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# MITIGATING THE TRADE-OFF BETWEEN EQUALITY AND DYNAMIC EFFICIENCY*

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*In a model where decisions on income distribution and investment are separated between two classes (workers and capitalists), Lancaster (1973) showed that dynamic inefficiency will occur. The reason is that investors do not internalise the external effects of investment. In two kinds of growth models, this paper proposes income distribution rules that reduce or eliminate these problems, by separating the considerations on efficiency and income distribution from each other. (JEL O41, D33)*

## 1. Introduction

The tradeoff between equality and efficiency is »our biggest socioeconomic tradeoff, and it plagues us in dozens of dimensions of social policy», states Okun (1975, p. 2). I believe that many economists would agree on this. Yet, as Blinder (1982) suggests, economists have paid fairly little attention to it, but instead extensively investigated only one of the aspects: efficiency. Meanwhile, the public discussion seems to be engaged mostly in issues on equality. An important explanation for the fact that economists seldom have attempted to study the problem in its entirety is that the tradeoff is hard to make operational. The reason is of course that the problem is simply huge; experts from several of the main branches of economics must be brought into cooperation, in order to make any significant progress.

The purpose of this paper is much more modest than this. I will confine myself to describe a quite special kind of tradeoff that may arise in a model of a growing economy where agents are not homogeneous. For this purpose, I will build on Lancaster's (1973) »differential game of capitalism», in which economic agents are divided into two groups, workers and capitalists. The former are assumed to completely control income distribution, whereas only capitalists have the ability to invest (for which means are to be taken from their own share of national income). The important outcome of this game is that investments are undertaken at a suboptimal level, from the viewpoint of society as a whole. This is due to the fact that capitalists receive only a *fraction* of the income generated by marginal investment, thus valuing capital lower than a benevolent social planner would do.

Since the problem was formulated as a differential game, it was natural to try to improve the outcome by refining the game situation. Hence Pohjola and others have proposed Stackelberg solutions and trigger strategies to achieve this (cf. Kaitala and Pohjola (1990) and the references therein). An underlying assumption seems to have been that it is unrealistic to assume long-time contracts on income distribution.

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In this paper I will, however, abolish the element of the game of the Lancasterian model[1] by relying on an assumption that there actually are implicit long-term agreements on relatively constant shares of income in industrialised economies. To motivate this somewhat ad hoc assumption, we assume that wages are the result from negotiations between one (monopolistic) trade union and one organisation representing the employees. These negotiations result in an agreement implying that all workers are employed and that a certain fraction of the gross national product is distributed to the trade union, which shares it equally among its members.[2] For simplicity we assume that nothing happens to the bargaining power over time, so that the income fractions remain constant.[3] Given these assumptions, it is easy to show that the capital stock is suboptimal in steady state. Inspired by Weitzman (1983) I then go on to suggest more refined remuneration systems that might improve welfare.

The special kind of tradeoff between equality and efficiency that is in the focus of the present paper will be discussed both in the context of a neoclassical and an endogenous growth model. In the former, the welfare improving rule for income distribution will have the property that the shares of total and marginal national products do not necessarily coincide. In particular, capitalists' marginal share must be an increasing function of output. If workers are not grudging towards capitalists, they might thus induce them to accumulate up to the golden rule level. In the endogenous growth model, the optimal growth rate is obtained if capitalists are confronted with the choice of either getting a (possibly constant) share of an optimally growing output, or a smaller share of a smaller (more slowly growing) output. In both cases the point is that there is a wide variety of income distributions that will lead to the optimal steady state capital stock (in the neoclassical case) or the optimal growth rate (in the endogenous case). Thus one can say that the remuneration systems will to some extent *separate the problem of efficiency from the problem of distribution*, hence the title of this paper.[4]

Sections 2 and 3 treat neoclassical and endogenous growth models respectively. In each of them the outcome of the problem of a benevolent social planner is first presented. Then the suboptimality accruing from the Lancasterian heterogeneity of agents is described. Both sections are then ended by suggestions on how these shortcomings could be remedied.

