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WHY THERE IS A LOWER BOUND ON THE CENTRAL BANK'S FOREIGN RESERVES*

ALPO WILLMAN

Bank of Finland, P.O. Box 160
SF-00101 Helsinki, Finland

This article examines the implications for the balance of payments of imposing a cash-in-advance constraint on financial market transactions. I show that with a welfare-maximizing government this constraint introduces a lower bound on the central bank's net foreign reserves; the depletion of the net foreign reserves below zero results in a welfare loss. I further show that either the private or public sector solvency constraint is violated, if the growth rate of domestic credit expansion exceeds a critical magnitude, which is somewhat below the foreign interest rate. Unlike in Buiter (1987), the violation of the solvency constraint does not depend on the way the credit expansion is used. The timing and the size of the speculative attack associated with an anticipated exchange rate regime shift are, however, dependent on the way the credit expansion is injected into the economy.

1. Introduction

In the literature on balance-of-payments crises the assumption that there exists a minimum level of net foreign reserves, below which they cannot be depleted, is of key importance from the point of view of analysis.[1] It is this assumption which leads to the analogy between balance-of-payments models and non-renewable resource models of speculative attack. However, the justification for this assumption can be challenged; a central bank facing a perfect world capital market can always create gross foreign reserves by borrowing and, hence, negative net reserve positions for defending a fixed exchange rate regime are feasible. Thus, one can question whether there is any lower bound on net foreign reserves.

Obstfeld (1986) showed that, in an idealized world, where taxes are not distortionary, a central bank's net foreign reserves can become infinitely negative without violating the consolidated government sector's intertemporal budget constraint, if the growth rate of domestic credit is below the world interest rate. Only if the growth rate of domestic credit expansion exceeds the world interest rate is the solvency constraint violated and does there exist a limit to the central bank's external borrowing.

Later Buiter (1986 and 1987) showed that the depletion of net foreign reserves can continue boundlessly even if the growth rate of domestic credit expansion is above the foreign interest rate. This is because the fiscal authori-

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[1] *See, for instance, Salant and Henderson (1980), Salant (1983), Krugman (1979), Flood and Garber (1984), Calvo (1987) and Willman (1987, 1988a, b).*

ty can use its credit from the central bank to accumulate foreign bonds. In this case the increase in domestic credit expansion results in a decrease in the central bank's net foreign reserves without any effect on the consolidated government sector's net foreign debt and solvency.

Hence, only if the growth rate of domestic credit expansion is above the foreign interest rate and the credit expansion is used to finance the government deficit is there a lower bound on the level of the central bank's net foreign reserves. In this case the continuous depletion of the central bank's net foreign reserves results, sooner or later, in a balance-of-payments crisis. However, a balance-of-payments crisis should be only a manifestation of a government solvency crisis, originating from a primary government deficit (net of interest on debt). The reason why in the real world, balance-of-payments crises have also occurred without the existence of persistent government budget deficit problems remains to be explained.

A conclusion which can be drawn on the basis of Obstfeld's and Buiter's studies mentioned above is that the world which their analytical framework describes is too idealized (i.e. no kind of friction exists) to be able to provide a rational explanation for the existence of the lower bound on the level of a central bank's net foreign reserves. With this consideration in mind, I extend their analysis by assuming a cash-in-advance constraint for asset transactions. This introduces a form of friction into the model.

Although a cash-in-advance constraint is a fairly conventional assumption in commodity market transactions, Helpman and Razin (1985) were the first to introduce this constraint into financial market transactions.[2] In particular, they analyzed the case of a debtor country running a current account deficit which has to accumulate foreign currency in advance in order to repay its foreign debt. Equally well, a cash-in-advance constraint could be faced by the lender (at the time of purchasing assets), as in the papers by Grilli and Roubini (1989) and Lucas (1990). However, for notational simplicity I assume for most of this paper, like Helpman and Razin (1985), that it is cash-in-advance constraints faced by borrowers which determine domestic demand for foreign currency. Further, as in all the cash-in-advance literature, I assume that the receiver of a payment (i.e. the receiver of a debt service payment or the seller of bonds) can adjust his/her portfolio at the instant the payment is received.

I show in this article that the cash-in-advance constraint in financial market transactions introduces a lower bound on the central bank's net foreign reserves. In addition, it is suggested that the critical level of the net foreign reserves is zero. The latter result is not, however, due to the violation of the public sector solvency constraint but rather to the fact that the depletion of the central bank's net foreign reserves below zero decreases welfare. This result is not dependent on the purpose for which the credit expansion is used. Hence, a balance-of-payments crisis need not to be a manifestation of a public sector solvency crisis.

Further, unlike in Buiter (1986 and 1987), a sufficiently large domestic credit expansion always results in the violation of either the public sector or the private sector solvency constraint. Finally, I show that the effects of the cash-in-advance constraint in financial transactions on the dynamics of the endogenous exchange rate regime shift are quite minor.

The paper is organized as follows. In section 2 I derive a model framework with choice theoretic foundations and in section 3 the intertemporal public sector and economy-wide budget constraints are derived. In section 4 the importance of the cash-in-advance constraint for the dynamics of the model is examined. In section 5 the issue of the lower bound on the central bank's net foreign reserves is discussed. The implications of the cash-in-advance constraint in financial transactions for the endogenous shift from the fixed to the floating exchange rate regime is studied in section 6. Finally, a few brief conclusions are offered in section 7.

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[2] *Helpman and Razin (1985) note that it was also recognized in earlier monetary literature that money is absorbed in financial transactions [see e.g. Friedman (1974)]. Later, Lucas (1990) points out that a significant proportion of money holdings is managed by financial intermediaries, which suggests that cash is required for financial market transactions.*

2. The Model framework

I employ the full employment, single good small open economy model. The good produced and consumed is non-storable. The model includes two domestic sectors, i.e. the household sector and the government sector.

2.1 The household sector

The special feature of the model is that there are two kinds of cash balances; cash balances needed for financial transactions and cash balances which satisfy the need for goods market transactions and other motives for holding money. As for the latter component of cahs balances, I do not specify the exact transaction technology but rather, include money directly in the utility function of the representative household.[3] If, instead, the Clower constraint in its simple form were applied to both goods market and financial market transactions, the main results of this paper would not change significantly.[4] The main advantages of the practice adopted in this paper are that the velocity of money is interest-elastic and the transaction concept determining the demand for cash balances includes both goods market and financial market transactions.

The representative household maximizes a discounted sum of future instantaneous utilities. Direct instantaneous utility is obtained from the consumption of the single commodity and from real money balances m.

To obtain a explicit solution, I follow Obstfeld (1986) and assume that the instantaneous utility function is logarithmic. Hence, the household's utility at time t is:

$$(1) \quad U(t) = \int_t^\infty e^{-\rho(v-t)}[\log c(v) + \gamma \log m(v)]dv; \; 0<\gamma<1$$

where ρ is the rate of subjective time preference.

The representative household can issue home currency denominated bonds D_p and buy and sell foreign currency denominated bonds F_p. I assume throughout that $D_p \geq 0$. All bonds are consols of infinite maturity. The interest rate on domestic bonds is i and on foreign bonds i^*.

