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AN ECONOMETRIC DIFFUSION MODEL OF EXCHANGE RATE MOVEMENTS WITHIN A BAND

Implications for interest rate differential and credibility of exchange rate policy*

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The paper presents a model of exchange rate movements within a specified exchange rate band enforced by central bank interventions. The model is based on the empirical observation that the exchange rate has usually been strictly inside the band, at least in Finland. In this model the distribution of the exchange rate is truncated lognormal from the edges towards the center of the band and hence quite different from the bimodal distribution of the standard target zone model.

The model is estimated using the daily observations of the changes in the FIM exchange rate since 1987. The estimated model is used to compute the expected future values of the FIM/ECU rate within the band for the period June 7, 1991 — September 7, 1992 and the corresponding interest rate differential between Finland and EMS countries. Subtracting this from the actual interest rate differential gives a measure of expected devaluation and devaluation risk premium. The premium was very large in autumn 1991 before the devaluation in November but afterwards it decreased considerably. After a few months the devaluation pressure increased again and finally on September 8, 1992 markka was allowed to float. (JEL F3)

1. *Introduction*

Many European countries have adopted a target zone exchange rate mechanism where the exchange rate is allowed to float only within a specified band enforced by central bank interventions. Examples are countries participating in the European Monetary System and Nordic countries. The regulated floating of exchange rates has also been established under the Louvre accord in 1987 and the subsequent consultations among the major industrial countries, although these target zones have not been publicly announced.

Whether the exchange rate movements within a known band can be forecast is an important question for macroeconomic forecasters and for the private sector agents who operate in the foreign exchange and financial markets. The usual procedure in macroeconomic forecasting is to set the future exchange rate at the currently prevailing level. Implicitly this means that exchange rate changes are assumed to follow a pure random walk process

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so that the best forecast for the future exchange rate is the spot rate. However, taking the exchange rate band into account implies that the asymptotic distribution of the random walk exchange rate process is uniform and the long-run mean of the exchange rate is the midpoint of the band.

On the other hand, a model incorporating the simple random walk with bounds set by the band has some undesirable properties. It does not fit very well to the empirical observations of exchange rate behaviour near the edges of the exchange rate band. This can be seen below in figure 1 which depicts a simulation experiment where the FIM/ECU rate is assumed to follow a random walk process within the band. Obviously, if the random walk model were the correct model of exchange rate movements, there should sooner or later emerge situations where the exchange rate remains very close to the edges of the band for fairly long periods of time. But this does not seem to be consistent with the observations from the real world. For example, the FIM/ECU rate remained clearly below the upper bound of the exchange rate band in autumn 1991 although there were quite visible devaluation speculations.

Much of the recent literature on target zone exchange rate behaviour assumes that the fundamental which drives the exchange rate follows a random walk process. The customary approach in target zone modeling is to assume that central bank intervenes only at the edges of the band. When these two basic assumptions are combined, the exchange rate movements can be modeled by a stochastic process which implies that the asymptotic distribution of the exchange rate is bimodal, i.e. U-shaped. In other words, the conventional target zone model implies that the exchange rate should tend to lie at the edges of the band even more clearly than the random walk process described in figure 1 with the asymptotically uniform distribution implies.

Contrary to the standard approach in target zone literature this paper begins the modeling of the exchange rate behaviour from the empirical observation that, when the band has existed, the exchange rate has lain strictly inside the band. This empirical fact has been taken into account in a stochastic diffusion model which is specified in section 3. In this model the distribution of the exchange rate is truncated lognormal from the edges towards the center of the band and hence quite different from the bimodal distribution of the first generation target zone model.

By a certain transformation this model can be very easily estimated econometrically. The model is applied to the exchange rate behaviour in Finland and the estimation results are presented in section 4.

Finally, in section 5 the model is used to compute the expected future values of the FIM/ECU rate within the band from the beginning of the unilateral pegging on June 7, 1991 until September 7, 1992 when markka was allowed to float. These are used in the estimation of the interest rate differential between Finland and EMS countries implied by the fixed exchange rate band. Subtracting this from the actual interest rate differential gives a measure of expected devaluation and devaluation risk premium.

The model implies that devaluation expectations and the devaluation risk premium started to increase in July 1991. In the autumn the premium was very large before the devaluation in November. Afterwards it decreased considerably for a period of about three months. Later in the spring 1992 the devaluation pressure began to grow again and finally in September markka was allowed to float.

2. *The target zone model of exchange rate movements*

Recently considerable attention in international economics has been devoted to modeling the behaviour of exchange rate movements in the case where the free floating of the rate is confined within a predetermined range. By now there is a considerable body of literature of the target zone model. Some examples are Krugman (1991), Froot and Obstfeld (1991) and Svensson (1991a, b, c, d).

Kontulainen, Lehmussaari and Suvanto (1990) have tested some of the main implications of the first generation target zone model in Finland, for instance the negative relationship between the interest rate differential and the exchange rate. Following Svensson (1991c), Vajanne (1991) and Vajanne and Pikkarainen (1992) have estimated linear mean reverting models for the exchange rate changes within the band. In the first generation target zone model and in the empirical applications there are, however, some questionable features in the implied distribution of the exchange rate which motivate the substantially different approach taken in this study.