## 2. *The neoclassical case*

Consider an economy with a population of size $N(t)$ that grows at the rate $n$. Assume that a constant fraction $\alpha$ of this population are capitalists, who run firms and invest. The remaining fraction $(1-\alpha)$ do nothing but work and consume their share of the only produced good. The production technology is described by a linearly homogeneous production function $Y=F[K,(1-\alpha)N]$, where $Y$ and $K$ represent output and capital respectively. This expression can be written on per capita form: $Y/N=y=F[K/N,(1-\alpha)]=f(k)$. Denote capitalists' consumption by $Z(Z/N=z)$, workers consumption by $X$ $(x)$ and their sum by $C(c)$. Using a dot to denote time derivative, capital accumulates according to the familiar differential equation $\dot{K}=F[K,(1-\alpha)N]-C$, which, upon division by $N(k=K/N)$, is equivalent to

$$(1)\quad \dot{k}=f(k)-c-nk$$

[1] *By making this simplification it is clear that we eliminate an interesting feature of Lancaster's model, namely the endogeneity of income distribution and its dependence on the institutional framework. Nevertheless I think it is fair to refer to Lancaster, because he highlighted the possibility of dynamic efficiency in a two-class model with intertemporal optimisation.*

[2] *For a suggestion how a certain combination of income shares and employment may be singled out, I refer to McDonald and Solow (1981). In a static context, they propose that such combinations can be uniquelly selected either if the union or the employers' organisation is dominant as to the bargaining power, or if »history has led to the notion that there is a 'fair' division of the net revenue between the workers and the employers» (p. 903).*

[3] *The important thing here is not that the fractions are constant but that they are exogenously given (i.e. results from negotiations). If capitalists' fraction of GNP were exogenously given as a fluctuating time path over the entire time horizon, we could still demonstrate that the capital stock would deviate from its optimal size.*

[4] *Many economists have recently shown interest in various aspects on the relationship between growth and equality. See for example Agell and Lommerud (1991), Alesina and Rodrik (1991) and Persson and Tabellini (1991).*

Suppose that a benevolent social planner has an instantaneous utility function u (c), satisfying the Inada conditions, and a discount factor ρ, then the *Planner's Problem* is to

$$(2) \quad \max_{c} \int_{0}^{\infty} u(c)\, e^{-\rho t} dt$$

subject to (1).

The solution of this problem is well known. There exists a »stable arm» such that the economy converges to a steady state, at which ρ+n=f'(k). When this equality holds, the so called modified »golden rule» is obeyd and the level of capital is k* (see Figure 1). The solution of this program will be referred to as

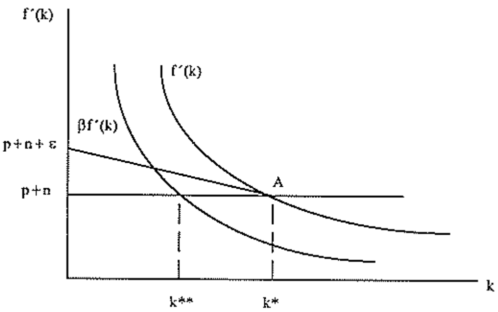


Figure 1.

the welfare maximizing one. Especially, the steady state capital stock and the output that it generates will be compared to other steady state results below.

Next, let us extend this model to a Lancesterian direction and assume decentralised decision making. Now the output is divided between workers and capitalists by a fraction β, which for simplicity is assumed to be constant over time. This parameter is not determined in the model, but is taken as given by negotiations and institutional and historical conditions. Workers will thus get (1–β)Y and are assumed to consume all of it, while capitalists receive βY. Capitalists now let the capital stock evolve according to $\dot{K}=\beta F[K,(1-\alpha)N]+(1-\beta)F[K,(1-\alpha)N]-Z-X=\beta F[K,(1-\alpha)N]-Z$ which on per capita (and per capitalist) form means that:

$$(3) \quad \dot{k} = \beta f(k) - z - nk$$

Sticking to the same utility function and time preference as the social planner, the *Capitalists' Problem* is to

$$(4) \quad \max_{z} \int_{0}^{\infty} u(z)\, e^{-\rho t} dt$$

subject to (3). As opposed to the Lancasterian research program, we do not specify any optimisation problem for the workers, which would have β as a control variable. Instead, β is assumed to be exogenously given.

The solution of the Capitalists' Problem is also characterised in Figure 1. The fact that β lies strictly between zero and unity makes the marginal product curve lie lower than in the standard case. From the steady-state condition βf'(k)=n+ρ it follows that the steady state capital stock now becomes k**, which is smaller than k* and we therefore take this solution to be inferior to the solution of the previous program. We should also note that the larger is β, the closer is k** to k*. Hence we could say that there is a tradeoff between equality and efficiency. The reason for this result is that there is an externality; workers will in the future benefit from investment today, but capitalists ignore this and pay attention only to how much they will receive for themselves. Hence, when they balance the value of investment to the value of consumption today, the former is too small, because the externality is not internalised. Clearly, any trick that reduces (or eliminates) this inefficiency is welcome, and I will now present one.

