not seem to pass the strict requirements for informational efficiency unless the predictable component of *ex post* monthly returns is due to a systematically time-varying risk premium. If this predictability is large enough to also allow for an excess profit trading strategy remains an open question for future research.

5. *Stability of changing volatility*

It is a well established fact that U.S. stock volatility has been changing over time.[18] Modeling stochastic volatility, in particular in

[17] *The calculation of the bond return series is described in detail in Frennberg and Hansson (1992a).*

[18] *Officer (1973) noted that stock volatility was extremely high during the great depression in the 1930s. Jones and Wilson (1989) found two periods of deviating volatility; the 1930s and the 1980s. Schwert (1989) related the changes in stock market volatility to macroeconomic fluctuations, like changes in the volatility of the industrial production, the money-supply and the rate of inflation.*

Table 5. Estimating and predicting monthly stock index returns with first lag and seasonal dummies.

Within-sample	1919–1990	1919–1939	1940–1959	1960–1979	1980–1990
R_{t-1}	$\underline{0.180}$	$\underline{0.207}$	0.031	0.054	$\underline{0.249}$
D1	$\underline{2.900}$	$\underline{1.959}$	$\underline{2.809}$	$\underline{3.919}$	3.040
D2	0.018	–1.207	0.497	0.221	2.419
D3	0.461	–0.965	0.452	$\underline{1.342}$	1.664
D4	0.949	1.416	0.549	0.850	1.293
D5	0.851	0.205	$\underline{1.919}$	0.323	1.267
D6	0.525	0.639	0.325	0.271	1.597
D7	$\underline{2.551}$	1.965	$\underline{2.941}$	$\underline{2.019}$	$\underline{4.204}$
D8	–0.733	–1.142	1.135	–1.275	–1.521
D9	$\underline{-1.083}$	$\underline{-2.110}$	0.448	–1.064	–1.813
D10	–0.187	–0.435	–0.318	–0.526	–1.268
D11	–0.174	–1.362	0.140	0.118	0.771
D12	$\underline{1.456}$	$\underline{1.810}$	$\underline{1.263}$	0.562	$\underline{2.829}$
R^2adj	0.088	0.083	0.061	0.079	0.046
Out-of-sample	1940–1990		1940–1959	1960–1979	1980–1990
α	$\underline{0.567}$		$\underline{0.838}$	0.155	0.818
β	$\underline{0.814}$		$\underline{0.469}$	$\underline{0.783}$	$\underline{1.170}$
X^2 ($\alpha = 0, \beta = 1$)	$\underline{8.464}$		$\underline{16.207}$	1.114	4.221
R^2adj	0.063		0.031	0.065	0.077

Heteroscedastic consistent (White 1980)) OLS-estimates of the following model for stock returns on the Swedish stock market:

$$R_t = a_1 R_{t-1} + b_1 D_1 + b_2 D_2 + \ldots + b_{12} D_{12} + e_t$$

The out-of-sample statistics refers to OLS-estimates of the following model:

$$R_{act,t} = \alpha + \beta R_{frc,t} + u_t$$

where $R_{frc,t}$ is the one-month ahead forecast generated by the estimated return model. The X^2-statistic is the test of the joint hypothesis that $\alpha = 0$ and $\beta = 1$ (unbiased forecast). Underlined coefficients are significantly different from zero at the 95 percent level.

the spirit of Engle (1982) (the ARCH model) and Bollerslev (1986) (the GARCH model), has become an intensive research area in recent years.[19] In this section we examine the hypothesis of constant volatility in the Swedish stock market (Section 5.1). Given the rejection of the stable volatility hypothesis, we estimate a simple model for stock volatility to examine to what extent stock volatility is predictable (Section 5.2).

5.1 Stable or evolutionary volatility?

An intuitively appealing way to get a general picture of the volatility is to plot the time-series of the cumulated squared deviations from the mean, CSD.[20]

$$(8) \quad CSD_t = \sum_{K=t_0}^{t} (R_k - \mu)^2$$

The slope of the CSD line (over any interval) is equal to the estimated variance over that interval. Thus, changes in the slope of the CSD line will be a good indication of shifting volatility. Another descriptive measure of the volatility is the 12-month rolling standard deviation [see e.g. Officer (1973)]. In Figure 3 both measures are plotted. To judge from the

However, the relations were weak. Hsu (1984) argues that stock volatility changes evolutionarily over time, reflecting changes in the economic and political environment.

