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DIFFERENTIAL INFORMATION AND EXCESSIVE VOLATILITY IN FINANCIAL MARKETS*

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It is analysed whether risk averse agents possessing different information have an incentive to trade in a zero-sum market. The key to generate trading in a zero-sum speculative market is whether expectations are »homogenized» through the trading process. If not, trading will take place and all agents expect to be able to exploit private information not fully revealed by market prices to make a speculative profit. The existence of a rational expectations equilibrium with heterogenous expectations is proven to exist, and shown to imply excessive volatility of prices and trading volumes.

1. The problem

Recent empirical research has rendered support to the view previously based on casual empiricism that capital markets seem to be excessively volatile as concerns both prices and trading volumes relative to what should be expected from hitherto known capital market models (see e.g. Shiller (1981) and Leroy (1989)). This has spurred a growing literature trying to account for these findings as a result of either speculative bubbles (see e.g. Blanchard and Fischer (1989) ch. 5) or different forms of »irrationality» in capital markets[1] known as »fads» (see e.g. Shiller (1984) and Summers (1986)) or »noise traders» (see e.g. DeLong et al. (1990)).

It is usually asserted that it is impossible to account for these volatility phenomena within a setting where all traders act rationally due to the socalled non-trade theorem (see Milgrom and Stokey 1982 and Tirole 1982). According to this proposition, purely speculative trading is not possible among rational risk-averse agents since they all perceive that trading is a zero sum game. A willingness to trade will signal access to information not possessed by others, since this is perceived by all, nobody will be willing to trade since losses are to be expected. Under these circumstances, arrival of new information will only cause price adjustment but no trade. With differences in information ruled out as a potential reason for trade in financial markets, one is left with trade caused by incentives to hedge against risk, and such precautionary actions are not

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[1] *This is related to recent sunspot-business cycle models (see e.g. Blanchard and Fischer (1989) for an introduction) generating fluctuations by having trade to depend on variables unrelated to market fundamentals.*

likely to account for excessive volatility in capital market.

This paper reconsiders this issue and shows that trade is possible in a zero-sum market when agents are differently informed.[2] The key to generate trading in a zero-sum speculative market is whether expectations are »homogenized» through the trading process. If not, trading will take place and all agents expect to be able to exploit private information not fully revealed by market prices to make a speculative profit. The existence of a rational expectations equilibrium with heterogenous expectations is proven to exist, and to imply excessive volatility of prices and trading volumes.

The present analysis renders, thus, support to the view of capital market endorsed by Working (1949), Keynes (1936) and Hirshleifer (1975) that the pricing of financial assets need not be closely related to market fundamentals. In particular, speculative profits can be made by acquiring new information, and trading may, thus, to a large extent be caused by speculation based on differences in information, rather than an incentive to hedge against risk. A welfare implication being that the incentives to acquire information may be excessive.

The paper is organised as follows. The zero-sum model of a speculative market is set-up in section 2, the working of this market for exogenous expectations is analysed in section 3, while a differential information structure is endogenously related to the information revealed by prices in section 4. Implications for price and volume volatility are considered in section 5, while section 6 gives a few concluding remarks.

2. *A zero-sum market*

There are N investors indexed by $i = 1,2, \ldots N$ in the market, each having initial wealth $\overline{W}_i$. The agents can invest in either a riskless asset with zero return[3] and a current price normalized to one or a risky asset with a current price P and a period t+1 price P^*. The period t+1 price P^* is assumed to be random and exogenously determined to the model. The i'th agent's portfolio decision problem must satisfy the following constraint:

$$(1) \qquad B_i + PS_i = \overline{W}_i$$

where B_i and S_i are the i'th agent's holding of the riskfree asset and the riskless asset, respectively. Restrictions on short selling of either the risky or the riskfree asset are assumed to be absent, and there is no possibility of default on these and, thus, no possibility of bankruptcy.

Period t+1 wealth is given by

$$W_i = B_i + P^*S_i$$

or using (1) as

$$(2) \qquad W_i = \overline{W}_i + (P^* - P)S_i$$

Agents are assumed solely to be concerned with period t+1 wealth, and to maximize expected utility conditional on the available information I_i (to be specified below). The analysis is confined to a two-period model in order to avoid bubble-phenomena and the like arising in dynamic models.[4]

The profit (Π_i) from investing S_i ($\gtreqless 0$) in the risky asset is given by

$$(3) \qquad \Pi_i = (P^* - P)S_i \qquad i = 1, \ldots N$$

We assume market clearing in the market, i.e.

$$(4) \qquad \sum_{i=1}^{N} S_i = 0$$

Given (1) and (2), we find another way of expressing that the market is a zero-sum market since

$$(5) \qquad \sum_{i=1}^{N} \Pi_i = (P^* - P) \sum_{i=1}^{N} S_i = 0$$

i.e. in the aggregate no profits can be made by trading in this market.