The following cash-in-advance constraints for the financial market transactions of the representative household are imposed: The transaction technology is such that debt-service costs must be paid in the form of cash, i.e. the following cash-in-advance constraints are faced by the representative household:

$$(2a) \quad N(t) \geq \int_t^{t+\tau} i(v)D_p(v)\,dv \approx \tau i(t)D_p(t) = \alpha(t)D(t)$$

$$(2b) \quad M_p^*(t) \geq -\int_t^{t+\tau} i^*(v)F_p(v)\,dv \approx -\tau i^*(t)F_p(t) = \beta(t)F_p(t), \text{ if } F_p \leq 0$$

$$M_p^*(t) \geq \int_t^{t+\tau} \dot{F}_p(v)\,dv = F_p(t+\tau) - F_p(t) \quad \text{if } F_p \geq 0 \text{ and } \dot{F}_p \geq 0$$

where N is domestic and M_p^* foreign currency cash balances needed for financial transactions. τ is the time interval before a financial transaction during which a payment is held in the form of cash. It can be seen that at the limit with τ approaching zero no cash balances are needed for financial transactions.

[3] *Incorporating money directly in the utility function has a long history starting from Samuelson (1947). An alternative way to introduce money into the analysis would be to specify explicitly the transaction technology and drop money from the utility function. However, these two approaches need not be exclusive of each other. For instance, in the cases of the Clower and the Baumol-Tobin transaction technologies, Feenstra (1986) showed that the maximization problem could be rewritten with money in the utility function and ignoring the transaction costs [see also the discussion by Blanchard and Fischer (1989, chapter 4, p. 192)].*

[4] *The Clower constraint without the Baumol-Tobin transaction technology would imply that balance-of-payment crises are pure real economic phenomena [for an example of this, see Calvo (1986)]. This is not supported by the conventional wisdom that balance-of-payments crises are essentially financial market phenomena. This is the main reason why the Clower constraint in its simple form is not adopted here. This problem would disappear if the Clower constraint with the Baumol-Tobin transaction technology were applied. However, besides making the analysis unnecessarily cumbersome, the extension of the transaction concept to include financial transactions would cause extra difficulties in formulating the model framework.*

As $D_p \geq 0$, domestic currency denominated cash balances are needed only for debt service payments.[5, 6] According to (2b), foreign currency denominated cash balances, in turn, are also needed for purchases of foreign bonds.[7] However, the cash-in-advance constraint for purchases of bonds plays an insignificant role from the point of view of the existence of a minimum level of net foreign reserves. Consequently, in most of what follows, I assume that cash is needed only for debt service purposes, i.e. $M_p^* = 0$, if $F_p \geq 0$. I shall return to this issue in section 6 in the context of an endogenous exchange rate regime shift.

Now the representative household faces the following budget identity:[8]

$$(3) \quad \dot{M} + \dot{N} + s\dot{M}_p^* + s\dot{F}_p - \dot{D}_p = i^*sF_p - iD_p + s(y + g - c)$$

where M is nominal money balances (equalling sm), y is real income, g is real net lump-sum transfers from the government to the representative household and s denotes the exchange rate. As I use a single good, small open economy model and the foreign price level is set equal to unity, the exchange rate s also represents the domestic price level.

Let us define real assets as $a = m + n + M_p^* + F_p - d_p$, where m, n and d_p denote real values of the variables M, N and D_p. Equation (3) can now be written as:

$$(4) \quad \dot{a} = i^*F_p - id_p - (m + n - d)e + y + g - c$$

where e denotes the rate of depreciation, i.e. $e = \dot{s}/s$. If cash balances are return-dominated by bonds, i.e. i and i^* are positive, exact equality holds in (2a) and (2b) and (4) can be written in the form:

$$(5) \quad \dot{a} = ra - (r + e)m - [(1 + \beta)r - i^*]F_p + y + g - c$$

where r is defined as follows:

$$(6) \quad r = i/(1 - \alpha) - e$$

r measures the representative household's real alternative costs of debt issuance. It can be seen that these costs are greater than the domestic real interest rate, i.e. $r > i - e$. This is because domestic debt issuance is always associated with an increase in non-interest-bearing cash balances N needed for debt service.

Integrating (5) and imposing the no-Ponzi-game condition, $a(t)e^{-rt} \to 0$ as $t \to \infty$, we note that under perfect foresight the following budget constraint must hold:

$$(7) \quad a(t) \geq \int_t^\infty e^{-\int_t^v r(u)\,du}\{[(1 + \beta)r - i^*]F_p + (r + e)m + c - y - g\}\,dv$$

Without the term associated with the foreign bonds F_p, equation (7) would be the familiar condition that the present value of future expenditure (i.e. consumption, c, plus the opportunity cost of holding money, $(r + e)m$) would be smaller than or equal to the present value of future receipts, $y + g$, plus the initial financial wealth a. However, this is also true in the present case, as the later analysis will show that the term multiplying F_p equals zero.

At each point of time t, the representative household chooses the paths of c, m and F_p which maximize (1) subject to (7), given a(t), (or equally subject to (5) and the no-Ponzi-game constraint).

Using the maximum principle, the Hamiltonian to be maximized is:

$$(8) \quad H = \log c + \gamma \log m + \lambda\{ra - (r + e)m - [(1 + \beta)r - i^*]F_p + y + g - c\}$$

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[5] *Like the simple Clower constraint in goods market transactions, when imposed in continuous time models, the cash-in-advance constraint for debt service payments in (2a and b) is only an approximation of actual payments. This does not, however, concern us here, because our results can be extended to a discrete-time-version of the model.*

[6] *As all bonds traded are consols of infinite maturity, the debt-service costs include only interest payments. Allowing maturities of bonds to be finite rather than infinite, would, in general, make the qualitative results presented in this paper quantitatively stronger but notationally the analysis would be more complicated.*

[7] *The way in which (2b) is written implicitly assumes that the representative household does not both buy and sell foreign bonds during the same interval τ. If we wanted to take this possibility into account, the integral of sales of bonds during the period τ would have to be added to the right-hand side of the lower row of (2b).*

[8] *A dot (˙) over a variable denotes its time derivative. In what follows, the dependence of variables on time will be deleted if it is not strictly necessary for clarity.*

where λ is the costate variable, which is interpreted as the shadow value, in utility terms, of real assets. The implied first-order conditions (at the interior maximum) are:

(9) $\partial H/\partial c = 1/c - \lambda = 0$

(10) $\partial H/\partial m = \gamma/m - \lambda(r+e) = 0$

(11) $\partial H/\partial F_p = \lambda[(1+\beta)r - i^*] = 0$

(12) $\dot{\lambda} = \rho\lambda - \partial H/\partial a = -(r-\rho)\lambda$

Equations (9) and (10) define the paths of consumption, c, and money, m, in terms of the costate variable λ:

(13) $c(v) = 1/\lambda(v)$

(14) $[r(v) + e(v)]m(v) = \gamma/\lambda(v)$

and (12) defines the path of λ as follows:

(15) $\lambda(v) = \lambda_t e^{\int_t^v [\rho - r(u)]du}$

where λ_t denotes the value of λ at time t, the point of time when optimization is carried out. It can be seen that, if the rate of subjective time preference is permanently greater (smaller) than the interest rate r, then λ converges towards infinity (zero) and c and m converge towards zero (infinity). Only if $\rho = r(t)$ is the path of consumption constant. If, in addition, the rate of depreciation e is constant, then the amount of money also remains unchanged in time.