[19] *Bollerslev, Chou and Kroner (1992) give a comprehensive review of the research area.*

[20] *This is similar but not equal to the CUSUM chart, which usually refers to the cumulated squared recursive residuals from some estimated model. The model in our case would simply be $R_t = \mu + e_t$, but the residuals we use are not recursive.*

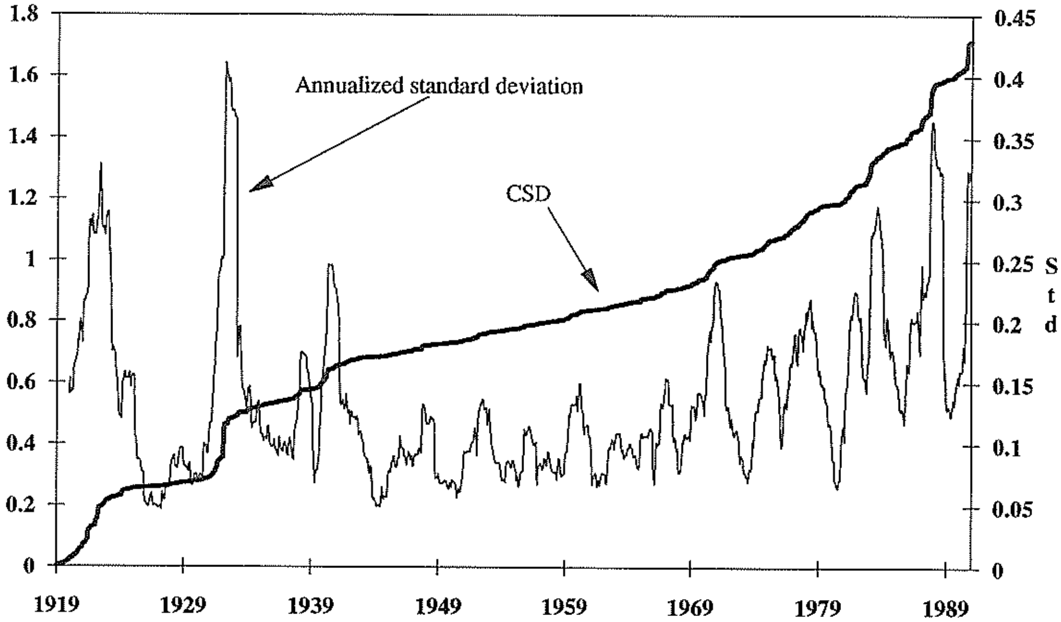

Figure 3. Cumulative squared deviations (CSD) and 12 months rolling annualized standard deviation.

shape of the plot it seems quite obvious that stock volatility has not been constant over time. The CSD chart begins with a steep slope for the period 1919–1923. It is then followed by a long period (1924–1969) with a relatively stable slope, interrupted by two short periods of extreme volatility. From 1970 and onwards the slope is slowly increasing, interrupted by a few periods of very high volatility. For a more formal analysis we need to introduce some statistical methods for detecting shifting variance.

A simple method for testing the hypothesis of constant variance in a time-series without specifying the potential breakpoints has been proposed by Hsu (1979). Hsu's test is based on the assumptions that the sequence of returns can be treated as independent and normally distributed random variables. Unfortunately, none of these conditions seem to be fulfilled by our data. A second method is the modified Levene's method.[21] This method has the advantage of being relatively robust against nonnormality and is more appropriate to our data. A third method, also relatively insensitive to departures from the standard assumptions of normality and serial independence, is to regress the squared deviation from the subsample means on dummies for the subperiods and test whether the dummy-coefficients are equal or not.[22]

In Table 6 we show the results of various tests of the hypothesis of stable volatility against the hypothesis of changing volatility.

Table 6. Test for shifting variance.

Time-period	Hsu's T^* Full samples	Hsu's T^* Outliers excluded	Modified Levene ($\alpha = 0.10$)	Shapiro-test
1919–90	<u>3.091</u>	<u>4.380</u>	<u>18.31</u>	<u>16.99</u>
1919–39	<u>–1.747</u>	–1.284	0.16	
1940–59	<u>–3.297</u>	<u>–1.726</u>	1.19	
1960–79	<u>3.544</u>	<u>3.544</u>	<u>19.98</u>	
1980–90	<u>2.831</u>	0.437	0.19	

Hsu's T* tests the hypothesis of stable variance within the sample. The modified Levene tests the hypothesis of equal variance in the first and second half of every sample. The Shapiro test is for the whole period where the null hypothesis is equal variance in all subperiods. Underlined values indicates a rejection of H_0 at the 95 percent level.

[21] *See Hsu (1979) p. 34–35.*

[22] *See Shapiro (1988) p. 1075.*

Table 7. Estimating and predicting stock index volatility with a seasonal AR(12)-model.