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[2] *Grossman and Stiglitz (1976, 1981) generate trade among differently informed agents in a model with an exogenous supply of the risky asset, i.e. a non zero-sum market.*

[3] *This assumption is not essential for the results to be derived.*

[4] *The problem of differential information in explicit dynamic models is rather subtle and has not been much analysed. Exceptions are Futia (1981) and Friedman and Aoki (1986).*

3. *Market equilibrium for exogenous expectations*

Homogeneous Information

Consider a market where all traders possess the same information on the future value/price of the asset. Let us first assume that traders are risk-neutral. The i'th agent will in this case find it worthwhile to participate in speculative trading if

$$(6) \quad E(\Pi_i \mid I) = (E(P^* \mid I) - P)S_i \geq 0$$

where I is the common information set for all traders.

If we consider the total expected profits by all traders, we find

$$(7) \quad \sum_{i=1}^{N} E(\Pi_i \mid I) = (E(P^* \mid I) - P) \sum_{i=1}^{N} S_i = 0$$

Since (7) holds in any equilibrium, we find by use of (6) that

$$(8) \quad E(\Pi_i \mid I) = 0 \qquad \text{for all } i$$

(8) says that nobody can expect to make a profit from trading in the speculative market and, hence, risk-neutral agents are indifferent as whether to trade or not. It is easily verified that arbitrage implies that $E(P^* \mid I) = P$, and that the amount individual agents are willing to trade at this price is indeterminate. This is just another way of expressing that when all traders agree on the expected price of the asset, risk-neutral agents are indifferent as to the amount traded. In this case the price adjusts instantaneously to new information, but this is not directly related to the trading process.

If we allow for risk-averse agents, we easily find that they would never trade in the market since[5]

$$(9) \quad E(\Pi_i \mid I) = 0$$

and

$$VAR(\Pi_i \mid I) = S_i^2 VAR(P^* \mid I) > 0$$

In conclusion, we find that with homogenous information in a purely speculative market risk-averse traders do not trade; risk-neutral agents may trade, but they do not expect any gain from their trade.

Heterogenous information

Assume risk-averse agents with constant absolute risk-aversion utility functions

$$(10) \quad V_i = -\exp(-a_i W_i) \qquad a_i > 0$$

The expected utility conditioned on available information (I_i) can be shown to be a monotone increasing function of

$$(11) \quad \tilde{V}_i = E(W_i \mid I_i) - \frac{a_i}{2} VAR(W_i \mid I_i)$$

Maximizing (11) subject to (5), we find

$$(12) \quad S_i = \frac{E(P^* \mid I_i) - P}{a_i \, VAR(P^* \mid I_i)} \gtreqless 0$$

The agents are assumed to have access to two information sources, i) private information (to be specified below) and ii) the current asset price P.

The participation constraint for the agent is that expected utility is non-negative. Inserting (12) into (11), we find

$$\tilde{V}_i = E(\Pi_i \mid I) - \frac{a_i}{2} VAR(\Pi_i \mid I_i)$$
$$= \frac{1}{2}\left(\frac{E(P^* \mid I^i) - P}{a_i \, VAR(P^* \mid I_i)}\right)^2 \geq 0$$

This shows that agents expect to be able to exploit their private information if it is not included in the market price either by selling or buying the risky asset.

The equilibrium condition implies that

$$\sum_i \lambda_i [E(P^* \mid I_i) - P] = 0$$

where

$$\lambda_i = (a_i \, VAR(P^* \mid I_i))^{-1}$$

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[5] *We presume that I does not represent full information, that is, $VAR(\bar{p} \mid I) > 0$.*

Solving for the equilibrium price, we find

$$P = \frac{\sum_i \lambda_i E(P^* \mid I_i)}{\sum \lambda_i} = \sum_i \rho_i E(P^* \mid I_i)$$

where

$$\rho_i = \lambda_i / \sum_j \lambda_j$$
$$\sum_i \rho_i = 1$$

This expression shows the well-known result that the asset price is a weighted average of the expected future price of the asset.

