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SPECTRAL ESTIMATION OF SECULAR AND CYCLICAL ELASTICITIES FOR BILATERAL TRADE*

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The paper uses spectral methods to estimate income and price elasticities for trade among Canada, Germany, Japan, the United Kingdom, the United States, other industrial countries, OPEC, and non-OPEC developing countries. By focusing on bilateral trade data, the paper avoids the aggregation biases associated with multilateral data. By differentiating between secular and cyclical elasticities, the paper recognizes that trade responses to secular expansions and to domestic bottlenecks need not be the same. Finally, by relying on spectral analysis, the paper isolates the secular and cyclical components of the data and thus avoids the drawbacks of time-domain analyses where a time trend proxies secular factors. To emphasize the usefulness of bilateral trade elasticities, the analysis re-examines the asymmetry in income elasticities noted by Houthakker and Magee and evaluates the loss of information associated with relying on multilateral trade data. (JEL C2, F4)

1. Introduction

This paper uses spectral techniques to estimate income and price elasticities for world trade. Spectral methods recognize that secular forces and cyclical factors might have different effects on trade, a useful distinction for policy makers. For example, the balance-of-payments projections underlying development plans and debt-rescheduling agreements of developing countries are best served by using elasticities reflecting long-term factors. Similarly, evaluating the international repercussions of alternative policies among industrial countries benefits from using elasticities embodying short-term considerations.

Interest in estimating secular and cyclical elasticities is not new but existing analyses suffer from two limitations. First, they use multilateral trade data which amounts to treating a country's pattern of trade as invariant to fluctuations in income and prices. Second, they proxy secular forces with time trends. But as Haynes and Stone (1983) note, time trends cannot proxy simultaneously the secular factors of both income and prices because of the ensuing collinearity. I eliminate these two limitations by applying Engle's Band Spectrum estimator (Engle, 1974) to *bilateral* trade data. The application yields a collection of esti-

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mated income and price elasticities that vary across trading partners and that differentiate between secular and cyclical factors.[1]

Section 2 presents the econometric model and describes the spectral-estimation technique. Using quarterly data for 1973 – 85, section 3 estimates the income and price elasticities for trade among Canada, Germany, Japan, the United Kingdom, the United States, other industrial countries, OPEC, and non-OPEC developing countries. The results reveal a substantial dispersion in bilateral elasticities which suggests that the structure of international trade responds to changes in income and relative prices. Given these estimates, I re-examine the asymmetries in income elasticities for U.S. trade noted by Houthakker and Magee (1969). I find significant asymmetries for bilateral trade but not for multilateral trade, a finding that highlights the tension between aggregating predictions and predicting the aggregate.

2. Econometric Formulation

I assume that imports of country k from country s behave according to

$$(1) \quad m_{kst} = \alpha_{0ks} + \alpha_{1ks} y_{kt} + \alpha_{2ks} p_{kst} + u_{kst},$$

where

m_{ks} = logarithm of the volume of imports of country k from country s,
y_k = logarithm of real GNP of country k,
p_{ks} = logarithm of the ratio of country s's export price (in country k's currency) to the domestic price of country k,
$u_{kst} \sim N(0, \sigma^2_{ks})$.

Equation (1) rests on the imperfect-substitute model (Goldstein and Khan, 1985) in which trade volumes respond directly to increases in income ($\alpha_1 > 0$) and inversely to increases in relative prices ($\alpha_2 < 0$).[2] Equation (1) also assumes homogeneity of degree zero in prices and excludes lags of income and prices as explanatory variables. Price homogeneity need not hold in the short run but assuming permanent money illusion is unrealistic. Lags of income and prices might be included in eq. (1) by modeling their secular components as weighted averages of their own past values. Spectral methods, as seen below, measure directly those components and avoid having to estimate them using distributed lags.

Estimating the parameters of eq. (1) with published measures of income and prices confuses secular with cyclical effects and thus cannot differentiate the corresponding elasticities. Engle's spectral technique identifies the secular and cyclical components of the data and uses only the relevant components for parameter estimation. Implementing this technique has two steps: obtaining the frequency-domain representation of eq. (1) and applying least squares to selected frequencies.[3] The first step involves premultiplying eq. (1) by the TxT matrix of Fourier elements Ω:

$$(2) \quad \Omega M_{ks} = \Omega\alpha_{0ks} + \alpha_{1ks}\Omega Y_k + \alpha_{2ks}\Omega P_{ks} + \Omega U_{ks},$$

where $\Omega_{jr} = (1/\sqrt{T})e^{i(2\pi j/T)r}$, $i = \sqrt{-1}$, and variables in upper case denote the Tx1 vector of observations of the associated variables in lower case. The observations in eq. (2) are the Fourier coefficients associated with the original time series. Each of these coefficients measures the relative importance of a frequency component in explaining the variance of the associated variable. Thus eq. (2) pairs observations by frequencies and not by dates as in time-domain least squares. In other words, instead of estimating α_1 and α_2 with T realizations of trade, income, and prices, Engle's technique relies on the Fourier coefficients associated with T spectral frequencies.[4]