Within-sample	1919–1990	1919–1939	1940–1959	1960–1979	1980–1990
$\Sigma \lvert e_{t-i} \rvert$	0.710	0.681	0.087	0.591	0.304
D_1	1.354	0.113	1.906	2.333	4.668
D_2	1.195	2.101	1.890	0.648	4.801
D_3	0.985	2.849	1.978	0.304	1.760
D_4	0.982	1.711	1.323	2.001	2.469
D_5	1.352	1.587	1.948	0.807	5.452
D_6	0.136	0.598	1.694	0.972	1.406
D_7	0.464	0.998	1.284	0.977	0.652
D_8	0.726	1.421	1.433	1.274	1.297
D_9	0.863	0.818	2.286	0.452	4.604
D_{10}	1.228	0.287	2.152	1.754	5.434
D_{11}	1.289	0.261	1.975	1.704	4.375
D_{12}	0.113	–0.360	1.328	0.562	1.689
R^2adj	0.145	0.091	0.000	0.125	0.105
Out-of-sample	1940–1990		1940–1959	1960–1979	1980–1990
α	0.666		1.020	0.909	1.885
β	0.815		0.524	0.711	0.690
X^2 ($\alpha = 0$, $\beta = 1$)	9.051		7.459	5.210	8.961
R^2adj	0.118		0.068	0.077	0.045

Heteroscedastic consistent (White 1980)) OLS-estimates of the following model (within sample) for volatility on the Swedish stock market:

$$|e_t| = \sum_{i=1}^{12} b_i \, |e_{t-i}| + \sum_{j=1}^{12} c_j D_j$$

The out-of-sample statistics refers to OLS-estimates of the following model:

$$|e_{act,t}| = \alpha + \beta |e_{frc,t}| + u_t$$

where $|e_{frc,t}|$ is the one-month ahead forecast generated by the estimated volatility model. The X^2 statistic is the test of the joint hypothesis that $\alpha = 0$ and $\beta = 1$ (unbiased forecast). Underlined coefficients are significantly different from zero at the 95 percent level.

The hypothesis of stable volatility is rejected for the full sample by all methods. This is not surprising bearing in mind the shape of the CSD chart and the 12-month rolling standard deviation. The omission of outliers does not affect the overall conclusion even though it changes the results for a few subperiods.

5.2 Predictability of stock volatility

The facts that, 1) stock volatility is not constant over time (Table 6), i.e. stock returns are heteroscedastic, and 2) absolute returns are strongly positively autocorrelated (Table 2), indicate that stock volatility can be predicted. In this subsection we examine to what extent stock volatility is predictable following the same methodology as in subsection 4.3.

Following the results in Frennberg and Hansson (1992c), where the ARCH and GARCH models, on a sample of monthly Swedish stock returns, were found to be outperformed in terms of forecasting power by a relatively simple AR model, we have estimated the following AR(12) model with seasonal dummies for stock volatility, based on monthly data:

$$(9) \quad |e_t| = \sum_{i=1}^{12} b_i |e_{t-1}| + \sum_{j=1}^{12} c_j D_j$$

The model explains 14.5 % of the in-sample variance for the period 1919–1990 (see Table 7). Note that for the subperiod 1940–1959, when volatility was extremely low and stable, the in-sample R^2 is 0.0 %. To examine the out-of-sample forecasting power of the model we have again estimated the following regression equation:

$$(10) \quad |e_{act,t}| = \alpha + \beta|e_{frc,t}| + u_t$$

where $|e_{act,t}|$ is the 'actual' residual from the successively estimated stock return model (6) and $|e_{frc,t}|$ is the one-month ahead forecast generated by the successively estimated volatility model (9). The interpretation of the estimated parameters α and β is the same as in section 4.3, but with the important exception that $|e_{act,t}|$ is itself only an estimate of the 'true' volatility in month t. The results in Table 7 show a forecasting power (R^2) of 11.8 % for the period 1940–1990, which is slightly higher than the return model in section 4.3. The hypothesis that $|e_{frc,t}|$ is an unbiased estimator of $|e_{act,t}|$ is rejected in all subperiods.

6. Conclusions

In this paper we have empirically examined some of the standard assumptions of stock returns on 72 years of monthly Swedish stock return data. Unfortunately, most of the assumptions are rejected by data. Stock returns are not log-normally distributed, serially independent, non-seasonal or homoscedastic. They are more likely to belong to a peaked and fat-tailed distribution, with positive first-order autocorrelation, surprisingly strong seasonality and with changing volatility over time. Our results are well in line with what has been reported in studies on other national stock markets. The non-normality and the positive first order autocorrelation are well known and typical for almost any stock market. The same is true for the January seasonality, which was one of the first anomalies reported in the literature. We also found a highly significant positive July seasonality.

More surprising and possibly also more at variance with the Efficient Market Hypothesis is the evidence of a negative autumn seasonality, since it implies that one could raise the expected return by systematically taking less risk in the autumn months. This phenomenon seems to be present in most international stock markets, though it has been especially pronounced in the Swedish stock market. Remarkably enough, we have not found any references in the literature mentioning this negative autumn seasonality.

We have finally found that the hypothesis of constant volatility can be rejected for Swedish stock index returns. Our data is more in favour of the »evolutionary» hypothesis, where stock volatility changes over time reflecting changes in the economic and political environment. Our study indicates that, given the failure of data to meet the usual distributional assumptions in finance, it may be worthwhile to pay more attention to modeling both the return and volatility generating processes for the market index, instead of simply assuming a strict random walk model. We have shown that even very simple models can be used to predict a small, but highly significant, proportion of the variability in both stock returns and stock volatility.

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