Finnish Economic Papers – Volume 6 – Number 2 – Autumn 1993

MARKETING OF PUBLIC DEBT: THE FIXED-PRICE TECHNIQUE*

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We develop a simple model in which government bonds are marketed at a present yield, rather than at a competitive one, and we study the consequences of this debt management choice on market equilibrium and the dynamics of debt accumulation. We show how equilibrium sequences with demand rationing are associated with interest costs that are higher than under competition and that supply rations are unsustainable. In the long run, if the debt manager follows an optimal policy in terms of service costs minimization and consistently responds to the signals represented by the rations, the economy converges to the competitive stationary state with no debt. (JEL E00, E6)

1. Introduction

The literature on debt management has devoted little attention to the practice of floating debt at a pre-set price, which is rather common in many countries as an alternative to placement techniques that involve competitive bidding. Most models simply assume that, under some constraints, the desired quantity of government debt can be placed by the Treasury at a competitively-determined price.

In this paper, we assume instead that government debt is issued at a pre-set yield, and we investigate the consequences of this debt management choice on the dynamics of debt accumulation. The fixed-price approach, which has been primarily used to analyze disequilibrium phenomena in the labor market,[1] is applied here to study rations on trades of financial assets, in a general equilibrium context where price rigidities are introduced as explicit institutional arrangements.

We consider a two-period, deterministic overlapping generations model with pure exchange, where goverment debt is the only investment opportunity. In such a framework, an equilibrium is obtained by introducing quantity constraints on trades of government securities, which absorb the imbalance between demand and supply resulting from presetting the interest rate; the dynamical structure of the model allows us to study the evolution of the economy under quantity rationing.

Our major results are the following:

(i) The selection of the yield affects the dynamics of public debt accumulation. In partic-

* *I would like to thank Costas Azariadis, Henning Bohn, Bruce Mizrach, an anonymous referee and workshop participants at Brown University, Boston College and the University of Pennsylvania for helpful comments.*

[1] *See Böhm and Puhakka (1988) for an analysis of rations in the labor market within a dynamic, general equilibrium model with overlapping generations; Puhakka (1985) and Azariadis and Smith (1989) study credit rationing in a similar framework.*

ular, demand-constrained equilibria are associated with sequences of pre-set interest rates which are higher than the ones prevailing under perfect competition, causing an acceleration of debt accumulation. On the other hand, supply-constrained equilibria with positive debt holdings cannot be sustained, because they are inconsistent with the satisfaction of the government's budget constraint and individual rationality.

These results suggest that, whenever yield setting is the debt manager's marketing choice, demand rations are the prevalent outcome and unnecessarily high interest costs are determined.

This conclusion is consistent, for example, with several episodes in the recent Italian experience in public debt management: government issues of medium- and long-term securities at a pre-set price have in fact often been associated with large amounts of unsatisfied demand and with »disequilibrium» rates of return (»too high» if compared to the ones paid on government debt in other countries, or to the domestic return on capital).

(ii) The steady-state equilibria in a demand-constrained economy will in general be associated with rations and will come in multiples; however, if the debt management policy is optimal in terms of long-run cost minimization, we can rule out many of the possible equilibrium paths and establish that in the long run the economy converges to a competitive stationary state, independently of the initial selection of the interest rate. In addition, if the debt manager consistently takes into account the information content of the rations, the economy will converge to the no-debt competitive stationary state: the process of price-adjustment therefore reduces the multiplicity of stationary solutions. This result provides a rationale for observed policy actions: as previously discussed, under yield setting rations can occur and may persist for a long time along the dynamic path the economy is following. However, while this phenomenon could be taken as evidence of irrationality of the authority, we have instead established that dynamic equilibria with rationing can be consistent with optimizing behavior and efficient information processing in a long-run perspective.

The paper is organized as follows. In Section 2 we provide some institutional background about the issue of marketing techniques for government debt. Section 3 describes a simple overlapping-generations model under the standard assumption of competition in the government asset market. In Section 4 we introduce a yield-setting mechanism in the sale of bonds and characterize the set of the quantity-constrained equilibria which are supported by sequences of pre-set yields; the properties fo demand-constrained equilibria are established in Section 5. Optimality questions are discussed in Section 6. Section 7 draws some conclusions and indicates a few possible extensions.

2. *Institutional Background and Some Stylized Facts*

Several types of marketing techniques are commonly used in the placement of public debt. For a given size of the issue,[2] we can classify them by referring to the way the yield (or price) is determined: auctions leave yield determination to market forces, while in »subscription» issues the yield is set by the authority. In this paper, we focus on the latter technique.