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[1] *Ball et al. (1966), Adams et al. (1969), Marston (1971), and Khan and Ross (1975) are among the first to estimate secular and cyclical income elasticities by applying time-series techniques to multilateral trade data. Haynes and Stone (1983) criticize previous studies and re-estimate these elasticities by applying spectral methods to multilateral trade data. Ball et al. (1966) recognize the usefulness of spectral techniques, but do not implement them.*

[2] *The variables in eq. (2) are expressed as deviations from their means with α_0 measuring rounding errors (see Engle, 1974).*

[3] *This discussion draws heavily from Engle (1974, 1976) and Fishman (1969).*

[4] *The usefulness of Engle's technique stems from the properties of Ω:*

$$\Omega^* \Omega = \Omega\Omega^* = I,$$

where + denotes the complex conjugate of the transpose and I is the identity matrix. These properties extend the validity of test statistics of time-domain analysis to the frequency domain. Specifically, $\Omega U_{ks} \sim N(0, I\sigma^2_{ks})$ and

Applying least-squares to selected frequencies involves premultiplying eq. (2) by a TxT selection matrix Γ:

$$(3)\quad \Gamma\Omega M_{ks} = \Gamma\Omega\alpha_{0ks} + \alpha_{1ks}\Gamma\Omega Y_k + \alpha_{2ks}\Gamma\Omega P_{ks} + \Gamma\Omega U_{ks},$$

where Γ contains ones on the diagonals corresponding to the selected frequencies and zeroes for the unwanted frequency components. Thus selecting a Γ that removes the cyclical frequency components from eq. (2) and applying least-squares gives the estimates associated with secular frequencies; the estimation of cyclical elasticities follows an analogous selection process. Note that I treat the income and price elasticities associated with a given frequency, secular or cyclical, as constants. However, »the» elasticity of trade with respect to the observed changes in either income or prices is not constant because it depends on the corresponding secular and cyclical elasticities the relative importance of which changes periodically.

3. *Estimation Results*

Based on quarterly observations for 1973Q1 – 1985Q2, the analysis estimates secular elasticities using a spectral band ranging from 0 to 0.125, implicitly assuming that secular factors take *at least* two years to complete a cycle. To estimate cyclical elasticities, the paper uses a spectral band ranging from 0.126 to 0.5, implicitly assuming that cyclical factors take *at most* two years to complete a cycle.[5]

The evidence reveals several features of interest (table 1). First, most of the secular income elasticities are positive, significant, and show a wide range of variation. For example, out of 56 elasticity estimates, 10 are negative, 26 vary between 0.1 and 2.0, and 20 are greater than 2. The estimated cyclical income elasticities exhibit a similar pattern but less than half of them are significant. This dispersion of estimates suggests that using a single elasticity for predicting bilateral trade can lead to large and systematic forecast errors. Second, the income elasticity for imports from OPEC is either negative or not significantly different from zero. The continuing technological improvements lowering oil requirements, as well as the potential inadequacy of the imperfect-substitute model, can account for this negative income elasticity.

Third, most of the price elasticities are negative and many of the estimates for the cyclical frequency are not statistically significant. Moreover, the secular price elasticities for Japan's bilateral imports are not significantly different from zero. This result, if taken literally, means the absence of suitable Japanese substitutes for foreign products. Finally, judging from the R_s^2, equation (1) offers an adequate explanation of the variation of the secular components of the data but is less suited to accounting for the cyclical, and more volatile, components of the data.

To examine the implications of these estimates for growth and trade, I test whether the bilateral income elasticities for U.S. exports and imports are equal. According to the results, income elasticities for U.S. trade are not significantly different from each other except for trade with Japan and developing countries. For these two countries, the income elasticities for exports are smaller than the income elasticities for imports (table 2). Thus a ceteris paribus secular expansion in world income induces a secular deficit in U.S. trade with Japan and developing countries whereas U.S. trade with other countries remains in balance.[6] But the overall balance of trade, by being the sum of bilateral trade balances, must deteriorate and in the absence of sufficiently large capital inflows, the resulting deficit implies a lower U.S. growth rate, a sustained deterioration of the terms of trade, or both.