On the basis of conditions (11) and (6), the following relation for the domestic interest rate is obtained:

(16) $i = (1-\alpha)(r+e) = (1-\alpha)[i^*/(1+\beta) + e]$

Equation (16) gives a modified uncovered interest parity relation. With the cash-in-advance parameters α and β equalling zero, it reduces to the conventional uncovered interest rate parity relation.

Now the costate variable λ can also be solved in terms of the fundamentals of the model. On the basis of (16), equation (7) reduces to:

(17) $a = \int_t^\infty e^{-\int_t^v r(u)\,du}\{[(r+e)m + c - y - g\}\,dv$

Substituting the right-hand sides of equations (13) and (14) for c and m in (17) and using (15), we end up with the relation:

(18) $1/\lambda = [\rho/(1+\gamma)]\{a + \int_t^\infty e^{-\int_t^v r(u)\,du}[y+g]dv\} = [\rho/(1+\gamma)]W$

where W is the real wealth of the representative household at time t, when the optimization calculations are done. On the basis of (15), real wealth evolves over time as follows:

(19a) $\dot{W} = (r-\rho)W$,

or

(19b) $W(v) = W(t)e^{(r-\rho)(v-t)}$

Now the consumption function (13) and the demand for money function (14) can be written in the familiar form:

(20) $c(v) = [\rho/(1+\gamma)]W(v) = [\rho/(1+\gamma)]W(t)\ e^{(r-\rho)(v-t)}$

(21) $m(v) = [\rho\gamma/(1+\gamma)]W(v)/(r+e) = [\rho\gamma/(1+\gamma)][W(t)/(r+e)]\ e^{(r-\rho)(v-t)}$

The good produced and consumed is nonstorable and, hence, real income y equals domestic output. With domestic output remaining at a constant, full employment level and private consumption being obtained as a solution of the household's optimization problem, net exports x are obtained from the identity

(22) $x(t) = y(t) - c(t)$

2.2 *The government sector*

Besides the household sector, our economy includes the central bank (monetary authority) and the central government (fiscal authority). The profit function of the central bank is specified as follows:

(23) $P = i(D_p + D_g) + i^*s(A - M^*_{cb}) - i^*sL + \dot{s}R$

where P denotes profit, D_g is the central bank's credit to the central government, A is gross and R net official foreign reserves, M^*_{cb} is the part of gross reserves which is held in a non-interest-bearing form (cash balances), and L is the central bank's foreign currency denominated debt. The central bank's profit consists of the net interest income and capital gains (or losses) on the stock of net foreign reserves resulting from changes in the exchange rate.

The central bank's gross foreign reserves A equal the sum of net reserves and foreign currency denominated debt (R + L). The condition $A \geq M^*_{cb} \geq 0$ must hold by definition. The interest rate on gross reserves in interest-bearing form as well as on the central bank's foreign currency denominated debt is the world market rate i^*. The cash-in-advance constraint in debt service states that part of the central bank's gross reserves may be in the form of cash balances. The cash-in-advance constraint is of the following form:

(24) $M^*_{cb} \geq -\theta L + \Theta F_g$,

where

$\theta = 0$, if $L = 0$ and $\Theta = 0$, if $F_g \geq 0$
$= -\tau i^*$, if $L > 0$ $\quad = -\tau i^*$, if $F_g < 0$

and F_g is the stock of foreign bonds held by the central government.[9] Equation (24) states that part of the central bank's gross foreign reserves must be in the form of non-interest-bearing cash balances, if either the central bank or the central government has foreign currency denominated debt. As the holding of the cash balances in unprofitable, it is assumed in the following that in (24) exact equality holds, and that as long as net foreign reserves $R \geq M^*_{cb}$ the central bank does not borrow from abroad (i.e. $L = 0$ and $A = R$). This implies that in the regime with $L > 0$, all gross foreign reserves are in the form of cash balances, i.e. $A = M^*_{cb}$, and in (24) the difference, $M^*_{cb} - R$, can be substituted for L. Now equation (24) can be transformed into the form:

(25) $M^*_{cb} = [\theta/(1+\theta)]R + [\Theta/(1+\theta)]F_g$

The central bank's balance sheet is written as:

(26) $sR + D_p + D_g = M + N + K$, or

(26') $sR + (1-\alpha)D_p + D_g = M + K$

where K is the central bank's own capital. K evolves over time as follows:

(27) $\dot{K} = \dot{s}R$

Equation (27) states that capital gains arising from exchange rate movements are added to the central bank's own capital. The rest of the central bank's profit is paid to the central government. Hence, the central bank and the central government do not possess separate budget constraints and the supply of money is determined by the relation:

(28) $\dot{M} = (1-\alpha)\dot{D}_p + \dot{D}_g + s\dot{R}$

In the present model the role of the central government is to distribute lump-sum transfers to (or collect lump-sum taxes from) the household sector.[10] Its other revenues or outlays consist of net interests payments on its debt and payments from the central bank. The budget deficit can be financed by either borrowing from the central bank or from abroad. Hence, the central government's budget identity can be written as follows:

(29) $sg + iD_g - i^*sF_g - (P - \dot{s}R) = \dot{D}_g - s\dot{F}_g$

The consolidated government sector's budget identity is obtained by inserting (23) into (29). Further, by using equations (27) and (28) and writing the budget identity in real terms, we obtain:

(30) $\dot{R} + \dot{F}_g + \dot{d}_p = i^*(R + F_g) + id_p - i^*M^*_{cb} + \dot{m} + \dot{n} + (m + n - d_p)e - g$

Finally, the balance-of-payments identity is obtained by expressing first the household sector budget identity (3) in real terms (i.e. deflated by s) and then adding it to the consoli-

[9] If we assume that the cash-in-advance constraint also applies to purchases of foreign bonds, and for simplicity that $\dot{R}(t) \leq 0$, then in the regime with $\dot{F}_g > 0$ and $F_g > 0$, $M^*_{cb} \geq -\theta L + F_g(t+\tau) - F_g(t)$.

[10] The model could easily also include government consumption (i.e. the distribution of goods to the private sector via the government sector). However, in our single good model the public good should be a perfect substitute for the private good in the representative household's utility function and all the results presented in this paper would remain unaffected.

dated government sector budget identity (30). Using identity (22), the balance-of-payments identity can be written in real terms as follows:

$$(31)\quad \dot{R} = x - (\dot{F}_g + \dot{F}_p + \dot{M}_p^*) + i^*(R + F_g + F_p - M_{cb}^*)$$

It is worth noting that equation (31) takes into account the fact that part of the foreign reserves, i.e. M_{cb}^*, is in non-interest-bearing form.