The empirical evidence clearly documents the relevance of »subscription» issues. An O.E.C.D. (1983) survey for the period 1969–81 shows that the technique of sale on a fixed-yield basis is very common for medium-term and long-term government securities. The system was in fact applied by twelve O.E.C.D. countries (Belgium, Canada, Finland, France, Germany, Italy, Japan, Luxembourg, New Zealand, Spain, Sweden and Switzerland). Australia abandoned this method in 1979, the United States in 1976. On the other hand, only six countries used some form of auction for selling notes and bonds (Germany, Japan, the Netherlands, Switzerland, the United Kingdom and the United States), while five used »tap» issues (Australia, Denmark, Germany, New Zealand and the United Kingdom) and two employed private placement techniques (Finland and Luxembourg).

In theory, the debt manager's task is to assess market conditions and, after taking into account legal and fiscal restrictions as well as broader economic considerations, to find the

[2] *The size of the issue can also be a variable, for example, in the case of a »tap» issue.*

securities and marketing forms which allow the financing of the deficit at the lowest possible cost for taxpayers. The possibility of a misjudgment in the selection of the yield is the reason why auctions are valuable techniques; however, in some instances the resulting yield may turn out to be unacceptable for the Treasury. This is more likely to occur when one of the following situations prevails: financial markets are not very efficient; the incidence of government debt to total financial wealth is very high; the menu of government securities is going through a period of fast innovation; or the demand is mainly composed of relatively unsophisticated investors. Hence, especially for medium- and long-term securities, for which »excessive» service costs have to be borne for a protracted period of time, more caution is justified with respect to the adoption of auctions, and fixed-price offerings may be preferable.

On the other hand, if all the terms of an offering are set by the Treasury, i.e., if both the price and the size of the issue are predetermined, the pricing choice reveals to be a successful one only when the volume of subscriptions received just covers the amount of securities offered: obviously a buyers' market would be evidence of a failure for the issue but, equally important, and excess demand cannot be considered a success, since it indicates that the issue could have been placed at lower cost.

Some of the theoretical issues on optimal mechanism designs as applied to a similar setup have been analyzed, for example, by Harris and Raviv (1988a,b) and Eden (1988). A more policy-oriented debate also arose in the U.S. in the early 1960s, when M. Friedman (1960) advocated the introduction of nondiscriminatory auctions for bills and the abandonment of the single-price method for notes and bonds.

In a different context, the importance of non-Walrasian allocation mechanisms in the explanation of capital markets anomalies is also documented in the literature on Initial Public Offerings of common stock, for which once again a single price is set by the underwriter (see Tinic, 1988).

3. Competitive Equilibrium

In this section we introduce the general features of the model, under the usual assumption that government bonds are marketed at the competitive interest rate.

We consider a deterministic overlapping-generations economy with stationary population and identical[3] individuals who live for two periods; they receive a time-invariant endowment of a perishable good $e=(e_1, e_2)$ and they derive utility from consuming an amount $c_t=(c_{1t}, c_{2t+1})$. We assume that the economy is in the so-called »Samuelson» case, i.e., is such that individuals save in the aggregate when the interest rate equals the growth rate. The only investment opportunity is a one-period, consumption-denominated government bond b_t, paying a competitively-determined interest factor R_{t+1}, to be received at maturity. No secondary markets for bonds exist under our assumptions. The government's budget constraint is $b_t=R_tb_{t-1}+g_t-\tau_t$, where g_t and τ_t are government consumption and lump-sum taxes on generation t, respectively. For simplicity, let us assume that the government keeps its primary budget permanently balanced, i.e., $g_t=\tau_t$, for each t: as a consequence, debt is issued simply to cover service costs, which are the focus of our analysis. A representative individual maximizes a utility function $u(c_{1t})+v(c_{2t+1})$ satisfying standard assumptions (u and v are strictly increasing, strictly quasi-concave and at least twice continuously differentiable), subject to the constraints $c_{1t}\leq e_1-\tau_t-b_t$, $c_{2t+1}\leq e_2+b_tR_{t+1}$, c_{1t} $c_{2t+1}\geq 0$; the solution is a saving function $s(R_{t+1})$ representing demand for bonds, to be equated in equilibrium to the supply of bonds b_t, such that the equality below holds for all periods.

(1) $b_t=s(R_{t+1})$

Let us introduce now the following definition.