The estimates of table 2 also have implications for forecasting international trade. Specifically, table 2 shows that U.S. *multilateral* income elasticities are symmetric ranging from 1.4 for exports to 1.8 for imports, a finding consistent with Haynes and Stone (1983). This

the application of least squares to eqs. (1) and (2) yields the same parameter estimates (see Engle, 1974).

[5] This choice of band width is consistent with conventional trade models that allow for 8-quarter lags for real exchange rates (see Goldstein and Khan, 1985). Using the same set of frequencies for all countries is restrictive but using country-specific frequencies undermines the comparability of results.

[6] This result assumes that all bilateral balances are zero prior to the expansion in world income.

Table 1. Income and Price Elasticities for Bilateral Trade[1] Band Spectrum Estimates: Secular and Cyclical Frequencies[2] 1973Q1 – 1985Q2.

Importer	Source	Secular Frequency					Cyclical Frequency				
		Income	Std. Dev.	Price	Std. Dev.	R^2	Income	Std. Dev.	Price	Std. Dev.	R^2
Canada	Germany	1.11	0.4	−0.65	0.2	0.62	1.37	0.7	−0.90	0.4	0.29
	Japan	2.64	0.5	−1.21	0.3	0.85	1.14	0.6	−1.62	0.5	0.42
	U.K.	−0.73	0.5	−0.46	0.2	0.55	−0.61	0.6	−0.78	0.4	0.11
	U.S.	1.33	0.2	−0.12	0.3	0.81	1.71	0.6	−0.51	0.7	0.21
	ROECD	0.92	0.3	−0.74	0.2	0.74	1.18	0.4	−0.91	0.2	0.50
	LDCs	2.49	0.3	−1.13	0.2	0.92	2.89	0.4	−0.93	0.3	0.62
	OPEC	−4.71	1.4	−1.74	1.1	0.78	−3.88	1.8	−1.23	0.7	0.44
Germany	Canada	2.00	0.5	−0.74	0.3	0.66	3.83	1.2	−1.45	0.4	0.33
	Japan	5.36	0.4	−0.04	0.4	0.94	4.74	0.7	−1.35	0.3	0.57
	U.K.	4.55	0.4	0.42	0.2	0.98	6.10	1.0	−0.84	0.4	0.55
	U.S.	1.99	0.3	−0.61	0.2	0.80	1.71	1.2	−0.77	0.4	0.09
	ROECD	2.23	0.1	−0.49	0.2	0.99	2.27	0.6	−0.64	0.5	0.38
	LDCs	2.26	0.2	−0.59	0.1	0.94	2.46	0.4	−0.83	0.2	0.60
	OPEC	−1.28	1.2	−0.52	0.2	0.80	−2.98	1.0	−0.31	0.1	0.72
Japan	Canada	0.46	0.1	−0.24	0.2	0.61	1.09	0.4	−0.93	0.4	0.23
	Germany	0.68	0.3	−0.60	0.4	0.64	1.51	0.2	−0.41	0.3	0.61
	U.K.	0.98	0.3	−0.46	0.5	0.45	0.60	0.5	−0.74	0.5	0.08
	U.S.	0.81	0.2	0.04	0.3	0.65	0.92	0.2	−0.46	0.2	0.41
	ROECD	0.79	0.2	−0.49	0.3	0.80	1.11	0.2	−0.82	0.3	0.56
	LDCs	1.41	0.1	0.13	0.5	0.94	1.71	0.3	−0.34	0.3	0.56
	OPEC	−0.33	0.2	−0.12	0.1	0.64	−0.17	0.5	−0.27	0.1	0.37
U.K.	Canada	−3.11	1.0	−0.36	0.3	0.52	−0.03	1.0	−0.52	0.4	0.05
	Germany	6.15	0.6	−0.76	0.2	0.96	4.22	0.8	−0.37	0.3	0.49
	Japan	5.71	1.2	−1.08	0.3	0.91	2.59	1.0	−0.50	0.5	0.19
	U.S.	3.02	0.5	−0.72	0.2	0.87	1.03	0.9	0.56	0.4	0.12
	ROECD	3.51	0.4	−0.34	0.1	0.96	2.33	0.6	−0.25	0.3	0.31
	LDCs	0.83	0.5	−0.11	0.2	0.34	1.13	0.7	0.25	0.3	0.08
	OPEC	−10.1	2.3	−0.83	0.3	0.88	−3.73	1.2	−0.78	0.1	0.74
U.S.	Canada	1.44	0.2	−1.01	0.2	0.95	1.84	0.5	−1.23	0.4	0.47
	Germany	1.77	0.5	−0.84	0.2	0.84	1.72	0.6	−1.16	0.2	0.65
	Japan	3.57	0.3	−1.04	0.2	0.97	3.08	0.7	−1.35	0.3	0.72
	U.K.	2.76	0.4	−0.84	0.2	0.87	2.09	0.7	−0.87	0.3	0.41
	ROECD	2.10	0.3	−0.88	0.2	0.94	2.62	0.5	−0.96	0.2	0.78
	LDCs	3.39	0.3	−0.50	0.2	0.94	3.88	0.3	−0.24	0.2	0.85
	OPEC	−2.08	2.0	0.01	0.4	0.15	−2.65	1.4	−0.05	0.2	0.19
ROECD	Canada	2.16	0.6	−0.68	0.3	0.65	1.39	0.8	−0.86	0.4	0.15
	Germany	2.36	0.2	−0.54	0.4	0.89	2.23	0.5	−0.18	0.5	0.43
	Japan	3.95	1.1	−0.14	1.0	0.57	3.01	0.7	−0.68	0.5	0.34
	U.K.	2.10	0.6	0.35	0.3	0.86	2.95	0.6	−0.43	0.4	0.41
	U.S.	2.64	0.4	−0.48	0.2	0.81	1.44	0.8	−0.01	0.4	0.11
	LDCs	3.20	0.5	−0.37	0.5	0.83	2.64	0.4	−0.45	0.3	0.54
	OPEC	−0.15	1.4	−0.55	0.1	0.75	1.57	1.0	−0.62	0.1	0.69
LDCs	Canada	0.57	0.1	−0.22	0.4	0.85	0.57	0.2	−1.19	0.7	0.20
	Germany	0.14	0.1	−0.50	0.2	0.63	0.46	0.1	0.15	0.3	0.33
	Japan	0.68	0.1	−0.89	0.4	0.92	1.00	0.1	0.07	0.4	0.65
	U.K.	−0.08	0.1	0.19	0.2	0.22	0.02	0.1	0.05	0.3	0.01
	U.S.	0.53	0.1	−0.65	0.5	0.70	0.70	0.1	−0.64	0.2	0.76
	ROECD	0.41	0.03	−0.18	0.2	0.93	0.63	0.1	0.18	0.4	0.41
	OPEC	0.07	0.3	−0.23	0.2	0.23	−0.81	0.4	0.04	0.2	0.32