In the standard formulation of the target zone model, pioneered by Krugman, the basic reason for exchange rate uncertainty is the stochastic fluctuation of the demand for money, especially the velocity of the money stock. Given the other factors of demand and the money supply, the velocity shocks would induce stochastic fluctuations in the exhange rate which would not remain in a finite band without offsetting central bank interventions.

The first generation target zone model, however, assumes that the central bank intervenes only at the edges of the band. The assumption that the infinitesimal interventions at the edges are sufficient is mainly based on the properties of the regulated Brownian motion by which the changes in the exchange rate and the underlying fundamental are modeled. The basic assumption behind the regulated Brownian motion process is that the cost function associated with the optimal controlling of the random walk Brownian motion is linear with respect to the control and the state of the system. This means that the controller can instantaneously change the state (see Harrison (1985)).

These basic assumptions are not necessarily realistic if one tries to apply the regulated Brownian motion to exchange rate movements and central bank interventions in the foreign exchange market. It is more likely that the cost of intervention increases nonlinearly when the exchange rate moves towards the edges of the band and may even jump at the edge of the band if the exchange rate happened to move outside the band and the central bank were forced to devalue.

The more recent literature has relaxed the assumption of interventions only at the edges of the band and also avoided the restrictive implications of the distribution of the exchange rate within the band. An important contribution among the econometric target zone exchange rate models have been made for instance by Chen and Giovannini (1992).

The starting point in the target zone model and any other model of regulated floating is the definition of the limits of exchange rate movements

$$(1) \quad L \leq S(t) \leq U$$

Accordingly, it is assumed that the spot price of foreign exchange, $S(t)$, is confined within a range defined by constant limits L and U.

Krugman assumes that the stochastic process of the log of the velocity, $v(t)$, which drives the float of the exchange rate is a random walk Ito process

$$(2) \quad dv(t) = \sigma dZ(t)$$

The Wiener process $dZ(t) = \varepsilon(t)\sqrt{dt}$ is the limit of the difference $Z(t+h) - Z(t) = \varepsilon(t)\sqrt{h}$, when $h -> 0$. Variable $\varepsilon(t)$ is a standard normal deviate and parameter σ is the constant standard deviation of the random shocks.

In the simplest case Krugman's model states that the log of the spot exchange rate at time t, $s(t) = \ln S(t)$, in the target zone equilibrium is the following function of the logs of the money supply, $m(t)$, and velocity

$$(3) \quad s(t) = m(t) + v(t) + A(e^{\phi v(t)} - e^{-\phi v(t)})$$

were $\phi = \sqrt{(2/\beta\sigma^2)}$ and β denotes the semi-elasticity of money demand to interest rate. Constant $A < 0$ is determined from the tangency of the exchange rate function (3) and the edges of the exchange rate band [see Krugman (1991)].

Changes in the money supply, $m(t)$, are used to prevent the exchange rate from moving outside of the bounds of the predetermined band. The target zone model implies that the distribution of the exchange rate is bimodal so that most of the mass of the distribution is concentrated near the edges of the band [see Svensson (1991b)]. This means that simply by looking at the actual exchange rate behaviour one should be able to observe time periods when the exchange rate remains continuosly close to the edges of the band.

One way to evaluate Krugman's model would be to use the traditional econometric estimation and testing procedures. However, the model is nonlinear and very difficult to estimate even though the underlying specification of the money market and foreign exchange market behaviour is very simple. The estimation problems can be seen clearly if stochastic differentiation is used to derive the exchange rate movements implied by equations (2) and (3). Therefore only a simple reference case will be illustrated here. In this experiment it is assumed that $\beta = \infty$ so that the exchange rate follows a random walk process within the bounds of the exchange rate band

$$(4) \quad dS(t) = \sigma S(t) dZ(t)$$

Figure 1 demonstrates a simulation where equations (1) and (4) have been used to generate a random walk target zone FIM/ECU rate starting from 4.94 FIM in the beginning of last year and ending with the devaluation November 14, 1991. The standard normal deviate $\varepsilon(t)$ was produced by a computer. Analogously to the target zone model, those shocks that would take the exchange rate outside the band have been cut off. The standard deviation parameter $\sigma = 0.002$ was estimated from the actual daily changes in the exchange rate.

In this experiment the simulated exchange rate goes very close to the upper edge of the band and at times it hits the upper bound in the latter part of 1991. However, in reality the exchange rate remained clearly below the upper limit of the band even in August — November when there was a considerable devaluation pressure on the FIM. At that time there were visible devaluation expectations which can be seen below in figure 6 which shows that the forward exchange rate rose above the upper edge of the band already in August 1991 and remained there until the devaluation.

Thus in contrast to the basic assumption of the first generation target zone model the central bank was forced to intervene perhaps exceptionally strongly when the exchange rate still was inside the band. Accordingly, the random walk model and the standard target zone model do not describe adequately the exchange rate movements, at least in Finland.