In this zero-sum market we have absence of speculative trading if

$$E(P^* \mid I_i) = \overline{P} \;\forall\, i$$

since this implies that

$$P = \sum_i \rho_i \overline{P} = \overline{P}$$

and, hence, no trading

$$S_i = 0 \;\forall\, i$$

We can, thus, conclude that if agents hold the same expectations, there is no trade in a zero-sum speculative market but if they do not hold the same expectations, there will be trade.

We shall next ask whether differences in expectations can survive in equilibrium. Not to bias our results, we shall specifically ask whether that is possible in a rational expectations equilibrium where agents are able to exploit the information revealed by the market price consistently with the underlying capital market model. Hence, by employing the rational expectations equilibrium concept we eliminate misperceptions of price signals as a reason for speculative trading.

4. *Endogenous expectations — differential information*

The information available to any trader prior to trading is summarized by the following forecast of P^*

$$P_i = P^* + \varepsilon_i \quad , \quad \varepsilon_i \sim N(0, \sigma_i)$$

The variance-covariance matrix $[\sigma_{ij}]$ for the individual forecast errors is denoted Ω, where

$$\sigma_{ii} = \begin{cases} \sigma_{ij} = \sigma_i^2 \text{ (variance)} & \text{for } i = j \\ \sigma_{ij} \quad \text{(covariance)} & \text{for } i \neq j \end{cases}$$

Agents are Bayesians updating their prior estimates of P^* by the information revealed by the market price. The market price is conjectured to be determined as

$$(13) \quad P = \sum_i x_i P_i$$

where

$$\sum_i x_i = 1$$

Hence, the information on P^* revealed by the market price can be written as

$$P = P^* + \varepsilon_M \quad , \quad \varepsilon_M \sim N(0, \sigma_M^2)$$

where

$$\varepsilon_M = \sum_i x_i \varepsilon_i$$

Using the information revealed by the market price, the posterior forecast of P^* by agent i is given as (see appendix)

$$(14) \quad E(P^* \mid P_i, P) = \alpha_i P_i + (1 - \alpha_i) P$$

where

$$\alpha_i = \frac{\sigma_M^2 - C_{iM}}{\sigma_M^2 + \sigma_i^2 - 2C_{iM}}$$
$$\sigma_M^2 = \sum_i \sum_j x_i x_j \sigma_{ij}$$
$$C_{iM} = \sum_j x_j \sigma_{ij}$$

and the conditional variance is given by

$$(15) \quad VAR(P^* \mid P_i, P) = \beta_i = \frac{\sigma_M^2 \sigma_i^2 - C_{iM}^2}{\sigma_M^2 + \sigma_i^2 - 2C_{iM}}$$

The private information P_i reflects belief about P^* before (prior to) any information from the market price has been obtained, whereas $E(P^* \mid P_i, P)$ reflects the updated belief about P^* after (posterior to) extracting information from the market price P. That is, (14) shows how the private information about P^* is combined with the market information about P^*. Similarly, (15) shows how the two information sources are combined to yield a conditional variance of the period t + 1 price P^*. It is seen that the information content of the market price is endogenously determined since it affects trade and, thus, the market price.

Inserting (14) and (15) into (12), we find the demand for the risky asset to be given by

$$(16) \quad S_i = \frac{\alpha_i}{a_i\beta_i}(P_i - P)$$

The trading in the risky asset is assumed to be a zero-sum game, i.e. there is no exogenous supply of the risky asset. The equilibrium condition for risky asset is, therefore,

$$(17) \quad \sum_i S_i = 0$$

Summing over (16) and using (17), we find the equilibrium price to be[6]

$$(18) \quad P = \sum_i \left(\frac{\alpha_i}{a_i\beta_j} / \sum_j \frac{\alpha_j}{a_j\beta_j}\right) P_i$$

For the price equation (13) to be consistent with the resulting price equation (18), we must have that

$$(19) \quad x_i = \frac{\alpha_i}{a_i\beta_i} / \left(\sum_j \frac{\alpha_j}{a_j\beta_j}\right) \qquad i = 1, 2, \dots N$$

It is seen from (19) that

$$(20) \quad \sum_i x_i = 1$$

(19) is not a closed form solution to the model since the α_i's and the β_i's depend on the x_i's, cf. (14) and (15). The equation system (19) constitutes a nonlinear system of equations in $(x_1, x_2, \dots x_N)$ and the solution corresponds to the equilibrium of the economy.