So far, the distribution mechanism has been the simplest possible; output has been divided in two parts at every level of k, that is, there has been no difference between »average» and »marginal» distribution. But it seems like welfare can be raised if such a difference is created. Take a look at Figure 1 once again. Total output is the area below the marginal product curve f'(k). Consider now a distribution rule that says that the allowance to capitalists corresponds to the surface below the curve AA, up to the actual k. It is clearly seen that capitalists marginal income is never smaller than n+ρ (which would stop investments) and, as we move to the right in the diagram, they get increasing shares of the national output. In particular, they get everything of the output generated by the unit of capital that makes the stock reach the desired golden-rule size. At the same time their »average share» is, of course, less than unity. It can be calculated by dividing the area below AA by the whole area below the marginal product curve.

More formally, this distribution rule implies that we design a compensation system that gives the following income to capitalists (CI):

$$(6) \quad CI = \int_0^k (\rho + n + \varepsilon - \frac{\varepsilon}{k^*}\tau)\, d\tau, \quad k \leq k^*$$

(Workers receive the residual income f(k)–CI.) If this is plugged into (3), instead of $\beta f(k)$, the rate of change of capitalists shadow price of capital ($\theta$) will be described by

$$(7) \quad \hat{\theta} = \rho + n - \left[\rho + n + \varepsilon - \frac{\varepsilon}{k^*}k\right]$$

which is nothing but (the negative of) the gap between AA and $\rho$+n in Figure 1. It is clear from (7) that $\theta$ stops falling at k*. Hence capitalists will now choose the same steady state capital stock and output as the planner would do.[5] (Note, however, that it is generally not the case that consumption per capita is equal in the two groups.)

In order to be faithful to the Lancasterian tradition, I should now have started out to investigate under what conditions we should expect this distributional rate to be accepted by both classes. My ambition is, however more modest: given the initial inefficiency, the rule represented by (6) should be thought of as some advice from outside, which the agents may or may not utilize. The question whether or not they will, is not answered here. Suffice it to say that the fact that the rule is quite flexible [$\varepsilon$ can be varied between 0 and $-f''(k^*)k^*$] makes it more likely that both groups can agree on it. The income distribution (i.e. $\varepsilon$) is still largely undetermined and will probably be subject to ever recurrent negotiations and renegotiations between the groups. Furthermore, in most western countries, capitalists' shares of national income range between 1/4 and 1/3. If we take such a number as the relevant value of $\beta$, then the difference between f(k**) and f(k*) would be fairly large, and the potential welfare gain would be substantial.

Before ending this section, I perform a numerical exercise in order to show that this new remuneration system can imply improvements for everyone, at least when only steady states are compared. Assume that the initial allocation is utilitarian, i.e. $\beta = \alpha$. This implies that equality in the distribution is feasible and realised. Using the Cobb-Douglas technology $Y = AK^{v}[(1-\alpha)N]^{1-v}$ [with $A = (1-\alpha)^{v-1}$], the capital stocks in the two steady states are

$$(8) \quad k^* = \left[\frac{v}{n+\rho}\right]^{\frac{1}{1-v}} \quad \text{and}$$

$$k^{**} = \left[\frac{v\beta}{\rho+n}\right]^{\frac{1}{1-v}}$$

The question is now whether there exist, for some rasonable values of n, $\rho$, $v$ and $\beta$, an $\varepsilon$ such that both parties are better off[6], that is, such that

$$(9) \quad k^*(\rho+n) + \frac{\varepsilon}{2}(k^*) > \beta(k^{**})^{v}$$

$$(10) \quad (k^*)^{v} - (\rho+n)k^* - \frac{\varepsilon}{2}k^* > (1-\beta)(k^{**})^{v}$$

For $v$ and $\beta$ I choose 1/3, which should not deviate too much from actual capitalists' shares. The value of $\rho$+n is empirically more uncertain. Therefore we will have to use more than one number. In Table 1 some computations are reported. For capitalists to be better off (in steady state) $\varepsilon$ must be larger than $\varepsilon_c$ at every chosen value of n+$\rho$. At the same time, workers benefit only if $\varepsilon < \varepsilon_w$. As we can see from the table, there are many values of $\varepsilon$ that satisfy this. Note that $\varepsilon < 0$ is not of interest.