3. An alternative model

It seems reasonable to assume that if there is a target zone the central bank tries to keep the exchange rate inside the band by intervening within the whole fluctuation range and not just at the edges. Therefore the exchange rate

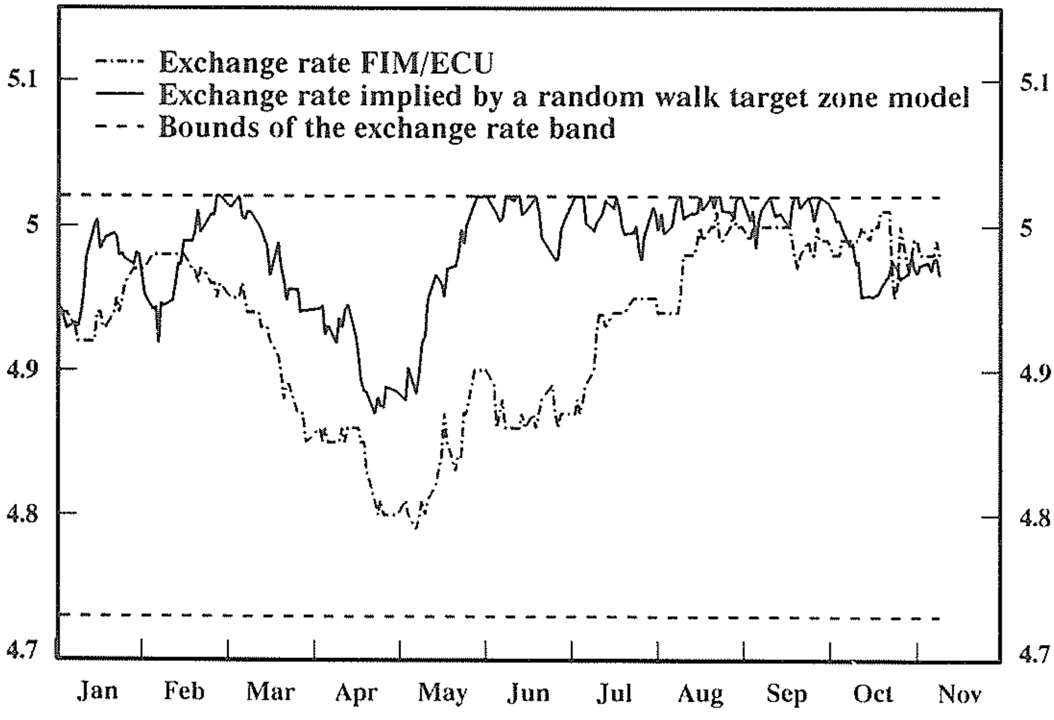

Figure 1. Exchange rate January 2 — November 14, 1991.

usually lies strictly inside the band. This empirical fact must be taken into account if one wants to make a realistic model of exchange rate movements within the band.

Of course, this has been just one elementary aspect of the behaviour of the FIM exchange rate. There may be many models which satisfy this necessary condition. When analyzing the alternative approaches, one might start from the theories of exchange rate determination and various central bank policy alternatives. However, there are so many potential behavioral modes that this is not very practical starting point for an econometric study. Instead, we may take a time series analytic point of view and look directly for such stochastic processes which satisfy the basic properties of the empirically observable exchange rate behaviour.

If the exchange rate is allowed to move freely to the edges of the band, the exchange rate behaviour can be modeled by the regulated Brownian motion. In this sense the bounds of the exchange rate band in Krugman's model are reflecting barriers of the exchange rate process. This means that the exchange rate may occasionally hit the edges of the band but then it is reflected back towards the center of the band. In principle there may be a large family of such processes.

Assume that the changes in the spot exchange rate can be described by the following Ito diffusion process

$$(5) \qquad dS(t) = \theta(S)dt + \sigma(S)dZ(t)$$

Let L denote the constant lower edge and U the constant upper edge of the band as before. Denote the conditional probability density function of the exchange rate by $f(S,t;S_0,t_0)$.

The limits L and U are reflecting barriers of process (5) if

$$(6) \qquad \partial[\sigma^2(S)f(S,t)]/\partial S - 2\theta(S)f(S,t) = 0$$

when $S(t) = L$ or $S(t) = U$. Specifically, the edges of the band may be called natural reflecting barriers of the exchange rate process if $\sigma(L) = 0$ and $\theta(L) > 0$ and, on the other hand, $\sigma(U) = 0$ and $\theta(U) < 0$ [c.f. Ingersoll (1987)]. Then at the lower edge the drift pulls the exchange rate upwards and at the upper bound downwards. Bertola and Caballero (1990) have analyzed the reflecting barrier properties of the target zone model. Contrary to the assumption of the standard target zone model we, however, assume that the exchange rate lies strictly inside the band and therefore $f(L,t) = f(U,t) = 0$ so that condition (6) is satisfied by assumption.

Modeling the behaviour of the exchange rate movements becomes easier if we assume that the distribution of the stochastic shocks is symmetric around the center of the exchange rate band. A priori there does not seem to be any reason to assume otherwise. Therefore in the following the symmetry of the distribution is assumed and we may start to build the model by denoting the absolute deviations of the exchange rate from the center of the band, C

$$(7a) \qquad X(t) = |S(t) - C|$$

$$(7b) \qquad 0 \leq X(t) \leq D$$

The width of the band is 2D so that $0 < D = U - C = C - L < C$.