Definition 1. A *competitive equilibrium* is a sequence of interest factors $\{R_{t+1}\}_{t=0}^{\infty}$ such that utility is maximized and markets clear, i.e., $b_t=s(b_{t+1}/b_t)$, where $s(b_{t+1}/b_t)$ is the solution of the consumer's problem.

If we assume that the savings function is monotonically increasing in the interest rate, such that $ds(R_{t+1})/dR_{t+1}>0$ and current and fu-

[3] *We could allow heterogeneity and the possibility of private loans; however, we would obtain identical results under the additional assumption of a 100% reserve requirement.*

ture consumption are gross substitutes, for convenience we can rewrite condition (1) as

(1') $b_{t+1} = z(b_t)$

which represents a reflected offer curve in the b_t, b_{t+1} space; since $z(b_t) \equiv s^{-1}(b_t)b_t$, $z(\cdot)$ is also monotonically increasing in its argument.

The properties of equation (1'), which together with an initial condition $b_0 > 0$ describes the set of dynamical equilibria for the economy, are well known. In particular, two stationary states satisfy the equation: $b = b^* > 0$ (the Golden Rule) and $b = 0$ (autarky). Under the assumption that savings are a monotonically increasing function of the interest rate, the Golden Rule is unstable, while autarky is stable (see Figure 1): therefore, if $b_0 > b^*$, any sequence $\{R_{t+1}\}_{t=0}^{\infty}$ will be such that $R_{t+1} > 1$, for all t, which implies that debt accumulation leads to a violation of the aggregate resource constraint; if $b_0 < b^*$, then $R_{t+1} < 1$, for all t, and the level of debt per capita decreases over time towards the autarkic steady state.

4. *Quantity-Constrained Equilibria*

In this section, we introduce a yield-setting technique for the placement of bonds in private portfolios; the imbalance between supply and demand that may result from this form of price rigidity is absorbed by a quantity constraint on trades in asset markets. Our purpose is to characterize the resulting equilibrium paths.

Under the yield setting choice, the government offers a consumption-denominated, one-period bond paying a pre-set coupon $\bar{R}_{t+1}$ at expiration. Denote by $\{\bar{b}_t\}_{t=0}^{\infty}$ the sequence of government debt under price setting. Under the hypothesis of a balanced primary deficit the government's budget constraint is $\bar{b}_t = \bar{R}_t \bar{b}_{t-1}$. The consumer's budget constraints take the form $c_{1t} \leq e_1 - \tau_t - \bar{b}_t$, $c_{2t+1} \leq e_2 + \bar{b}_t \bar{R}_{t+1}$. The solution to the consumer's problem is a savings function $s(\bar{R}_{t+1})$, which we assume to be monotonically increasing in the interest rate.

In the construction of equilibrium sequences of pre-set interest rates and rations on bond trading, we shall introduce the following conditions for Drèze equilibria [see Drèze (1975)]:

(i) rationing may affect either supply or demand, but may not affect both simultaneously;

(ii) no quantity rationing is allowed unless price rigidities are binding.

Demand-Constrained Equilibria

Let us introduce the following definition.

Definition 2. A *demand-constrained equilibrium* is a sequence of demand rations and pre-set interest factors $\{(\sigma_t, \bar{R}_{t+1})\}_{t=0}^{\infty}$ such that

(i) $s(\bar{R}_{t+1}) \geq \sigma_t$, for all t, with strict inequality for some t;

(ii) $\min\{s(\bar{R}_{t+1}), \sigma_t\} = \bar{b}_t$, for all t, and

(iii) $\bar{b}_t = \bar{R}_t \bar{b}_{t-1}$, for all t,

where $s(\bar{R}_{t+1})$ is the solution of the constrained consumer's problem.

Condition (i) states that, in a demand-rationing equilibrium, the yield rigidity is binding and the public is constrained in its demand for bonds; (ii) shows how market clearing is obtained with the introduction of a ration on demand, such that uncostrained supply is equal to constrained demand; (iii) means that the government satisfies its budget constraint, i.e., that no simultaneous supply rationing occurs.

We can now state the following proposition.

Proposition 1. A sequence $\{(\sigma_t, \bar{R}_{t+1})\}_{t=0}^{\infty}$ supports a demand-constrained equilibrium if and only if

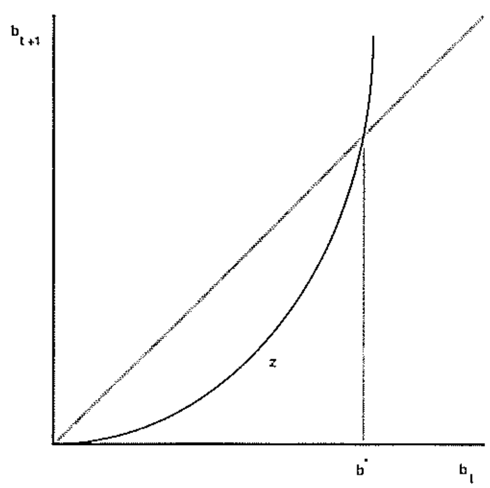

Figure 1.