[6] *Implicitly assuming $\sum_j \frac{\alpha_j}{a_j\beta_j} \neq 0$, cf. proof of theorem 1.*

Theorem 1: There exists a rational expectations equilibrium to the model unless $a_i = a \; \forall \, i$, $\sigma_i^2 = \sigma^2 \; \forall \, i$ and $\sigma_{ij} = 0 \; \forall \, i, j \; (i \neq j)$.

Proof: See appendix.

This result essentially says that no equilibrium exists if all agents are structurally identical in the sense of having the same objective functions and independent signals generated by the same distribution (of course the actual realizations and thus private information differ). In this case it does not matter who receives a given piece of information since they will all react to it in the same way. If so, the price would reveal all available information leading to homogeneous expectations and thus no trade, precluding the existence of an equilibrium.

Given the existence of equilibrium, the interesting question is whether expectations are homogenized through the information revealed by prices and thus whether there would be any trade.[7]

Theorem 2: The rational expectation equilibrium existing under the assumption of Theorem 1 is not fully revealing.

Proof: See appendix.

Having proved that agents in general hold different expectations in equilibrium, we turn next to the implications for volatility in prices and trading volumes.

5. Volatility in prices and trading volumes

The volatility of the market price can be evaluated by use of (13), implying that

$$P = P^* + \varepsilon_M$$

and hence,

$$\mathrm{Var}(P) = \mathrm{Var}(P^*) + \mathrm{Var}(\varepsilon_M)$$

[7] *This result may seem to contradict Grossman (1976), but this is not the case since an exogenous supply of the risky asset is assumed in contrast to the present case of a purely speculative market.*

or

(21) $\mathrm{Var}(P) > \mathrm{Var}(P^*)$

The equilibrium price under differential information fluctuates, thus, more than the full information price.[8]

The reason for this excess volatility is to be found in the imperfect information on the future value of the asset which at the market level is captured by the term ε_M. This term may be interpreted as aggregate misinformation or noise in the sense that it is a reflection at the aggregate level of the noise in the individual signals on the »true» value of the assets.

Noise in this sense does not necessarily carry a negative implication since it is the result of individuals' endeavors to collect information, and it so happens that insufficient information is available to predict the future price of the asset perfectly. Noise is, thus, not necessarily related to irrelevant factors or traders living in a world of their own, but simply a reflection of the fact that traders may know something but not everything about the future. The agent and the market are doing their best, but it happens not to be good enough, and it leaves agents with different perceptions about the future.

In the present model the excessive price volatility is related to volatility in traded volumes since there would be no trading under full information, but trading as a result of differential information and, thus, heterogenous expectations in equilibrium.

A measure of trading is

(22) $$\sum_i S_i^2 = \sum_i \frac{(E(P^* \mid I_i) - P^*)^2}{a_i \mathrm{Var}(P^*(I_i))} > 0 \text{ if } \exists\, i: \quad E(P^* \mid I_i) \neq P$$

That is, trading is excessive relative to the zero-trade under full information.

Volatility tests

The logic of volatility tests is as follows (Shiller, 1981). In a particular form of a capital market model the market price P_t is given as $P_t = E_t(P_t^*)$, that is, as the mathematical expectation conditioned on all available information at time t on the full information price P_t^*. The forecast error is now defined as

$$U_t = P_t^* - P_t$$

and since the forecast error U_t must be uncorrelated with the forecast P_t, it follows that the covariance between U_t and P_t is zero. Hence,

$$\mathrm{Var}(P_t^*) = \mathrm{Var}(P_t) + \mathrm{Var}(U_t)$$

or

(23) $\mathrm{Var}(P_t^*) > \mathrm{Var}(P_t)$

which is another way of saying that the actual price should fluctuate less than the hypothetical price prevailing under full information. Empirical studies have come up with the reverse inequality to that of (23).

It is seen that the preceding logic presumes the model to be informatively efficient, that is, by observing the current market price you obtain a sufficient statistic on all information relevant for predicting the price of the asset. Volatility tests, thus, depend on a joint hypothesis involving the particular formulation of a capital market model and an assumption of informationally efficient markets.