This confines the exchange rate within the bounds of the band but how would we expect variable X(t) to behave? There are, of course, many alternatives but a simple specification which takes into account the special feature of the reflecting barrier process defined above is

$$(8) \qquad dX(t) = \alpha[D - X(t)]dt + \sigma[D - X(t)]dZ(t)$$

where parameters $\alpha \leq 0$ and $\sigma \geq 0$ are constants.

Then at the edges of the band, $X(t) = D$, the standard deviation $\sigma[D - X(t)]$ goes to zero and as a continuous function of time Ito process (8) cannot pass the limits set by the band despite the random shocks. Moreover, if $\alpha < 0$, the drift term tends to push the exchange rate back towards the center of the band. In this sense the bounds of the band are reflecting barriers although strictly speaking $X(t) < D$ so that the edges are asymptotes of exchange rate process (8). In the limiting case where $\alpha = 0$ the expected change is zero and the exchange rate follows a random walk process.

One important reason for the rather simple specification (8) is the econometric and mathematical tractability of the model. For other specifications the econometric estimation and mathematical analysis of the model would soon become much more difficult. But even model (8) is not easy to estimate because of the heteroscedastic error term. However, we get an estimating model of exchange rate movements by performing the following logarithmic transformation

$$\text{(9a)}\quad Y(t) = \ln[D - X(t)]$$

$$\text{(9b)}\quad \equiv \ln[D - |S(t) - C|]$$

Then we can use Ito's lemma to see that (8) and (9a) yield

$$\text{(10a)}\quad dY(t) = -(\alpha + \sigma^2/2)\,dt - \sigma\, dZ(t)$$

$$\text{(10b)}\quad \equiv \mu dt - \sigma\, dZ(t)$$

Accordingly, the transformed exchange rate Y(t) is normally distributed and therefore the exchange rate S(t) itself follows a truncated lognormal distribution defined from the edges towards the center of the band. The maximum likelihood estimators of the parameters of random walk process (10b) are (c.f. Lo (1986))

$$\text{(11)}\quad \mu = \Sigma(Y_t - Y_{t-1})/n$$

$$\text{(12)}\quad \sigma^2 = \Sigma(Y_t - Y_{t-1} - \mu)^2/n$$

where n is the number of observations. The parameter estimates are simply the mean and variance of the differences of the transformed data ln(D − |S(t) − C|). The precision of estimators (11) and (12) increases with the number of observations [see Ingersoll (1987)]. Therefore estimation with daily observations is preferable to less frequently sampled data.

4. Estimation results

Table 1 presents estimation results obtained from daily observations of the changes in the FIM exchange rate from January 2, 1987 until September 7, 1992. The estimation results for different band regime subperiods are also presented in tables 1 and 2.

Model (10b) implies that the residual random shock, ε_t, should be a standard normal deviate if the model is correct. Therefore the sum of the squared standardised residuals is tested by a chi-square test. The critical value at the five percent level is chi² .95 ≈ $(N(1.96) + \sqrt{(2n-3)})^2/2$, where N(·) denotes the standard normal distribution function.

In some periods the exchange rate has fluctuated in a way which is not fully explained by model (10b). There are some serially correlated changes in the transformed data which can be explained by an autoregressive model

Table 1. Estimation results with daily observations 1987 – 1992.

Estimation period	2.1.1987 – 7.9.1992	2.1.1987 – 29.11.1988	30.11.1988 – 16.3.1989	17.3.1989 – 6.6.1991
α	−0.012	−0.005	−0.024	−0.010
μ	−0.002	−0.003	0.003	0.002
σ^2	0.028	0.016	0.042	0.016
chi²	1435	485	73	558
chi² .95	1488	516	86	592
n	1437	487	75	560
β_1	−0.08	−0.17		−0.18
	(0.03)	(0.04)		(0.04)
β_2	−0.06			−0.09
	(0.03)			(0.04)
β_3	−0.07			
	(0.03)			
L		101.3	100.5	96.5
U		106.0	106.8	102.5
C		103.65	103.65	99.5
D		2.35	3.15	3.0

$$(13) \quad Y_t - Y_{t-1} = \mu(1-\Sigma\beta_i) + \Sigma\beta_i(Y_{t-i} - Y_{t-i-1}) + \sigma\varepsilon_t$$

Coefficients β_i are negative and statistically significant in some band regimes with a maximum lag of three days, $i=3$. The estimated coefficients with the standard errors in parentheses are presented in tables 1 and 2. The statistically significant autocorrelation is not very high as compared to the total variance of the differences of Y_t, as can be seen in figures 2 and 3.

Fortunately, the observed high frequency oscillation does not complicate the computation of expected future exchange rates for longer future horizons, say for a few months or longer periods ahead. During such longer periods the expected changes in Y_t converge to the long-rum mean μ, as can be seen from equation (13).

Generally the results seem to support the model. Estimates of parametar α are negative although slightly different in different estimation periods.

There are some estimation problems associated with the exact size of parameter α and its stability when measured from a high frequency sample. Both the mean, μ, and the variace, σ^2, seem to have varied in different estimation periods. However, to some extent these changes have been opposite so that the effect on parameter $\alpha = -\mu - \sigma^2/2$ is smaller than it otherwise might be.