(2) $\bar{b}_{t+1} \geq z(\bar{b}_t)$

for all t, $t=0, 1, \ldots$, with strict inequality for some t, starting from an initial condition $\bar{b}_0$, $0<\bar{b}_0 \leq b^*$

Proof: If $\{(\sigma_t, \bar{R}_{t+1})\}_{t=0}^{\infty}$ supports a demand-constrained equilibrium, then there exists a sequence of demand rations $\{\sigma_t\}_{t=0}^{\infty}$ such that $\sigma_t \leq s(\bar{R}_{t+1})$; this implies $\bar{b}_t = \sigma_t \leq s(\bar{R}_{t+1})$ and, using the government's budget constraint, $\bar{b}_t \leq s(\bar{b}_{t+1}/\bar{b}_t)$. Under gross substitutability, the latter inequality can be expressed as $\bar{b}_{t+1} \geq z(\bar{b}_t)$, where $z(\bar{b}_t) \equiv s^{-1}(\bar{b}_t)\bar{b}_t$. The converse easily follows.

Graphically, the set of demand-constrained equilibrium sequences defined by (2) is contained in the shaded area in Figure 2. For $\bar{b}_0 > b^*$ the sequence $\{R_{t+1}\}_{t=0}^{\infty}$ will be bounded strictly above one and debt accumulation will lead to a violation of the aggregate resource constraint.

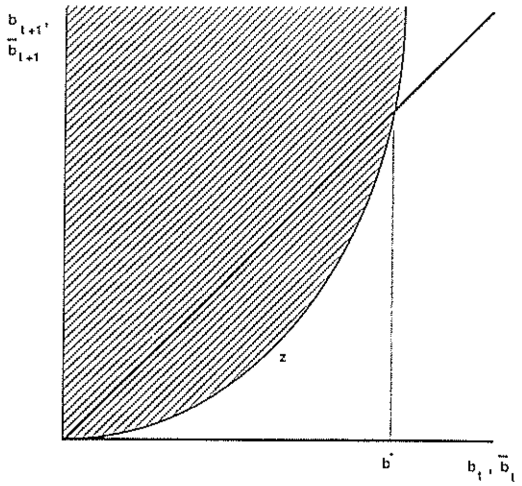

Figure 2.

Supply-Constrained Equilibria

A supply-constrained equilibrium is defined symmetrically as follows.

Definition 3. A *supply-constrained equilibrium* is a sequence of supply rations and preset interest factors $\{(\beta_t, \bar{R}_{t+1})\}_{t=0}^{\infty}$ such that

(i) $\beta_t \leq \bar{b}_t$ for all t, with strict inequality for some t;

(ii) $\min\{\beta_t, \bar{b}_t\} = s(\bar{R}_{t+1})$ and

(iii) $\bar{b}_t = \bar{R}_t \bar{b}_{t-1}$, for all t,

where $s(\bar{R}_{t+1})$ is the solution of the constrained consumer's problem.

Condition (i) states that the government is constrained in the supply of bonds; (ii) is a market-clearing condition which introduces a ration on supply and implies that demand is not simultaneously rationed; (iii) is the government's budget constraint.

The proposition below shows the unsustainability under yield-setting of a supply-constrained equilibrium associated with positive holdings of government debt.

Proposition 2. There exists no sequence $\{(\beta_t, \bar{R}_{t+1})\}_{t=0}^{\infty}$ supporting a supply-constrained equilibrium with positive debt.

Proof: From the definition of supply-constrained equilibrium, the existence of a sequence of supply rations $\{\beta_t\}_{t=0}^{\infty}$ requires, from conditions (i) and (ii), $\beta_t = s(\bar{R}_{t+1})$; but, by (iii), $s(\bar{R}_{t+1}) \leq \bar{R}_t \bar{b}_{t-1}$, with strict inequality for some t, i.e., the government's budget constraint is not satisfied for some t.