[5] *More generally, the integrand in (6) could be any function $\varphi(k)$ satisfying $n + \rho \leq \varphi[k] \leq f'(k)$ $\forall k \in [0, k^*]$ and $\varphi'(k) < 0$. The last condition makes CI concave, which is necessary for the Optimal Control problem to be well-defined.*

[6] *If we assume that the trade union is dominant (cf. McDonald and Solow (1981)) it suffices that it is made better off by this rule.*

Table 1. Admissible values of $\varepsilon$.

| $\rho+n$ | $\varepsilon_c$ | $\varepsilon_w$ |
|---|---|---|
| 0.1 | –0.08 | 0.17 |
| 0.2 | –0.17 | 0.33 |
| 0.3 | –0.25 | 0.51 |
| 0.4 | –0.33 | 0.67 |

Assuming, thus, that an agreement on (6) is reached, the conclusion is as follows. Suppose k is smaller than k*. If workers want to have their income increased, they must accept that capitalists receive increasingly greater shares of the additional national income. This may in part be compensated by successive decreases of $\varepsilon$, but it also seems necessary to suppress envy.

## *3. The Endogenous Growth Case*

A central message from the so called new growth models is that changes in economic incentives affect not only the level at which growth occurs, but, more importantly, the growth *rate*. Thus the difference between two time-invariant incentive structures is not that the difference in capital per capita (in steady state) will be constant (as in the neoclassical growth model) but will *increase* over time. If these models describe actual economies more accurately than the older ones did, it becomes even more important to be careful when deciding upon income distribution, since this affects the incentive to invest, and hence the *rate* of economic growth.

We can analyse the Lancasterian efficiency problem by using the model in Rebelo (1991). This model is probably the simplest possible endogenous growth model, but, as Sala-i-Martin (1990) notes, more sophisticated models like Barro (1990), Romer (1989), Lucas (1988) and even Grossman and Helpman (1991) can be viewed as variations or microfoundations of it[7].

Let us start by studying the case with homogeneous agents. The production function in this model is simply linear in capital:

$$(11)\quad Y=F(K,N)=AK$$

i.e. the work force/population is normalised to one and assumed to be constant. The special feature of this function is of course that the marginal product of capital is constant. The law of diminishing returns is thus suspended, which implies that capital accumulation will never cease, provided that the economy is productive enough, that is, provided A is not too small.

We will solve this model explicitly and represent the instantaneous felicity from consumption by the Constant Intertemporal Elasticity of Substitution utility function $C^{1-\sigma}(1-\sigma)^{-1}$. The *Problem of the Benevolent Planner* is therefore to

$$(12)\quad \max_{c}\int_{0}^{\infty} e^{-\rho t}\frac{C^{1-\sigma}}{1-\sigma}dt$$

subject to

$$(13)\quad \dot{K} = AK - C$$

A convenient way to do this is to use the current value Hamiltonian

$$(14)\quad H_0 = \frac{C^{1-\sigma}}{1-\sigma} + \theta(AK - C)$$

We here denote the shadow price of K by $\theta$. From this it is straightforward to derive the conditions

$$(15)\quad C^{-\sigma} = \theta$$

$$(16)\quad \dot{\theta}/\theta = \rho - A$$

$$(17)\quad \lim_{t\to\infty} e^{-\rho t}\,\theta K = 0$$

Logarithmic differentiation of (15) gives:

$$(18)\quad -\sigma(\dot{C}/C) = \dot{\theta}/\theta$$

This toghether with (16) implies that

[7] *Some readers might find it inappropriate to call the old well-known linear model endogenous, especially since there is no explicit modelling of* intentional *technological progress in it. However, I use this term in order to be consistent with linguistic use today.*

(19) $\dot{C}/C = (A-\rho)/\sigma \equiv \gamma$

Thus C always grows at a constant rate. To see how fast K grows in steady state, divide (13) by K:

(20) $\dot{K}/K = A - C/K$

Since $\dot{K}/K$ is constant by the definition of a steady state, we can conclude that C, Y and K grow at the common rate $\gamma$. Furthermore, it turns out that there is no transitional dynamics in this model; the transversality condition is satisfied only when the growth rate of capital is $\gamma$ all the time [cf. Sala-i-Martin (1990)]. Equation (19) has interesting implications. It is seen that $\gamma$ is large when $\rho$ and $\sigma$ are small, that is, when agents are thrifty. Similarly the economy grows rapidly when the returns to investments are substantial, i.e. when A is large.