3. *The intertemporal public sector and the economy-wide resource constraints*

All sectoral flow budget identities and balance sheet identities must hold at any point of time. These identities imply intertemporal sectoral budget or solvency constraints. Equation (7), or equally well equation (18), presents the intertemporal budget constraint of the household sector. The intertemporal public sector budget constraint can be derived from the flow budget identity (30). Rearranging and solving equation (30) forward, we obtain:

$$(32)\quad R(t) + F_g(t) + d_p(t) = [R(T) + F_g(T) + d_p(T)]e^{-i^*T} + \int_t^T e^{-i^*v}\{-(\dot{m} + me) - \alpha\dot{d}_p - [i - i^* - (1-\alpha)e]d_p + i^*M_{cb}^* + g\}dv$$

where $T > t$. We next assume that in the long run the stock of assets held by the public sector cannot be depleted at a rate faster than the foreign interest rate, i.e.:

$$(33)\quad \lim_{T\to\infty} [R(T) + F_g(T) + d_p(T)]e^{-i^*T} = 0$$

This is the no-Ponzi-game transversality condition and it rules out situations where the government deficit is financed through continued and cumulative borrowing. Now the intertemporal consolidated government sector budget constraint reduces to:

$$(34)\quad R(t) + F_g(t) + d_p(t) \geq \int_t^\infty e^{-i^*v}\{-(\dot{m} + me) - \alpha\dot{d}_p - [i - i^* - (1-\alpha)e]d_p + i^*M_{cb}^* + g\}dv$$

It states that the sum of the present discounted value of government deficits and the drain on public sector revenues caused by non-interest-bearing reserves and low-interest-rate debt granted to the private sector plus the present value of future resources appropriated by printing money should be smaller than or equal to outstanding net public assets.

It is useful to also derive the intertemporal economy-wide budget constraint, which is obtained by solving forward the balance-of-payments identity (31). As for the government, it is assumed that the representative consumer does not finance consumption indefinitely through Ponzi schemes. If solved for the present value of consumption, the intertemporal economy-wide resource constraint can be written as follows:

$$(35)\quad \int_t^\infty e^{-i^*v}c(v)dv < y/i^* + R(t) + F_g(t) + M_p^*(t) + F_p(t) - \int_t^\infty e^{-i^*v}[i^*(M_{cb}^* + M_p^*)]dv$$

It gives the maximally sustainable present value of consumption, which cannot exceed the sum of the nation's initial net stock of foreign assets and the present value of the income stream (y minus interest income lost due to the cash-in-advance constraint in financial transactions and capital controls).

The economy-wide budget constraint (35) is not independent of the household and public sector intertemporal budget constraints (7) and (35). If, for instance, the intertemporal public sector budget constraint is not violated but the economy-wide budget constraint is violated, then the intertemporal private sector budget identity must also be violated.

4. *The behaviour of the model*

The dynamics of the model presented in section 2 is simplified considerably if we make a heroic but rather conventional assumption that the rate of subjective time preference ρ equals the foreign interest rate i^*.[11]

Now the model can be written as follows:

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[11] *This assumption has been used by e.g. Drazen and Helpman (1985), Obstfeld (1986), Calvo (1987), Claessens (1988) and Frenkel and Klein (1989). As an exception to this practice, see e.g. Puumanen (1986).*

$$(36)\quad W(0) = a(0) + \int_0^{\infty} e^{-\int_0^t r(u)\,du}\{y + g\}\,dv$$

where

$$a(0) = m(0) - (1-\alpha)d_p(0) + (1+\beta)F_p(0)$$
$$= d_g(0) + R(0) + (1+\beta)F_p(0)$$

and $\beta = 0$, if $F_p \geq 0$
$= -\tau i^*$, if $F_p < 0$

$$(37)\quad \dot{W}(t) = (r - i^*)W(t)$$

$$(38)\quad r = i/(1-\alpha) - e = i^*/(1+\beta),$$

where $\alpha = \tau i$

$$(39)\quad c(t) = [i^*/(1+\gamma)]W(t)$$

$$(40)\quad m(t) = [i^*\gamma/(1+\gamma)]W(t)/[r + e(t)]$$

$$(41)\quad \dot{m}(t) = (1-\alpha)\dot{d}_p(t) + \dot{d}_g(t) + \dot{R}(t) - [m(t) - (1-\alpha)d_p(t) - d_g(t)]e(t)$$

$$(42)\quad g(t) = \dot{d}_g(t) + d_g(t)e(t) - \dot{F}_g(t) + i^*[R(t) + F_g(t) + d_p(t)] + [i(t) - i^*]d_p(t) - i^*M^*_{cb}(t)$$

$$(43)\quad \dot{R}(t) = y - c(t) - [\dot{F}_g(t) + \dot{F}_p(t) + \dot{M}^*_p(t)] + i^*[R(t) + F_g(t) + F_p(t) - M^*_{cb}(t)]$$

$$(44)\quad M^*_p(t) = \beta\, F_p(t);\ \beta = 0,\ \text{if } F_p \geq 0;\ = -\tau i^*,\ \text{if } F_p < 0$$

$$(45)\quad M^*_{cb}(t) = [\theta/(1+\theta)]R(t) + [\Theta/(1+\theta]F_g(t);$$

where

$\theta = 0$, if $R \geq M^*_{cb}$ and $\Theta = 0$, if $F_g \geq 0$
$= -\tau i^*$, if $R < M^*_{cb}$ $\quad = -\tau i^*$, if $F_g < 0$

Equation (36) defines the present value of net private wealth at time $t = 0$. For notational simplicity, time $t = 0$ denotes the present time. It is worth noting that $a(0)$ is predetermined by the past history of asset accumulation whereas the variable $W(0)$ and the division of $a(0)$ between its components can change at time $t = 0$, if information concerning future developments in the variables on the right-hand side of equation (36) changes.

Equation (37) defines the change in net private wealth as a function of the difference between the rate of subjective time preference (equalling i^*) and the real rate of domestic financing costs. Equation (38) is the modified uncovered interest parity condition, which states that real financing costs arising from domestic (defined by equation (6)) and foreign sources must be equal.

Equation (39) defines the flow of consumption and equation (40) the stock of money demanded as shares of net life-time wealth. Equation (41) is the money supply equation (28) expressed in real terms.

Equation (42) is the public sector flow budget identity solved for the government lump-sum transfers to the house-hold sector. It is obtained from (30) after inserting (41). Together with relations (44) and (45) determining the domestic sector's foreign currency holdings, the balance-of-payments identity (43) closes the model.

From the point of view of the dynamics of the model it is crucial if the private sector is the net investor or the net debtor in the foreign financial markets. In the first case, i.e. $F_p \geq 0$, the parameter $\beta = 0$ and $r = i^*$. In this interest rate regime equations (37) and (39) imply that wealth $W(t)$ and consumption $c(t)$ remain constant in time. As long as the depreciation rate e in (40) is constant, this is also the case as regards the stock of money demanded.