Intuitively, in a supply-constrained equilibrium the ration imposed on the government will not permit the satisfaction of its budget constraint, implying a default of the commitments taken with respect to the previous generation. But this is inconsistent with voluntary trades and perfect foresight, since no guarantee would be offered at any time t about the pre-set rate being in fact paid to bondholders: as a consequence, the autarkic equilibrium always prevails.[4]

Propositions 1 and 2 taken together indicate the existence of an asymmetry with respect to the characterization of quantity-constrained equilibria in a dynamic setting, and they suggest a prevalence of demand rations and downward rate rigidities in the government asset markets, whenever yield setting is introduced as an institutional arrangement.

It should be noticed that there is a difference between the possible unsustainability of a demand-constrained equilibrium in the explosive case (i.e., when $b_0 > b^*$), which is due to violation of the resource constraint, and the unsustainability of supply constrained equilibria with positive debt, which is attributed to the

[4] *Under certain conditions, supply-contsrained equilibria with positive holdings could be obtained by introducing a partial default probability.*

insolvency of the government's financial obligations, even in situations where the physical constraint is not binding.

To conclude, the reader should not be led to believe that Proposition 2 is inconsistent with the empirical evidence, which witnesses the possibility of situations in which the government is not able to place the entire issue and is, therefore, rationed. Our extreme non-existence result rests on the simplified structure of the model: for example, a broader set of fiscal policy instruments would allow the existence of supply-constrained equilibria, since unsuccessful issues could be associated with tax raises or expenditures cuts, rather than with debt default; in addition, a more realistic maturity structure would introduce more flexibility within the government's budget constraint, allowing supply-constrained equilibrium to exist whenever the ration is limited to a specific maturity.

Stationary States with Rationing

In the model under examination, a stationary equilibrium is not necessarily associated with market clearing: in fact, excess demand situations may prevail even in the long run. We restrict the attention to stationary states with demand-rationing, which are defined as follows.

Definition 4. A pair $(\sigma, \bar{R})$ is a *stationary demand-constrained equilibrium* if

(i) $s(1) \geq \sigma$;
(ii) $\min\{s(1), \sigma\} = \bar{b}$ and
(iii) $\bar{R} = 1$.

According to the definition above, a demand-constrained stationary equilibrium is any point along the 45° line, such that $\bar{b} \leq s(1) = b^*$: even when the corresponding competitive economy displays a unique steady state with positive debt, the set of the stationary states for the economy with rationing is given by the interval $S = [0, s(1)]$, containing infinitely many points which are characterized by persistence of excess demand. Toward which stationary state with rationing the economy will converge depends on the choice of the ration: in other words, the selection of the interest rate made by the fiscal authority will determine the long-run level of government debt.

5. *Properties of Demand-Constrained Equilibria*

In this section, we confine our attention to demand-constrained equilibria; since a large multiplicity of such equilibria appears, we can establish their properties by selecting a specific one. Graphically, Figure 3 shows a sequence of competitive equilibria $b_{t+1} = z(b_t)$ and one of the many possible sequences of demand-constrained equilibria that satisfy $\bar{b}_{t+1} \geq z(\bar{b}_t)$: the latter is defined by a continuous function $\theta : [0, b^*] \rightarrow [0, b^*]$, such that $\bar{b}_{t+1} = \theta(\bar{b}_t)$ and $\theta(B) \geq z(B)$ for any B in $[0, b^*]$, with strict inequality for some t. Suppose now that the sequences described by θ and by z start from the same initial condition $b_0 = \bar{b}_0 < b^*$: then, the trajectory of θ will lie above the trajectory of z. This implies that the level of government debt under demand rationing is higher than the level of debt under competition, as formally proved in the next proposition.

Proposition 3. If $\{R_{t+1}\}_{t=0}^{\infty}$ is a competitive equilibrium for $b_{t+1} = z(b_t)$ and $\{(\sigma_t, \bar{R}_{t+1})\}_{t=0}^{\infty}$ is a demand-constrained equilibrium for $\bar{b}_{t+1} = \theta(\bar{b}_t)$, and the two sequences start from the same initial condition $b_0 = \bar{b}_0 < b^*$, then $\bar{b}_t \geq b_t$ for all t, $t = 1, 2, \ldots$, with strict inequality for some t.