Table 1. Continued.

Importer	Source	Secular Frequency					Cyclical Frequency				
		Income	Std. Dev.	Price	Std. Dev.	R^2	Income	Std. Dev.	Price	Std. Dev.	R^2
OPEC	Canada	0.80	0.3	−1.22	0.2	0.80	−0.18	0.3	−0.08	0.2	0.06
	Germany	0.94	0.6	−1.05	0.3	0.61	−0.26	0.2	−0.22	0.1	0.37
	Japan	0.57	0.4	−1.08	0.2	0.80	−0.41	0.2	−0.24	0.1	0.42
	U.K.	0.73	0.4	−0.90	0.3	0.49	−0.21	0.1	−0.15	0.1	0.28
	U.S.	0.81	0.3	−0.90	0.2	0.63	−0.09	0.1	−0.23	0.1	0.32
	ROECD	0.64	0.4	−1.10	0.3	0.80	−0.35	0.1	−0.25	0.1	0.53
	LDCs	−0.38	0.4	−0.80	0.2	0.87	0.32	0.1	−0.76	0.1	0.76

Notes:

[1] The appendix describes the data definitions and the associated sources. ROECD stands for the Rest of the OECD Countries.

[2] The spectral band for secular frequencies is [0, 0.125], which includes frequency components taking *at least* two years to complete a cycle; the spectral band for cyclical frequencies is [0.126, 0.5] which includes frequency components taking *at most* two years to complete a cycle.

Table 2. Structural Asymmetries in Secular Income Elasticities[1] United States.

Trading Partner	Exports		Imports		Asymmetries	
	Estimate	Std. Dev.	Estimate	Std. Dev.	Estimate	Test[2]
Canada	1.33	0.2	1.44	0.2	−0.11	−0.39
Germany	1.99	0.3	1.77	0.5	0.22	0.38
Japan	0.81	0.2	3.57	0.3	−2.76	−7.65
U.K.	3.02	0.5	2.76	0.4	0.26	0.41
ROECD	2.64	0.4	2.10	0.3	0.54	1.08
LDCs	0.53	0.1	3.39	0.3	−2.86	−9.04
OPEC	0.81	0.3	−2.08	2.0	2.89	1.43
Multilateral Elasticities						
This study[3]	1.38	0.1	1.84	0.3	−0.46	−1.28
Haynes and Stone[4]	1.50		1.46			

[1] The source for the data is table 1.