If the private sector is a net foreign borrower, i.e. $F_p < 0$, the dynamics of the model is quite different. Now equation (38) implies that $r = i^*/(1 - \tau i^*)$, which is greater than the subjective time preference rate i^*. In this case the private sector is the net saver accumulating its wealth. The time patterns of the consumption and the stock of money demanded are also rising.

Hence, we see that there are two possible interest rate regimes in the model. It can be shown that the low-interest-rate regime, $r = i^*$, is utility superior to the high-interest-rate regime, $r > i^*$.[12]

[12] *With the nation's initial net stock of foreign assets given, one can see from the intertemporal economy-wide resource constraint (35) that in the high-interest-rate regime $r > i^*$ and $F_p < 0$ the term $\int e^{-i^*v} i^* M^*_p > 0$ and, hence, consumption is at a lower level than in the low-interest-rate regime. In addition, on the basis of equations (39) and (40); $m = [\gamma/(r+e)]c$ from which it is easy to see that m is also at a lower level in the high-interest-*

However, if the economy is initially in the high-interest-rate regime, it cannot instantaneously jump to the low-interest-rate-regime. This is because at least part of the initial financial wealth is in the form of money, which must always be at a positive level. Hence, as foreign assets are the flexible component of financial wealth, it is quite possible that $F_p(0)<0$.

In appendix 1, I show that the interest rate which prevails in the economy at each instant of time depends on the division of wealth between predetermined financial wealth, a, and the present value of future income streams. If the share of financial wealth is high enough, the prevailing interest rate is $r=i^*$, otherwise the interest rate regime is $r=i^*/(1-\tau i^*)$. I then go on to show that although initially the economy would be in the high interest rate regime, the internal dynamics of the model pushes the economy into the low-interest-rate regime.

Hence, as the interest rate regime $r=i^*$ is the one to which the steady state solution of the model is any case attracted, in the rest of this paper I restrict the analysis to that regime.

5. *The lower bound on the level of the central bank's net foreign reserves*

The assumption of a lower bound on the level of the central bank's net foreign reserves, which triggers an speculative attack on the currency, plays a crucial role in the analysis of balance-of-payments crises. In this section, I examine whether such a lower bound exists and, if it does, what is the critical level of the central bank's net foreign reserves.

One can approach this issue by asking (a): does there exist a lower bound on the net foreign reserves below which they cannot be depleted without violating sectoral solvency constraints, or by asking (b): is there some level of foreign reserves below which the central bank is unwilling to allow the foreign reserves to be depleted although, from the point of view solvency, further depletion would be possible?

These issues have previously been studied in the light of the first question by Obstfeld (1986) and Buiter (1986 and 1987). They considered whether a continuous domestic credit expansion results in the violation of the public sector solvency (or intertemporal budget) constraint. In this respect their results were inconclusive; it was dependent on the way the credit expansion was used. In addition, as pointed out by Buiter (1987), balance-of-payments crises would always be manifestations of government solvency crises. Also in cases where credit expansion results in solvency problems, the question to be answered is still what is the minimum level of the net foreign reserves. In a complete model of a balance-of-payments crisis, the lower bound on the net foreign reserves should also be an endogenous variable of the model.

The question of the lower bound on the net foreign reserves has not previously been studied in the light of the second question. This is done in this section by studying not only solvency but also the welfare effects of diminishing net foreign reserves with and without the cash-in-advance constraint in financial market transactions. I show that the cash-in-advance constraint in financial transactions results in a welfare loss, if the net foreign reserves are allowed to be depleted below the zero level. Hence, for a welfare-maximizing government this is a lower bound below which the government is unwilling to allow the net foreign reserves to be depleted. I further show that the solvency constraint of either the private or the public sector is violated if the growth rate of domestic credit expansion exceeds a critical magnitude, which is below the foreign interest rate. Unlike in Buiter (1987), the violation of the solvency constraint does not depend on the way the credit expansion is used.

To present these results, the public sector flow budget identity (42), the public sector and the economy-wide intertemporal budget constraints (34) and (35), respectively, and the utility function (1) are needed.

Given a fixed exchange rate, the public sector flow budget identity can be written as:

$$(46) \quad g(t)=\dot{d}_g(t)-\dot{F}_g(t)+i^*[R(t)+F_g(t)] + id_p(t)-i^*M^*_{cb}(t)$$

rate regime than in the low-interest-rate regime. Hence, the low-interest-rate regime must be utility superior to the high-interest-rate regime.

The corresponding intertemporal public sector budget constraint is:

$$(47)\quad R(0)+F_g(0)+d_p(0)\geq \int_0^\infty e^{-i^*t}\{-\alpha\dot{d}_p(t) -(i-i^*)d_p(t)+i^*M^*_{cb}(t)\}dt$$

where $\alpha=\tau i$. The economy-wide resource constraint is:

$$(48)\quad c_o=i^*\int_0^\infty e^{-i^*t}\,c(t)\,dt \leq y+i^*[R(0)+F_g(0)+F_p(t)] -i^*\int_0^\infty e^{-i^*t}\,i^*M^*_{cb}(t)\,dt$$

Without loss of generality, assume that F_g is greater than or equal to zero. Now the central bank's cash holdings of foreign currency (45) can be written in the form:

$$(49)\quad M^*_{cb}(t)=[\theta/\{1+\theta\}]R(t);\ \text{where}$$

$$\theta=0,\ \text{if } R\geq 0$$
$$\theta=-\tau i^*,\ \text{if } R<0$$

Domestic credit can be expanded either in the form of central bank credit to the central government d_g or in the form of credit to the household sector d_p. Expansion in d_g can be used either for investing in foreign assets F_g or for lump-sum transfers to households. Each of these cases is studied in turn.

Case 1: the central government invests in foreign assets by borrowing from the central bank, i.e. $\dot{d}_g=\dot{F}_g$.

The cash-in-advance constraint in financial market transactions is of importance in the model as long as $R(t)>0$ (or $L(t)=0$). For this reason it is convenient to select the time $t=0$ so that the net foreign reserves are depleted to zero, i.e., $R(0)=0$. For notational simplicity I further assume that $F_g(0)=d_p(0)=0$. As the demand for money is constant in the fixed exchange rate regime, $\dot{R}=\dot{d}_g$ and, hence, $R(t)=-F_g(t)$. The public sector flow budget identity (46) and the intertemporal budget constraint (47) are simplified to:

$$(50)\quad g(t)=-i^*M^*_{cb}(t)$$

$$(51)\quad 0\geq\int_0^\infty e^{-i^*t}\{g(t)-i^*M^*_{cb}(t)\}dt$$

By inserting (50) into (51), it can be seen that the right-hand side of (51) equals zero and the public sector solvency constraint is not violated. This is due to the fact that the additional interest payments resulting from the cahs-in-advance constraint in debt service can be covered by lump-sum taxes collected from households. Hence, the burden caused by additional interest payments is shifted to the household sector. The importance of this phenomenon can be studied by examining the economy-wide budget constraint which (with $F_p(0)=0$) reduces to:

$$(52)\quad c_o\leq y-i^*\int_0^\infty e^{-i^*t}\,i^*M^*_{cb}(t)\,dt$$

Assuming that d_g developes as $\dot{d}_g(t)=\mu d_g(t)$, implying $R(t)=[1-e^{\mu t}]d_g(0)$ we end up with:

$$(53)\quad c_o\leq y+i^*_{cb}d_g(0)\{1+[i^*/(i^*-\mu)] [\lim_{T\to\infty} e^{(\mu-i^*)T}-1]\}$$

where $i^*_{cb}=\tau i^{*2}/(1+\tau i^*)>0$, when $R<0$, otherwise $i^*_{cb}=0$.[13] It is easy to see that the right-hand side of (53) approaches to minus infinity if the growth rate of domestic credit μ is above the foreign interest rate i^*, implying the violation of the intertemporal economy-wide (and hence the private sector) budget constraint. The explanation for this is that the present value of the net lump-sum transfers needed to cover the additional interest payments becomes minus infinity, also reducing private wealth to minus infinity.