Proof: Construct two equilibrium sequences starting from an identical initial condition $b_0 = \bar{b}_0$; then, $\bar{b}_1 = \theta(\bar{b}_0) = \theta(b_0) \geq z(b_0) = b_1$. The same holds for all t: in fact, if $\bar{b}_t \geq b_t$,

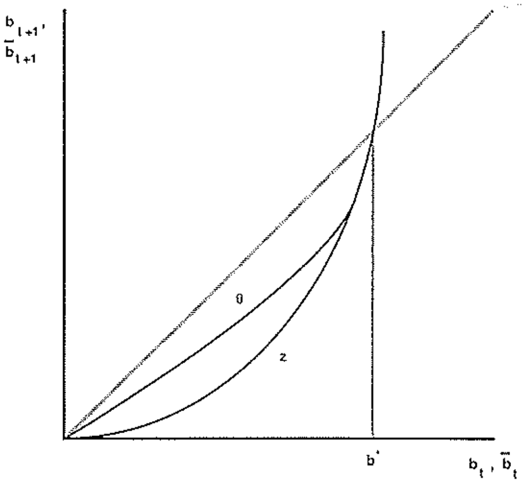

Figure 3.

then $\bar{b}_{t+1}=\theta(\bar{b}_t)\geq z(\bar{b}_t)\geq z(b_t)=b_{t+1}$, with strict inequality for some t.

A corollary to Proposition 3 is that along alternative equilibrium paths pre-set yields tend to be higher than competitive ones, causing »excessive» service costs and an acceleration of debt dynamics.[5]

Corollary. If $\{R_{t+1}\}_{t=0}^{\infty}$ is a competitive equilibrium sequence for $b_{t+1}=z(b_t)$ and $\{(\sigma_t, \bar{R}_{t+1})\}_{t=0}^{\infty}$ is a demand-constrained equilibrium sequence for $\bar{b}_{t+1}=\theta(\bar{b}_t)$, and the two sequences start from the same initial condition $b_0=\bar{b}_0<b^*$, then $\bar{R}_{t+1}\geq R_{t+1}$, for all t, $t=1, 2, \ldots$, with strict inequality for some t.

Proof: It follows immediately from Proposition 3 and the government's budget constraint. Proposition 3 established that $\bar{b}_1\geq b_1$; but $\bar{b}_1=\bar{R}_1 b_0$ and $b_1=R_1 b_0$, i.e., $\bar{R}_1=\bar{b}_1/b_0$ and $R_1=b_1/b_0$; thus, $\bar{R}_1\geq R_1$. Analogously, for all $t=1, 2, \ldots$, $\bar{R}_{t+1}=\bar{b}_{t+1}/\bar{b}_t=\theta(\bar{b}_t)/\bar{b}_t$ and $R_{t+1}=b_{t+1}/b_t=z(b_t)/b_t$; but both $\theta(\bar{b}_t)/\bar{b}_t$ and $z(b_t)/b_t$ are increasing in $\bar{b}_t$ and b_t, respectively; therefore, being $\bar{b}_t\geq b_t$, $\bar{R}_{t+1}\geq R_{t+1}$ for all t, $t=1, 2, \ldots$, with strict inequality for some t.

6. Optimal Yield-Setting Policies

Within the debate on debt management policy, many seem to hold the belief that an increase in the degree of competitiveness in government asset markets, to be achieved through a wider use of auctions, is a condition for welfare improvement. However, the predictions of the standard normative analysis of rationing equilibria indicate that there is no clear theoretical basis for advocating such reforms: in a static framework, Böhm (1984) shows that there exist fixed-price equilibrium allocations which are not Pareto dominated by Walrasian equilibrium allocations; Böhm and Puhakka (1988), in an overlapping-generations model, also establish that competitive equilibria may be dominated by equilibria with rationing.

Moreover, recent work on the theory of debt management has questioned the relevance of standard welfare analysis as an explanation of observed policy actions suggesting that actual government behavior can in fact be better interpreted as the solution of the maximization of an objective function which differs from the representative agent's utility; the question is which objective function is the relevant one when debt management is the issue.

Our results from Section 5 allow the introduction of a natural, simple objective function for the debt manager: long-run minimization of service costs, given an initial value for the stock of debt and for the level of the interest rate. Since, as shown in Section 5, demand rations imply sequences of interest rates which are higher than in a competitive equilibrium, the cost-minimizing solution is to let the level of debt converge to the competitive level in the long run.[6] It should, in fact, be clear that, if the initial choice of the yield implies a demand constraint, a one-shot adjustment to the competitive yield is never feasible, since it would imply a supply constraint in the following period.

In a dynamic setting, for economies which are on a demand-constrained equilibrium path, we therefore introduce the following definition of an optimal yield-setting policy as a policy that minimizes service costs in the long run.

Definition 5. A yield-adjustment policy is *optimal* in the sense of long-run cost minimization if the corresponding demand-constrained equilibrium sequence $\{(\sigma_t, \bar{R}_{t+1})\}_{t=0}^{\infty}$ converges to a competitive stationary state, i.e., $\lim_{t\to\infty}\bar{b}_t=\lim_{t\to\infty}b_t$.