The time period from the ECU linkage on June 7, 1991 until the devaluation on November 15, 1991 is divided in two quite different subperiods, as can be seen in figure 2. The variance of the daily changes in the FIM/ECU rate increased considerably as speculation regarding a devaluation intensified starting on August 9, 1991. The estimation results for the FIM/ECU rate are presented in table 2.

It seems as if the variance of the exchange rate fluctuations increases when the exchange rate is near the edges of the band for a long time because of revaluation or devaluation pressure. An example of the former case is the period of November 30, 1988 — March 16, 1989 and of the latter case period August 9 — November 14, 1991. This seems to be in contrast with the standard target zone model, which implies that the variance goes to zero at the edges of the band. However, the exchange rate has not been at the edges like the target zone model implies so that actually we have no empirical observations of the variance at the edges.

5. *The expected future exchange rate and the interest rate differential*

The target zone model can be applied to as-

Table 2. Estimation results for the FIM/ECU exchange rate.

Estimation period	7.6.1991 – 8.8.1991	9.8.1991 – 14.11.1991	7.6.1991 – 14.11.1991	21.11.1991 – 7.9.1992
α	0.015	−0.038	−0.015	−0.002
μ	−0.017	−0.016	−0.019	0.001
σ^2	0.004	0.109	0.068	0.001
chi²	38	66	112	194
chi² .95	50	78	128	214
n	42	68	114	196
β_1		−0.25	−0.22	
		(0.12)	(0.10)	
β_2		−0.22	−0.17	−0.15
		(0.13)	(0.10)	(0.07)
β_3		−0.27	−0.22	
		(0.13)	(0.10)	
L			4.72953	5.39166
U			5.02207	5.72516
C			4.87580	5.55841
D			0.14627	0.16675

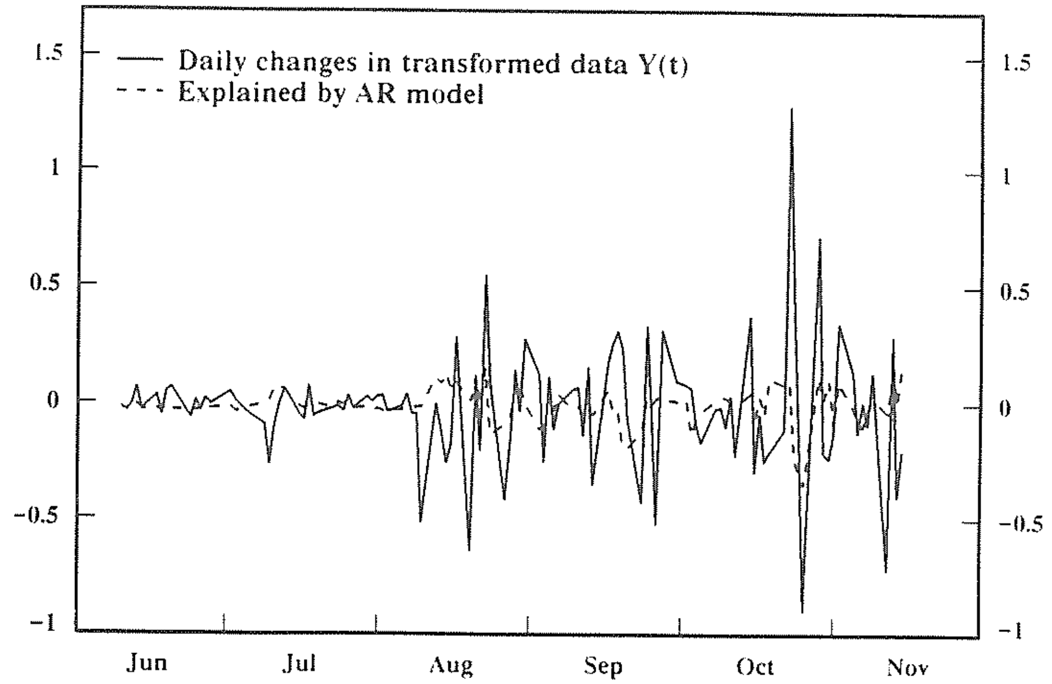

Figure 2. Changes in FIM/ECU June 7 — November 14, 1991.