If the growth rate of domestic credit μ is in the interval $0\leq\mu<i^*$, the right-hand side of (53) is finite, equalling $y-\mu i^*_{cd}d_g(0)/(i^*-\mu)$. This implies that the maximum sustainable level of consumption is the smaller the closer to i^* is μ. Further, the non-negativity of consumption requires that μ must be smaller than $i^*/(1+x)$ where $x=i^*_{cb}d_g(0)/y>0$.

In addition, it can easily be seen from (53) and (52) that the level of private consumption decreases if the net foreign reserves $R(t)$ are

[13] *It can be seen that $i^*_{cb}=0$, if $\tau=0$ (i.e. no cash-in-advance constraint). In this case consumption is unaffected by the depletion of the net foreign reserves towards minus infinity.*

frozen at some negative level or if they are even temporarily below the zero level. From the utility function (1), it can be seen directly that, as m is a constant share of c, the reduced level of consumption implies the lower level of utility.

Hence, in summarizing we can conclude that government borrowing from the central bank to finance its investment in foreign assets does not affect the solvency of the public sector. This is the result reported by Buiter (1987). Given a cash-in-advance constraint in financial market transactions, however, this policy worsens the solvency of the private sector whenever the net foreign reserves have been depleted below zero. The non-violation of the solvency of the private sector and keeping the level of private consumption above zero require that the growth rate of domestic credit is below the foreign interest rate. Welfare losses occur, if the net foreign reserves are allowed, even temporarily, to be depleted below zero.

Case 2 (the case of expansive fiscal policy): The central government borrows from the central bank to finance lump-sum-transfers to households.

The money supply identity with m fixed implies that $\dot{R}(t) = \dot{d}_g(t)$ and (49) implies $M^*_{cb} = 0$ until R is depleted to zero. Hence, we can assume that at $t = 0$ $R(0) = 0$. For notational simplicity, assume that also $F_g(t) = d_p(t) = F_p(0) = 0$.

Now the public sector flow budget identity (46) and the intertemporal budget constraint (47) can be written as follows:

$$(54)\quad g(t) = \dot{d}_g(t) + i^*R(t) - i^*M^*_{cb}(t)$$

$$(55)\quad 0 \geq \int_0^\infty e^{-i^*t}[g(t) + i^*M^*_{cb}(t)]\,dt$$

Assume again that $\dot{d}_g(t) = \mu d_g(t)$ and insert g(t) defined by (54) into (55) to obtain:

$$(56)\quad 0 \geq \int_0^\infty e^{-i^*t}[\mu d_g(t) + i^*R(t)]\,dt$$

The fact that the term $i^*M^*_{cb}(t)$ disappears from the right-hand side of (56) implies that the cash-in-advance constraint in financial market transactions has no effect on the public sector solvency problem. The reason for this is the same as in case 1, i.e. that the burden caused by the central bank's additional interest payments on its foreign debt can be shifted to the household sector as increased lump-sum taxes.

Using the identity $R(t) = m(0) - d_g(t) = d_g(0)[1 - e^{\mu t}]$ equation (56) reduces to

$$(57)\quad 0 \geq d_g(0)\{1 + [\lim_{T\to\infty} e^{(\mu - i^*)T} - 1]\}$$

Equation (57) repeats the result obtained by Obstfeld (1986); the public sector solvency constraint is violated if the growth rate of the domestic credit $\mu \geq i^*$, but it is not violated if $\mu < i^*$. In the first case, the right-hand side of (57) approaches infinity and in the latter case it equals zero.

Owing to the cash-in-advance constraint in financial market transactions, the depletion of the net foreign reserves below the zero level worsens the solvency of the household sector. This can be seen from the intertemporal economy-wide budget constraint, which reduces to the same form as in case 1, i.e. to (52) or equally to (53). Hence, we can draw the same conclusions; even in cases where the foreign reserves are only momentarily below the zero level, a welfare loss is caused to the household sector. As earlier, the non-negativity of consumption requires that μ must be smaller than $i^*/(1 + x)$ where $x = i^*_{cb}d_g(0)/y > 0$.

Case 3 (the case of expansionary monetary policy): The central bank buys bonds d_p issued by the household sector.

I now assume that $d_g(t) = F_g(t) = 0$ and that also $R(0) = 0$. Hence, $R(t) = (1 - \alpha)[d_p(0) - d_p(t)]$ where $\alpha = \tau i$. The increase in the household sector's domestic credit is used for investments in foreign assets F_p so that $\dot{F}_p = -\dot{R}$ and for the accumulation of cash balances N needed for debt service. The public sector flow budget identity (46) and intertemporal solvency constraint (47) can be written as:

$$(58)\quad g(t) = i^*R(t) + id_p(t) - i^*M^*_{cb}(t)$$

$$(59)\quad d_p(0) \geq \int_0^\infty e^{-i^*t}\{-\alpha\dot{d}_p(t) - (i - i^*)d_p(t) + i^*M^*_{cb}(t) + g(t)\}\,dt$$

where $i - i^* = -\tau i^{*2}/(1 + \tau i^*) < 0$ and, hence, the domestic nominal interest rate is below the world market interest rate i^*. Assuming $\dot{d}_p(t) = \mu d_p(t)$ and inserting (58) into (59), we end up with:

$$(60)\quad d_p(0) \geq \int_0^\infty e^{-i^*t}\{-\alpha\mu d_p(t) + i^*[d_p(t) + R(t)]\}dt = d_p(0)[1 - \alpha \lim_{T\to\infty} e^{(\mu - i^*)t}]$$

It can be seen that the right-hand side of (60) equals the left-hand side, if $\mu < i^*$, and approaches minus infinity, if $\mu > i^*$. Hence, expansive monetary policy does not result in solvency problems for the government sector. Rather, it tends to loosen the solvency constraint (the case where $\mu \geq i^*$). This is so in spite of the fact that the interest rate on the central bank's growing foreign debt is higher than the domestic nominal interest rate which the household sector pays on its debt to the central bank. The interpretation is that the pace at which the net foreign reserves diminish is slower than the pace at which the household sector's debt to the central bank grows. This is due to the fact that, owing to the cash-in-advance constraint in debt service payments, the leakage from the domestic credit expansion is not complete; a fraction $\alpha = \tau i$ of it is used for accumulating cash reserves N.