While the set of stationary states for the economy with rations is given by the interval $S=[0, b^*\}$, under the definition above the set is restricted to $b=\{0, b^*\}$. In other words, under an optimal yield-setting policy, a demand-constrained sequence of public debt stocks is equivalent to a competitive sequence, as formally proved in the proposition below.

Proposition 4. Under an optimal yield-adjusting policy, a demand-constrained equilibria sequence is equivalent to a competitive equilibrium sequence.

Proof: It follows by simple application of the definition of equivalent sequences. Two

[5] *To be precise, under the assumption of stationary population, any equilibrium sequence which is associated with a negative interest rate displays decreasing values of debt: under yield setting debt accumulation will, therefore, decelerate at a slower pace.*

[6] *Quoting Tobin (1963): »The problem of debt management, then, can be put in these terms: How are long-run interest costs on a given volume of federal debt to be minimized, given the contribution that debt management and monetary policy jointly make to economic stabilization? ...»*

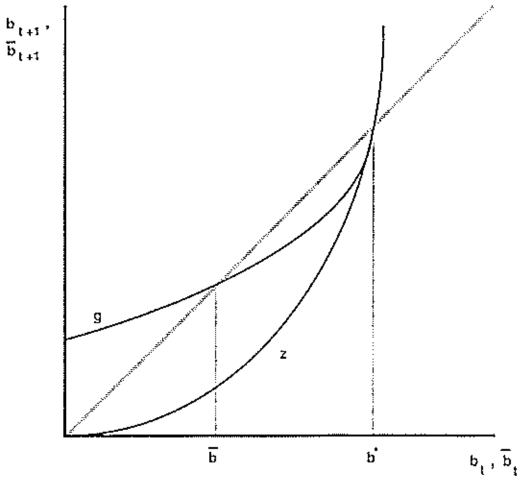

Figure 4.

sequences, say $\{\bar{b}_t\}_{t=0}^{\infty}$ and $\{b_t\}_{t=0}^{\infty}$, are equivalent if $\lim_{t\to\infty}(\bar{b}_t - b_t)$ exists and is equal to zero. This condition is clearly satisfied if the yield-adjusting policy is optimal.

Since the shape of the demand-constrained equilibrium trajectory depends on the yield-setting policy, we have seen how imposing a restrictive condition means ruling out some possible demand-constrained equilibrium paths. For example, Figure 4 illustrates a demand-constrained equilibrium sequence which exhibits two stationary states, $\bar{b}$ and b^*; b^* corresponds to the Golden Rule, while $\bar{b}$ cannot be optimal according to our definition, since it differs from the autarkic solution.

However, the policies that satisfy our optimality criterion still represent a large set. Our next task is to restrict it by introducing plausible additional assumptions on the decision process of the debt manager in the selection of the pre-set rate.

Following Böhm and Puhakka (1988), we impose that the yield choice satisfies the following definition of consistency.

Definition 6. A yield-adjustment policy associated with a demand-constrained equilibrium sequence $\{(\sigma_t, \bar{R}_{t+1})\}_{t=0}^{\infty}$ is *consistent* if it follows a stable yield-adjusting process according to the law of demand and supply.

Consistency means that, whenever a ration on demand is observed, the price of the bond is increased, i.e., the interest rate is reduced; this implies a decreasing sequence of debt. In other words, rations are signals of an imbalance which a rational adjustment process can correct.

By applying the above definition we can derive the following result concerning the long-run properties of demand-constrained economies.

Proposition 5. An optimal yield-adjustment policy is consistent if and only if the associated demand-constrained equilibrium sequence $\{(\sigma_t, \bar{R}_{t+1})\}_{t=0}^{\infty}$ converges to the no-debt stationary state, i.e., $\lim_{t\to\infty}\bar{b}_t=0$.

Proof: By consistency, $\bar{b}_{t+1}<\bar{b}_t$, for all t, for any b_0 in $(0, b^*)$; however, by optimality, we imposed $\lim_{t\to\infty}\bar{b}_t=b$, where $b=\{0, b^*\}$, starting from any initial condition $\bar{b}_0<b^*$; therefore, the autarkic steady state is the only optimal solution which is consistent.