The latter assumption is crucial in accounting for the fact that the present model has a different volatility implication than that presumed in most empirical analysis. The present model is one where the market is not informationally efficient. The market price contains some information but not enough to render the information content of private information signals superfluous. Hence, the market price is a weighted average of the price expected by traders conditional on their private information and the information revealed by the market price.

It may be useful to note that the present model, of course, has the implication that the forecast error is uncorrelated with the infor-

[8] *Black and Tonks (1990) show how asymmetric information may create excessive market volatility in a setting with informed, uninformed and noisy traders, and a possibility to retrade upon the arrival of new information.*

mation used for forecasting $I_i = \{P, P_i\}$. To see this, notice that the forecast error of agent i is

$$\begin{aligned} U_i &= P^* - E(P^* \mid I_i) \\ &= P^* - \alpha_i P_i - (1-\alpha_i)P \\ &= P^* - P - \alpha_i P_i - \alpha_i P \\ &= \varepsilon_M - \alpha_i P_i - \alpha_i P \end{aligned}$$

Hence,

$$E[U_i \mid I_i] = E(\varepsilon_M \mid I_i) - \alpha_i P_i - \alpha_i P$$

But

$$\begin{aligned} E[\varepsilon_M \mid I_i] &= E[P^* \mid I_i] - P \\ &= \alpha_i P_i + \alpha_i P \end{aligned}$$

and it follows that the forecast error is not correlated with the available information.

$$E[U_i \mid I_i] = 0$$

However, if it is considered how forecast errors are related to the price, we find

$$E[U_i \mid P] \neq 0$$

since all relevant information is not captured by the market price, that is, the market is not informationally efficient.

6. Concluding remarks

It has been shown that trade will take place in a purely speculative market where risk-averse agents hold heterogenous expectations. This is sustainable in a rational expectations equilibrium implying that imperfect information about market fundamentals (the future price) may cause excessive volatility in prices and trading volumes. Excessive market volatility in financial markets may, thus reflect that markets are not informationally efficient.[9] To some extent the market can be said to be trading on noise in the sense of the imperfection of private information signals. The noise is not, however, a reflection of any irrational behaviour on the part of any private investors, but simply a reflection of the fact that agents have different perceptions.

Major shortcomings of this paper are the static nature of the model as well as assuming the most in respect to the ability of agents to process information. These assumptions preclude interesting dynamics arising from learning over time. However, in the present context where the aim is to show that differential information may create excessive volatility in financial markets, these assumptions may be seen as a strength. This is so because they enable us to show that even under the most idealized assumptions the market system has problems with respect to aggregation and dissemination of information.

[9] *See Bray (1985) and Andersen (1985) for a survey of theoreticl models of informational efficiency in capital markets.*

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Appendix

I. Derivation of Posterior Mean and Variance[10]

Let

$$P_i = P^* + \varepsilon_i$$

and

$$P = P^* + \varepsilon_M$$

P_i and P are bivariate normally distributed with mean (P^*, P^*) and variance-covariance matrix

$$V_i = \begin{bmatrix} \sigma_i^2 & C_{iM} \\ C_{iM} & \sigma_M^2 \end{bmatrix}$$

Let

$$\rho_i = \frac{C_{iM}}{\sigma_M \sigma_i}$$

$$k_i = \frac{\sigma_M}{\sigma_i}$$

We know that

$$f(P^* \mid P_i) \propto \exp\{-(P - P_i)^2/2\sigma_i^2\}$$

[10] *This appendix is based on Jaffee and Winkler (1976).*

The likelihood function is proportional to $f(P \mid P^*)$ which is normally distributed with mean $P^* + \rho_i k_i (P_i - P^*)$ and variance $k_i^2 \sigma_i^2 (1 - \rho_i^2)$

$$f(P \mid P^*, P_i) \propto \exp\{-[P - (P^* + \rho_i k_i (P_i - P^*))]^2/2k_i^2\sigma_i^2(1 - \rho_i^2)\}$$

Thus, the posterior distribution is

$$f(P^* \mid P, P_i) = f(P^* \mid P_i) f(P \mid P^*, P_i)$$

This can (cf. Jaffee and Winkler (1976)) be written

$$f(P^* \mid P, P_i) \propto \exp\{-(P - E(P^* \mid P, P_i))^2/2\beta_i\}$$

where

$$E(P^* \mid P, P_i) = \frac{k_i(k_i - \rho_i)P_i + (1 - \rho_i k_i)P}{k_i^2 - 2\rho_i k_i + 1}$$

$$\beta_i = VAR(P^* \mid P, P_i) = \frac{k_i^2(1 - \rho_i^2)\sigma_i^2}{k_i^2 - 2\rho_i k_i + 1}$$