The intertemporal economy-wide budget constraint reduces into equation (53). Again the right-hand side of (53) approaches minus infinity, if $\mu > i^*$, implying the violation of the intertemporal private sector budget constraint. Likewise, our conclusions concerning the welfare effects of unduly expansive monetary policy, which depletes the net foreign reserves below zero, are the same as in cases 1 and 2 and, hence, the depletion of the central bank's net foreign reserves below zero decreases the utility of the representative household.

We can conclude that without the cash-in-advance constraints in financial market transactions, the sectoral solvency constraints set no lower bound on the central bank's net foreign reserves, if the domestic credit expansion is used to accumulate either the central government's (case 1) or the private sector's (case 3) foreign assets. Likewise, in the second case, in which the credit expansion is used to finance lump-sum transfers to the households, there is no lower bound on the net foreign reserves, if the growth rate of credit expansion is below the foreign interest rate. Only if the growth rate of credit expansion exceeds the foreign interest rate is the solvency constraint violated. Under all these policy alternatives, the welfare of the household sector remains unaffected. These results are in accordance with those obtained by Obstfeld (1986) and Buiter (1987).

The cash-in-advance constraint in financial market transactions changes the conclusions stated above in two important ways: Firstly, all three policy alternatives become similar in the sense that either the public sector or the private sector solvency constraint is violated, if the rate of domestic credit expansion exceeds a critical magnitude, which is somewhat below the foreign interest rate. However, the depletion of the central bank's net foreign reserves towards minus infinity is still possible in all three cases, if the rate of credit expansion is below the critical magnitude. The faster rate of credit expansion would not be sustainable with a positive level of consumption. Secondly, the cash-in-advance constraint introduces the welfare aspect into the analysis; the welfare of households is decreased, if the net foreign reserves are depleted below the zero level.

Especially the latter conclusion is important, because its implication is that a welfare-maximizing government is unwilling to allow the central bank's net foreign reserves to be depleted below zero. Hence, for a welfare-maximizing government, the critical lower bound on the net foreign reserves is zero.

6. The endogenous regime shift from the fixed exchange rate regime to the floating exchange rate regime

I showed above that, with a cash-in-advance constraint in financial market transactions, a welfare-maximizing government does not allow its official foreign reserves to be depleted below zero. Within the present framework, however, no argument can be found which would motivate the government to continue expansive credit policy and allow the exchange rate to float after the net foreign reserves have been exhausted to zero; although consumption would be unaffected by the exchange rate regime shift the increased rate of inflation would decrease the demand for money (see equation (40), which on the basis of the utility function (1) would decrease welfare.[14] Hence, instead of allowing the exchange rate regime to shift, the welfare-maximizing government would stop credit expansion at the latest when the net foreign reserves have been depleted to zero.[15]

To fully rationalize the endogenous regime shift from the fixed exchange rate regime to the floating exchange rate regime, an additional argument is required which would motivate permanently expansive credit policy. One argument for such an inflationary policy could be that suggested by Barro and Gordon (1983); if the welfare-maximizing government treats expected inflation as exogenously given, a positive rate of inflation can be a property of the time consistent solution.

Distortionary taxation could provide another argument for expansive monetary policy; inflation can also be treated as a tax. In the literature on seigniorage it has been shown that attaching a positive inflation rate to distortionary taxation can be part of the optimal tax system.[16] Hence, the extension of the present framework to include distortionary taxation might result in reasonable justification for an expansive credit policy and the endogenous exchange rate regime shift at the moment the central bank's net foreign reserves have been depleted to zero. Further analysis of this issue is, however, beyond the scope of this article.

Instead I take expansionary credit policy as given and, on the basis of the analysis of the previous section, I assume that the exchange rate regime shift occurs at the moment the central bank's net foreign reserves reach zero. The aim is to examine what kind of effects the cash-in-advance constraint has on the size and timing of the speculative attack on the currency associated with the exchange rate regime shift. I also discuss about the case in which the cash-in-advance constraint is extended to include purchases of foreign bonds. I start the analysis by studying the behaviour of the model in the floating exchange rate regime.

6.1 The behaviour of the model in the floating exchange rate regime

In deriving the behaviour of the model in the floating exchange rate regime, I closely follow the exposition of Puumanen (1986). Assume that the supply of money rule is $\dot{M} = \mu M$. Hence, the growth in real money balances is determined as follows:

$$(61) \quad \dot{m}/m = \mu - e$$

Denote the right-hand side of the demand for money function (40) as

$$(62) \quad \phi(t) = i^* \gamma W(0)/[(1+\gamma)(i^* + e(t))]$$

Insert (62) into (40) and differentiate logarithmically. As $r = i^*$, the real wealth in (40) is constant, i.e. $W(t) = W(0)$, and we obtain:

$$(63) \quad \dot{\phi}/\phi = \mu - e$$

Solve (62) for the depreciation rate e and insert it into (63) to obtain:

$$(64) \quad \dot{\phi} = (\mu + i)\phi - \gamma i^* W(0)/(i+\gamma)$$

This linear differential equation has an unstable root $(\mu + i^* > 0)$ and the saddle-path stability implies that, if the growth rate of the supply of money changes, there is an instantaneous jump in ϕ to its new stationary value defined as:

$$(65) \quad \phi^* = \gamma i^* W(0)/[(i+\gamma)(\mu + i^*)]$$

Hence, as in (63), $\dot{\phi} = 0$ the depreciation rate e must equal the growth rate of the supply of money, i.e. $e = \mu$. This implies that real money balances remain constant and that the level of the exchange rate is determined as $s(t) = M(t)/m(t)$.

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[14] *The property that consumption is unaffected by the exchange rate regime shift is due to the fact that the utility function (1) is separable in money and consumption. If it were inseparable, then consumption would also be affected by the exchange rate regime shift* [*see Claessens (1988)*].

[15] *As long as the central bank's net foreign reserves are positive, it does not matter from the point of view of welfare how expansive the domestic credit policy is. This kind of policy would only change the time pattern of the lump-sum transfers without any effects on the present value of these transfers.*

[16] *See Phelps (1973) and, for instance an overview by Spaventa (1989) and the literature mentioned there in.*

6.2 The timing of the exchange rate regime shift and the size of the speculative attack

Assume for notational simplicity that the central bank's own capital $K(0)=0$ and that in the fixed exchange rate regime the exchange rate is set equal to unity. Now the money supply identity (28) can be written in stock form as follows:

$$(66)\quad M(t)=(1-\alpha)D_p(t)+D_g(t)+R(t);\ \alpha=\tau i$$

It can be seen that domestic credit can be injected into the economy through two alternative channels, i.e. through central bank credit to the central government, D_g, or through central bank credit to the household sector, D_p. Assume first that the credit expansion is created via the household sector and for simplicity that $D_g(0)=0$. Specify the following money supply rule:

$$(67)\quad D_p(t)=D_p(0)e^{\mu t}$$

In the fixed exchange rate regime the demand for nominal money balances is constant, i.e. $M(t)=M(0)$. Now, on the basis of (66) and (67), the central bank's net foreign reserves evolve in the fixed exchange rate regime as follows:

$$(68)\quad R(t)=M(0)-[M(0)-R(0)]e^{\mu t}$$

Equation (68) states that within some finite time interval the net foreign reserves have been depleted to zero and, hence, there is a regime shift from the fixed exchange rate regime to the floating exchange rate regime.