The above proposition shows how the interaction between the yield-adjustment process and dynamics helps reducing the multiplicity of equilibrium paths under demand rationing. Graphically, the functions which satisfy our definitions of consistency and optimality are given by deformations of the competitive equilibrium trajectory. Thus, Proposition 5 rules out situations like the one depicted in Figure 5. In Figure 5, the demand-constrained equilibrium sequence is defined by the mapping f; under f, the competitive stationary equilibrium with positive debt b^* is stable, implying a sequence $\{\bar{b}_t\}_{t=0}^{\infty}$ converging to it; however, this sequence is increasing, and therefore inconsistent. Figure 6 shows two possible sequences, h and j, which are both consistent and optimal, and therefore converge to 0.

In summary, if independently of the initial pre-setting choice, the debt manager follows an optimal adjustment rule and the information content of rations is rationally taken into account, in the long run rations have to vanish and the economy approaches a competitive stationary state. Our results provide a rationale for observed policy actions: while dynamic equilibria with rationing do imply excessive service costs and could therefore be taken as evidence of irrationality within a short-run perspective, we have instead established that, under some restrictions, they can be reconciled with the assumptions of optimizing behavior and efficient information processing in the long run.

To conclude, it should be stressed that as mentioned at the beginning of this section, not

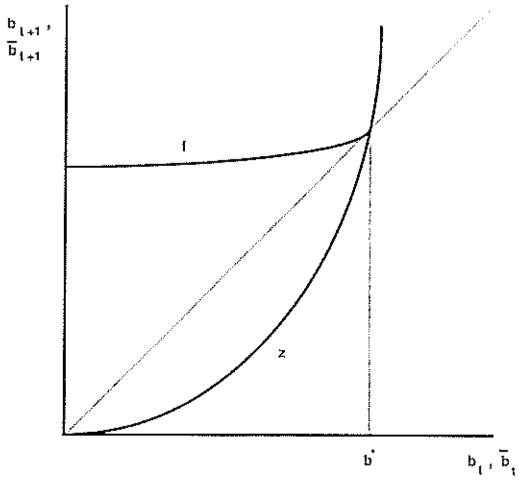

Figure 5.

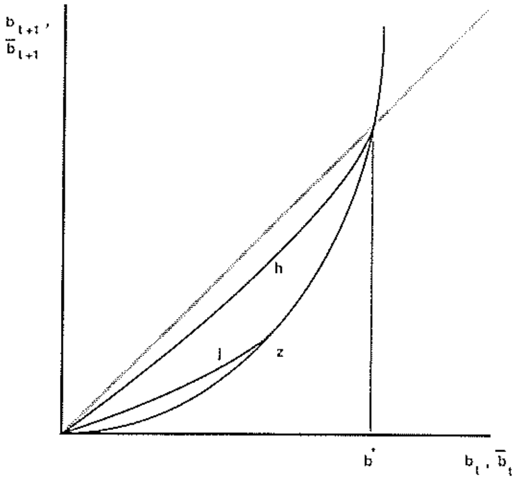

Figure 6.

necessarily a policy which is optimal in the sense of cost minimization implies maximization of individual welfare. In fact, the policies which satisfy our conditions of consistency and optimality are certainly not going to be welfare-maximizing, since they all converge to the autarkic, sub-optimal stationary state. In addition, while steady-state welfare comparisons can be established for yield-setting policies, a Pareto ranking may instead turn out to be impossible for non-stationary allocations, because of the presence of two potential sources of inefficiency: the intertemporal structure of the economy and the public debt marketing choice.

7. *Conclusions and Directions for Future Research*

We have shown how equilibria with bond demand rationing can be supported by sequences of pre-set bond yields, and how in this case pre-set yields are higher than competitive ones, introducing an element of acceleration in the dynamics of public debt. On the other hand, equilibria with supply rationing and positive debt cannot exist under the solvency constraint imposed on the government. If the debt manager minimizes service costs in the long run and consistently responds to the signals represented by excess-demand situations, the rations will vanish and the level of debt will converge to the competitive steady-state level of zero.

This paper represents a preliminary step toward a complete theory of debt management under alternatice placement techniques. In particular, extensions would be possible by removing the following simplifying assumptions:

(i) It is unsatisfactory to treat the government's initial selection of the marketing scheme as exogenous, rather than the result of an optimal choice from a large class of feasible allocation mechanisms, including auctions;

(ii) By limiting our attention to government bonds as the only investment opportunity, we ignored the possible interactions between quantity constraints in the bond market and equilibrium in other markets (spillover effects).

(iii) In the current paper, we have abstracted from the consideration of uncertainty. In a companion paper (Bertocchi (1993)) we study the behavior of a Bayesian-learning debt manager who optimally uses the information produced by rations in order to update his estimates of the unknown parameters in a stationary world.

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