Inserting we find

$$E(P^* \mid P, P_i) = \alpha_i P_i + (1 - \alpha_i)P$$

where

$$\alpha_i = \frac{\sigma_M^2 - C_{iM}}{\sigma_M^2 + \sigma_i^2 - 2C_{iM}}$$

and

$$\beta_i = \frac{\sigma_M^2 \sigma_i^2 - C_{iM}^2}{\sigma_M^2 + \sigma_i^2 - 2C_{iM}}$$

II. Proof Theorem 1

From (19) we have that

$$\text{(A-1)} \quad x_i = z_i / \sum_j z_j \quad , \quad \sum_i x_i = 1$$

under the assumption that

$$\text{(A-2)} \quad \sum_i z_i \neq 0$$

where

$$z_i = \frac{\alpha_i}{a_i \beta_i} = \frac{\sigma_M^2 - \sigma_{iM}}{a_i(\sigma_M^2 \sigma_i^2 - \sigma_{iM}^2)}$$

$$\sigma_M^2 = \sum_i \sum_j x_i x_j \sigma_{ij}^2$$

$$\sigma_{iM} = \sum_j x_j \sigma_{ij}$$

Hence, the equation system (A-1) can be written as

$$x_i = f_i(x_i, \ldots x_n) \qquad i = i \ldots n$$

or in a more compact form simply as

$$x = F(x)$$

where $x = (x_i, x_2 \ldots x_n)$

The z_i variable can be written

$$z_i = \frac{\frac{1}{\sigma_i^2} - \frac{\rho_{iM}}{\sigma_i \sigma_M}}{(a_i - \rho_{iM}^2)}$$

We have that

(i) $0 < \sigma_M^2 \leq \sum_i \sigma_i^2$

(ii) $-1 \leq \rho_{iM} \leq 1$

Hence, it follows that $x_i \in [\underline{x}_i, \overline{x}_i]$. Note that $\rho_{iM} \rightarrow \pm 1$ implies that $x_i \rightarrow 1$.

Hence, x is defined on the bounded set $\chi = \prod_{i=1}^{n} [\underline{x}_i, \overline{x}_i]$ and it follows that F is defined on a compact and convex set.

F is a continuous mapping from χ into itself and by Brouwer's fixed point, then it follows that there exists a $x^* \in \chi$ for which $x^* = F(x^*)$.

This presumes (A-2) to be fulfilled and we next have to check whether the solution violates this condition.

Given that $0 < a_i < \infty$ and $0 < \sigma_M^2 \sigma_i^2 - C_{iM}^2 < \infty$, we have as a necessary and sufficient condition for (A-2) to be violated that

(A-3) $\sigma_M^2 = C_{iM} \; \forall \, i$

The solution to (A-1) must thus not imply that (A-3) is fulfilled. Since (A-1) depends on risk-aversion and the latter does not, there will not, in general, be any coincidence of the two conditions except in pathological probability zero events.

However, if agents have the same aversion to risk ($a_i = a \; \forall \, i$), (A-1) becomes independent of risk-aversion and this argument is not necessarily valid. Expression (A-3), is seen to be fulfilled in this case if $\sigma_i^2 = \sigma^2 \; \forall \, i$ and $\sigma_{ij} = 0 \; \forall \, i, j \; (i \neq j)$ which from (A-1) implies $x_i = x \; \forall \, i$, but this leads to a contradiction since (A-1) is only valid when (A-3) is not fulfilled.

III. Proof Theorem 2

A fully revealing equilibrium is characterized by

$$E(P^* \mid I_j) = E(P^* \mid I_i) \; \forall \, i, j \; (i \neq j)$$

This implies

$$P = E(P^* \mid I_i) \; \forall \, i$$

The expectation of P^* is given by

$$E(P^* \mid I_i) = \alpha_i P_i + (1 - \alpha_i) P$$

Hence, for the equilibrium to be fully revealing, we must have

$$\alpha_i (P_i - P) = 0 \; \forall \, i$$

which, in general, requires α_i equal to zero for all i.

From (14) we find that $\alpha_i = 0 \; \forall \, i$ if

$$\sigma_M^2 = C_{iM}$$

but this is the condition precluding the existence of equilibrium, hence the equilibrium is non-revealing.