[2] Entries in this column are computed as $[\hat{\alpha}_{1sk} - \hat{\alpha}_{1ks}]/\sqrt{\hat{\text{var}}(\hat{\alpha}_{1sk} - \hat{\alpha}_{1ks})}$.

[3] The aggregation of the bilateral estimates of table 1 uses the mean of bilateral trade shares as weights.

[4] Haynes and Stone (1983), table 3, p. 93. The secular elasticity refers to the gain at ∞ year cycle; these elasticities are significant at the five percent level.

symmetry occurs because the two large and negative asymmetries cancel out the remaining small but positive asymmetries in the process of aggregation. This canceling, however, creates a conflict for predicting how the U.S. balance of trade responds to expansions in world income: Aggregating predictions based on disaggregated elasticities points to a deterioration of the trade balance whereas predictions based on the aggregate elasticities conceal that deterioration.

4. *Conclusions*

Bilateral trade elasticities play an important role in designing trade policies and predicting how the structure of international trade

responds to changes in income and relative prices. However, the empirical evidence on these elasticities is scant, outdated, and limited in the country-coverage. To relax these limitations the paper estimates income and price elasticities for bilateral world trade using Engle's Band Spectrum estimator. The paper finds that bilateral trade elasticities exhibit enough of a dispersion to suggest that the pattern of trade is sensitive to changes in income and prices.

Although the analysis is subject to several limitations — reliance on single equation methods, aggregation across commodities and countries, and other data considerations — eliminating them is likely to strengthen the main implication of this paper: that sole reliance on multilateral elasticities entails an important loss of information for both policy applications and empirical analyses of international trade whereas reliance on bilateral elasticities entails no such loss.

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Data Appendix

The data for trade flows are in billions of U.S. dollars, F.O.B., quarterly at annual rates. To estimate the volume of imports, I deflate the dollar value of imports by the associated dollar import price, $P_s e_s$ where P_s is the multilateral export unit value of country s in local currency with 1972 = 1 and e_s is the U.S. dollar exchange rate index ($U.S./local currency) with 1972 = 1. For country blocs, I use aggregate, dollar export prices. Specifically, export prices for the blocs of industrial and developing countries are geometric means of the dollar export prices of the countries included in each of these two aggregates. The weights are sample means of exports of each member of the group in total exports of that group. The countries included in the aggregate of developing countries (fixed weights in parentheses) are South Korea (32 percent), Mexico (36 percent), and Taiwan (32 percent). The countries included in the bloc of other OECD countries are Austria (5 percent), Belgium (15 percent), France (26 percent), Italy (18 percent), Netherlands (17 percent), Norway (4 percent), Sweden (8 percent), and Switzerland (7 percent). For OPEC, P_s is the oil market price, in dollars.

Data for the relative price for imports of country k from country s, P_{ks}, are constructed as $P_{ks} = e_s(1+\tau)P_s/P_{yk}$, where P_{yk} is the GNP Deflator with 1972 = 1 and τ is an ad-valorem tariff rate. In view of data difficulties, the paper assumes that the GNP/GDP deflator for the Rest of OECD, non-OPEC LDCs, and OPEC is their export price. Thus, for example, exports of the United States to OPEC use OPEC's export price as a proxy for OPEC's GNP price deflator. Because of data limitations, the data for the tariff rate are available only for the United States and are constructed as the share of tariff receipts in total U.S. imports.

Real income for Canada, Germany, Japan, the United Kingdom, and the United States is defined as real GNP measured in domestic currency. For both industrial and developing countries, real income is measured as a geometric mean of industrial production of the countries listed above using the same weights. Finally, in view of data difficulties, the paper assumes that OPEC's income equals OPEC's real exports.

The data sources are Bank of Canada; Bank of England; Bank of Japan; *Canadian Statistical Review,* published quarterly by Statistics Canada; Deutsche Bundesbank; *Lange Reihen der vierteljahrlichen volkswirtschaftlichen Gesamtrechnung für die Bundesrepublik Deutschland,* published quarterly by Deutsches Institute für Wirtschaftsforschung, Berlin; *Economic Trends,* published monthly by U.K. Central Statistical Office; National Income Accounts in *Survey of Current Business,* published monthly by U.S. Department of Commerce, Bureau of Economic Analysis; *International Financial Statistics* and the *Direction of Trade,* published by the International Monetary Fund; International Energy Agency.