A problem of dynamic inefficiency arises if we again divide agents into two classes. Assume as in section 2 that workers receive and consume $(1-\beta)Y$, while capitalists get the remainder $\beta Y$. The number of workers is constant equal to one, although they are now only a fraction of the population. The optimising behaviour that rules the economy is given by *Capitalists Problem #2:*

(21) $$\max_Z \int_0^\infty e^{-\rho t}\frac{Z^{1-\sigma}}{1-\sigma}dt$$

subject to

(22) $\dot{K} = \beta AK - Z$

where Z is capitalists' consumption (in total). Following the same procedure as before, one sees that the growth rate now is

$$\gamma' = \dot{Z}/Z = \dot{K}/K = (\beta A - \rho)/\sigma$$

which is smaller than $\gamma$. Workers' consumption, $X = (1-\beta)AK(t)$, of course grows at the same rate. Just as in the previous section, this lower growth rate is a result of the fact that capitalists valuate investments too low, because they do not perceive the good that they do to workers. Again, we have a kind of trade-off between equality and efficiency, because $\gamma'$ approaches $\gamma$ as $\beta$ comes close to unity.[8]

One way to reestablish the efficient growth rate $\gamma$ is to let capitalists get all the output, i.e. to let $\beta$ equal unity[9]. However, if the social preferences are utilitarian, this is not a good policy. It may instead be wise to try to construct a remuneration system that induces capitalists to behave in a way that makes the growth rate equal to $\gamma$, even though the national income is (more or less) evenly distributed among all inhabitants of the economy.

One way to accomplish this is to let capitalists' income be

(23) $CI(t)' = AK(t) - \delta AK_0 e^{\gamma t} \quad \delta \in (0,1)$

where $K_0$ is the initial capital stock. We may interpret this as follows: capitalists are expected to accumulate capital at the rate $\gamma$. If they do so, they can safely rake in $(1-\delta)Y(t)$. If they do not, the system will punish them by giving them a smaller fraction of a smaller national product in every moment. We can see that it is optimal for them to do the former, by solving the *Modified Capitalists' Problem:*

$$\max_Z \int_0^\infty e^{-\rho t}\frac{Z^{1-\sigma}}{1-\sigma}dt$$

subject to

(24) $\dot{K} = AK(t) - \delta AK_0 e^{\gamma t} - Z$

Again, the most straightforward way to solve this problem is to use a Hamiltonian:

(25) $$H_1 = \frac{Z^{1-\sigma}}{1-\sigma} + \theta(AK - \delta AK_0 e^{\gamma t} - Z)$$

Note that the middle term in the parenthesis contains nothing that is subject to a choice of

[8] *For a similar result, in a Barro (1990) – type model, see Alesina and Rodrik (1991).*

[9] *To see that a planners solution would imply a growth rate equal to $\gamma$, note that for him (22) is equivalent to $\dot{K} = \beta AK + (1-\beta)AK - Z - X = AK - C$. Thus the relevant Hamiltonian is (14), with its implied growth rate $\gamma$.*

the individual capitalist. The conditions turn out to be the same as for the representative agent:

$$Z^{-\sigma}=\theta$$

$$\dot{\theta}/\theta=\rho-A$$

$$\lim_{t\to\infty} e^{-\rho t}\theta K=0$$

Consequently, by the same arguments as before, the growth rate is

$$\gamma = \dot{X}/X = \dot{Z}/Z = \dot{K}/K = (A-\rho)/\sigma$$

This growth rate is the efficient one because the benevolent planner would choose it, striving to maximise the utility of all citizens (cf. footnote 7). Since $\delta$ can take on any value in the (open) unit interval, it is of course possible to pick one such that every citizen of this economy has the same level of consumption. In this case, therefore, the tradeoff between equality and efficiency seems to be totally abolished.

## 4. Conclusions

Lancaster showed that, in a model where decisions on income distribution and investments are separated between two classes (workers and capitalists), dynamic inefficiency will occur. The reason is that investors do not internalise the external effects of investments. I have here displayed this dilemma in two growth models, and suggested income sharing contracts that reduce or abolish the inefficiency. The contracts work by separating efficiency and distributive issues from each other, at least to some extent. An interesting extension of the problem would be to allow for international capital movements. In some cases this would certainly put restrictions on which remuneration systems that can be used, without driving capital out of the country.

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