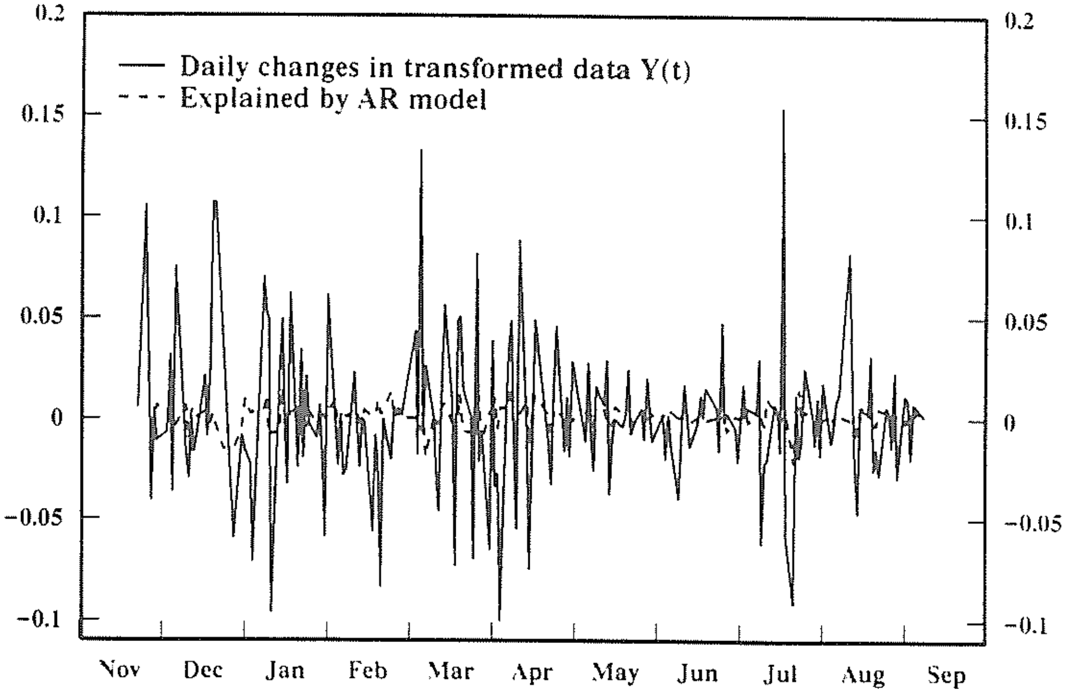

Figure 3. Changes in FIM/ECU November 21, 1991 — September 7, 1992.

sess the credibility of a target zone exchange rate policy. The model gives the future path of the expected exchange rate within the band, which can be used to compute the corresponding interest rate differential with respect to foreign countries. Comparing this to the actual interest rate differential at different maturities gives an idea of the financial market agents' expectations concerning the timing and size of a shift of the exhange rate band.

Usually this analysis has been based on the uncovered interest rate parity [see Svensson (1991b, d)]. However, the uncovered interest rate parity should not be applied unconditionally because there may be other reasons for the interest rate differential than just an expected change in the exchange rate. An interest rate differential may also reflect the risk premium associated with unexpected exchange rate movements. Moreover, even in the case of completely fixed exchange rates and totally free capital movements there may be interest rate differentials between countries because of the credit risk premium resulting from foreign indebtedness [see Rantala (1990)].

The risk premium associated with exchanges rate changes must be taken into account when the exchange rate movements have an impact on the economic activity, in other words when the changes in the exchange rate correlate with changes in output, consumption etc. This is certainly true in the case of an exchange rate policy which changes the position of the exchange rate band, i.e. in the case of an active devaluation or revaluation. Indeed, the purpose of such an exchange rate policy is to influence the real economic activity, at least in the short run.

But in the case of a relatively narrow band the exchange rate movements within the band may be assumed to be generally independent of the real economic activity, at least of the fundamental cyclical changes in the economy. The exchange rate fluctuations within a fixed band result mainly from the shocks in domestic and international financial markets and the central bank interventions in the foreign exchange and financial markets.

In fact, we can decompose the changes in the exchange rate into two components so that one component is the change within the band and another component is the shift of the band. This was one reason for estimating the parameters of the diffusion process separately for different band periods.

Completing model (5) in this way means that it can be rewritten in the from

$$(14)\quad dS(t) = \theta(S)dt + \sigma(S)dZ(t) + \pi(F)dQ(t)$$

The last term in equation (14) takes into account the shifts of the band associated with devaluations or revaluations which depend on a vector of fundamentals F(t). The changes in the exchange rate band typically take place discretely in time so that variable dQ(t) may be assumed to be a Poisson process. Variable $\pi(F)$ measures the size of the shift, which may also be assumed to be a stochastic variable.

The following analysis concerns the interest rate differential implied by expected exchange rate changes within the band. These are assumed to be independent of the changes in real economic activity so that we can apply the uncovered interest rate parity. In fact we assume that such exchange rate changes are comparable to stochastic price level movements in a closed economy general equilibrium model where monetary policy and the price level have no impact on the real economic activity [c.f. Cox, Ingersoll and Ross (1985)]. In complete analogy with such a model we may assume that the interest rate differential associated with expected exchange rate movements within the band does not include any risk premium term because the stochastic exchange rate changes do not correlate with those fundamentals which determine agents' utility.

Denote the short-term foreign interest rate by r(t). Then the equilibrium price of a domestic zero-coupon financial instrument maturing at time $T \geq t$, $P(S,r,t,T)$, is determined by the price of a foreign zero-coupon instrument, $P'(r,t,T)$, and the conditional expectation of the exchange rate, $E_t(S(T))$, so that

$$(15a)\quad P'(r,t,T) = P(S,r,t,T)E_t[S(T)]/S(t)$$

$$(15b)\quad P'(r,T,T) = P(S,r,T,T) = 1$$

Accordingly, we assume that the uncovered interest rate parity holds in this case. It does not hold in general if the changes in the exchange rate correlate with the fundamentals of real economic activity and portfolio choice.

Because the relationship between the interest rate $R(\cdot)$ and the price $P(\cdot)$ of a zero-

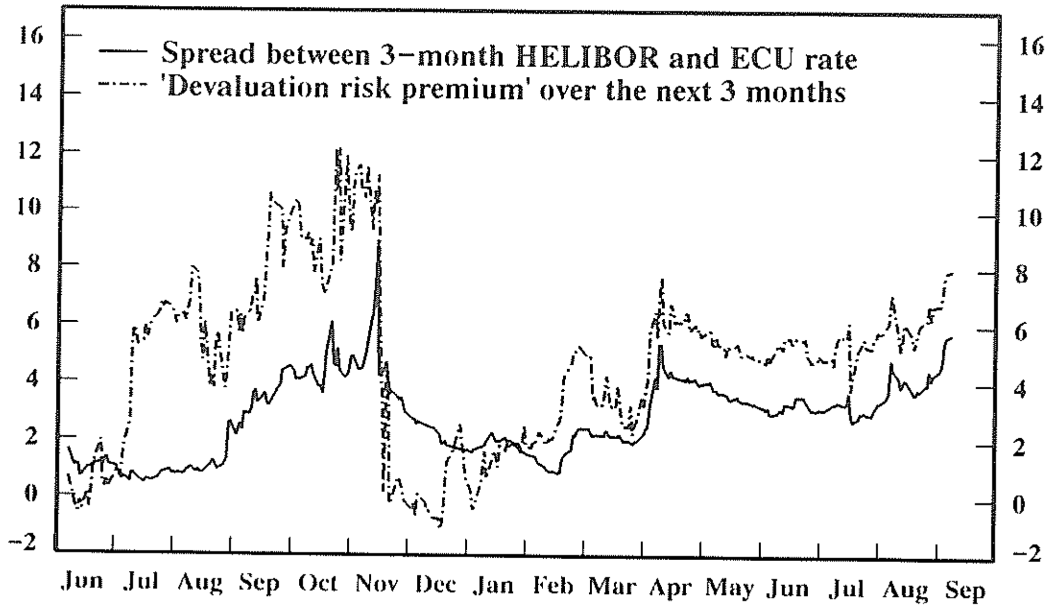

Figure 4. Interest rate differential June 7, 1991 — September 7, 1992, %.

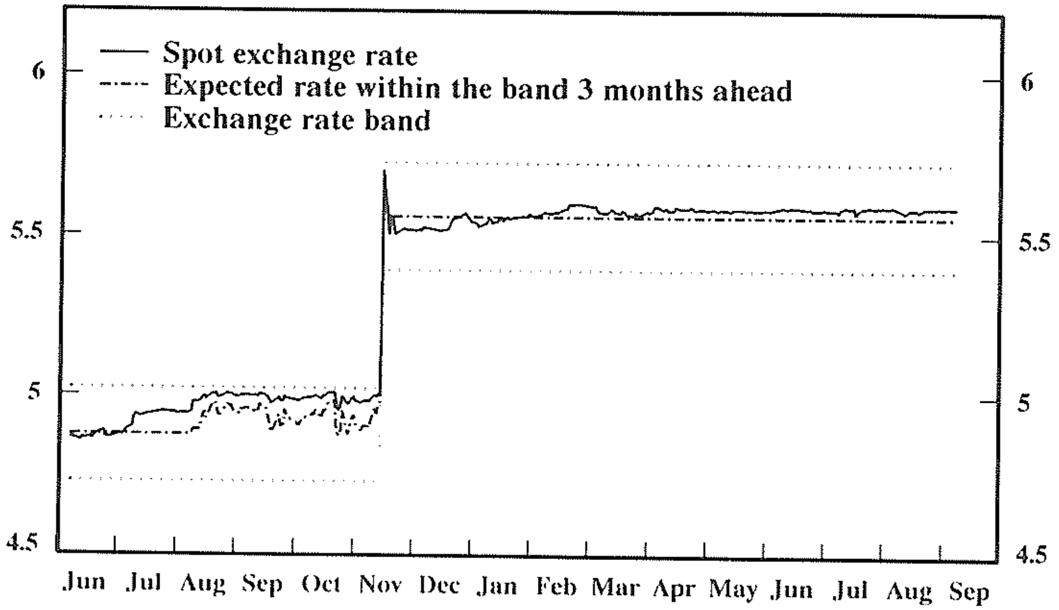

Figure 5. Spot and expected future exchange rate FIM/ECU.

coupon instrument with a term to maturity of $T-t$ is by definition $P(\cdot)=\exp(-R(\cdot)(T-t))$ the interest rate differential is

$$R(S,r,t,T)-R'(r,t,T)=\{\ln E_t[S(T)]-\ln S(t)\}/(T-t) \quad (16)$$

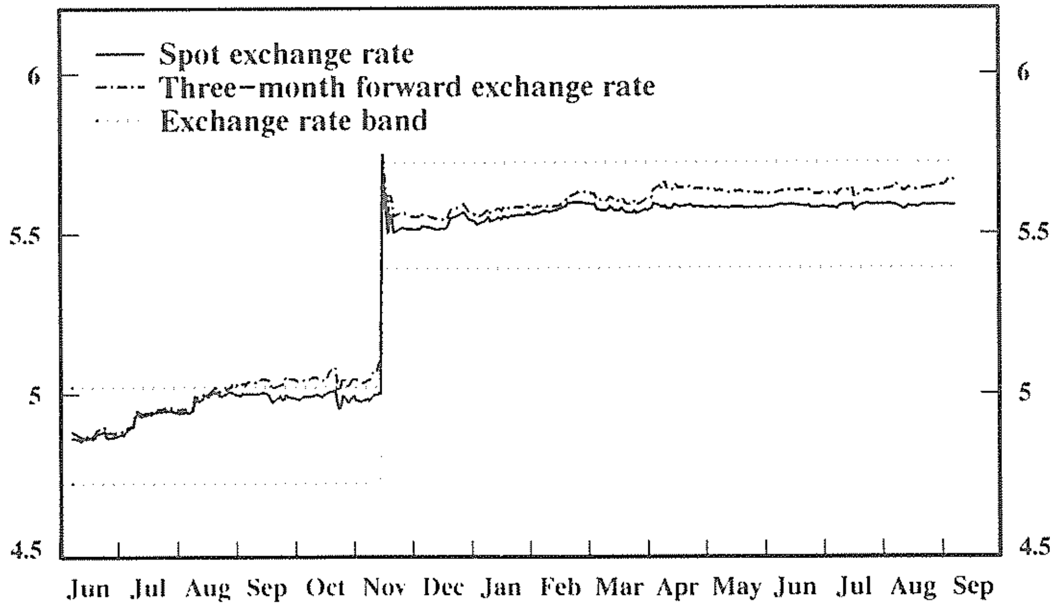

Figure 6. Spot and forward exchange rate FIM/ECU.

In the case of the exchange rate model developed earlier in this study the exchange rate changes in the lower and upper part of the band are mirror images of each other. Solving equations (8) and (9) for a predetermined spot exchange rate, S (t), gives the future value of the exchange rate, S(T), at time $T \geq t$

$$\text{(17a)}\quad S(T) = \min\{C, L + [S(t) - L]e^{\mu(T-t)-\sigma Z(T)} \mid L < S(s) \leq C,\ s \geq t\}$$

$$\text{(17b)}\quad S(T) = \max\{C, U + [S(t) - U]e^{\mu(T-t)-\sigma Z(T)} \mid C \leq S(s) < U,\ s \geq t\}$$

The conditional expected value is

$$\text{(18a)}\quad E_t[S(T)] = \min\{C, L + [S(t) - L]e^{-\alpha(T-t)} \mid L < S(s) \leq C,\ s \geq t\}$$

$$\text{(18b)}\quad E_t[S(T)] = \max\{C, U + [S(t) - U]e^{-\alpha(T-t)} \mid C \leq S(s) < U,\ s \geq t\}$$

Using equations (16) and (18a − b) we can compute the interest rate differential implied by the expected movement of the exchange rate within the band. Figure 5 depicts the expected changes in the FIM/ECU exchange rate for a three-month forecasting horizon in June 7, 1991 — September 7, 1992. These are obtained from equations (18a − b) using parameter value $\alpha = -0.012$, which is the estimate for period January 2, 1987 — September 7, 1992 given in table 1.

The expected exchange rate has been used to compute the interest rate differential according to equation (16). This has been subtracted from the actual differential between three-month Helibor and ECU rates to obtain the »devaluation risk premium», which is shown in figure 4. In fact, this variable measures primarily the annualized expected devaluation. It may also contain a credit risk premium induced by the foreign indebtedness but the dominant part has probably been the interest rate differential associated with the expected devaluation.[1]

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[1] *The ECU interest rate is a weighted arithmetic average of the Euromarket rates of the currencies making up the ECU basket. A referee's point that the foreign interest rate should be computed as a harmonic mean, as pointed out by Pikkarainen (1991), holds for one-period optimal portfolio diversification. The underlying portfolio behaviour here is, however, one of trading in continuous time. The arithmetic mean is consistent with the optimal continuous-time portfolio choice.*

The premiums for three months and other short maturities were very large in autumn 1991 before the devaluation. At the same time the forward FIM/ECU rates rose above the upper bound of the exchange rate band. This can be seen in figure 6 which depicts the three-month forward rate computed from the covered interest rate parity.

The interest rate differential decreased considerably after the devaluation and the forward rates returned into the exchange rate band in late November 1991. The risk premium began to grow again at the end of February 1992.

6. Conclusion

The model of exchange rate movements within a fixed hand developed above is an alternative approach to the standard target zone model. It is, of course, only a step towards more realistic econometric treatment of exchange rate behaviour in a monetary policy regime where the float of the exchange rate is confined within predetermined limits. Specifying a special kind of reflecting barrier model which is consistent with some empirical observations of exchange rate behaviour has not yet given a model which would explain the exchange rate movements perfectly by statistical standards.

However, the results give some support to the model. The distribution of the exchange rate has more mass near the center of the band than implied by the pure random walk model with an asymptotically uniform distribution and much more than in Krugman's target zone model with a bimodal distribution.

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