The condition which connects the fixed exchange rate regime to the floating exchange rate regime is that the exchange rate cannot jump discretely when the regime shift occurs. This is the continuity condition, which the perfect foresight solution of the model must satisfy in order to be unique [see Calvo (1977)].

Denote by z the timing of the regime shift. At time z, all the remaining net foreign reserves are exhausted to zero. On the basis of equation (66), the stock of money immediately before the attack is:

$$(69)\quad M(z_-)=M(0)=[1-\tau i(z_-)]D_p(z)+R(z_-)$$

where z_- indicates the instant before the attack. As $R(z_+)=0$, where z_+ denotes the instant after the attack, $R(z_-)$ equals the size of the attack. The speculative attack has caused the discrete jump in the depreciation rate of the exchange rate from zero to μ (also implying a rise in the domestic nominal interest rate). As shown in the previous section, this is due to the fact that in the floating exchange rate regime the depreciation rate of the exchange rate equals the growth rate of the supply of money M. It is easy to see from (66) that in the floating exchange rate regime, with $R(t)=0$, M grows at the same rate as D_p, i.e. at the rate μ.

As the level of the exchange rate remains unchanged at time z, equation (40) implies that the real as well as the nominal money balances drop at time $t=z$ from the level $M(0)$ to the level $[i^*/(i^*+\mu)]M(0)$. Now the stock of money immediately after the speculative attack can be written as follows:

$$(70)\quad M(z_+)=[i^*/(i^*+\mu)]M(0)=[1-\tau i(z_+)]D_p(z)$$

On the basis of equation (16), the domestic nominal interest rate jumps from the level $i=i^*/(1+\tau i^*)$ to the level $i=(i^*+\mu)/[1+\tau(i^*+\mu)]$ at time $t=z$. This implies that:

$$(71)\quad \Delta i_z=i(z_+)-i(z_-)=\mu/\{[1+\tau(i^*+\mu)](1+\tau i^*)\}$$

As the denominator on the right-hand side of (71) is greater than one, the nominal interest rate rises less than the rise in the depreciation rate of the exchange rate.

Subtract (70) from (69) and solve for the size of the speculative attack to obtain:

$$(72)\quad R(z_-)=[\mu/(i^*+\mu)]M(0)-\tau\Delta i_z D_p(z)$$

It can be seen that the size of the speculative attack is smaller than the stock-shift in money balances, M, which the first term on the right-hand side of (72) measures. The second term on the right-hand side of (72) is introduced by the cash-in-advance constraint in financial market transactions. This constraint implies that the rise in the domestic nominal interest rate increases the cash balances, N, needed for debt service. This partly compensates for the effect of the diminished demand for money, M, on the size of the speculative attack.

What if the cash-in-advance constraint is extended to include purchases of foreign bonds? The analysis presented above would not change at all. The only difference would be that at the instant the attack and the exchange rate regime shift occurs, the portfolio shift from money, m, into foreign bonds, F_p, would occur sequentially via foreign cash balances M_p^*; at time z domestic currency would be changed into foreign cash balances and at time $z+\tau$ in the floating exchange rate regime these money balances would be changed into interest-bearing assets.[17]

Using equations (66) and (68), the exact timing of the exchange rate regime shift can be solved from (72). We obtain:

$$(73) \quad z=(1/\mu)\{\log\,[i^*M(0)/(i^*+\mu)] - \log\,[M(0)-R(0)] - \log\,[1-(1+\tau i^*)\tau\Delta i_z]\}$$

The last term on the right-hand side of (73) is introduced by the cash-in-advance constraint in financial transactions and it is negative. Hence, the cash-in-advance constraint delays somewhat the timing of the endogenous exchange rate regime shift.

Assume next that the credit expansion is injected via credit to the central government D_g and that $D_p=0$. It is easy to show that in this case the size of the speculative attack equals the stock shift in the amount of money M and, hence, the term Δi_z in (72) and (73) disappears. This implies that the size of the speculative attack on the currency is greater and the exchange rate regime shift occurs earlier than in the case where credit is expanded through D_p. This is due to the fact that the cash-in-advance constraint concerns the financial transactions between the private sector and the central bank but not the transactions made within the consolidated government sector. Hence, in this respect, the cash-in-advance constraint in debt service introduces a kind of asymmetry into the model.

[17] *The fact that part of the nation's financial resources is in non-interest bearing form for the period τ decreases the wealth of the representative household. This effect is, however, taken into account at the instant the expansive monetary policy rule is announced.*

7. Conclusions

In the balance-of-payments crisis literature the analysis of endogenous exchange rate regime shifts is based on the assumption that there is a lower bound on the central bank's net foreign reserves below which the reserves are not allowed to be depleted. Typically, the lower bound of the reserves is postulated to equal zero. Besides being analytically simple, this assumption accords well with commonsense wisdom. A worrying feature of this assumption is, however, that even in models with choice theoretic foundations it is made ad hoc.

In this article the basic choice-theoretic framework was extended to include cash-in-advance constraints in financial market transactions. The most important result was that, owing to the cash-in-advance constraint, the depletion of the central bank's net foreign reserves below zero (even temporarily) causes a welfare loss. Hence, for a welfare-maximizing government a lower bound on the central bank's net foreign reserves below which the reserves are not allowed to be depleted.

This result does not depend on the way in which the recipients of domestic credit (the central government or households) use the credit nor is the violation of sectoral solvency constraint required. Hence, a balance-of-payment crisis can occur independently of a government sector solvency crisis.

As to the sectoral solvency constraints, we found that with the cash-in-advance constraint in financial transactions either the household or the government sector solvency constraint is violated if credit expansion exceeds a critical magnitude, which was shown to be somewhat below the foreign interest rate. The way in which the credit expansion occurs does not matter, as far as this result is concerned. However, from the point of view of the timing and the size of the speculative attack on the currency, it does matter whether the credit expansion takes the form of central bank credit to the private sector or to the central government; if channelled via the private sector the regime shift occurs somewhat later than if credit is channelled via the central government sector.

In order to fully rationalize on choice — theoretic foundations an endogenous regime shift from the fixed to the floating exchange rate regime (or recurrent devaluations), it is still necessary to show why an expansive credit policy is optimal. It was suggested that distortionary taxation together with the cash-in-advance constraint in financial market transactions may offer one solution to this problem. To prove this explicitly is a topic for further research.

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Appendix 1:

The internal dynamics of the model

There are two possible interest-rate regimes in the model defined by equations (36)–(45) in section 4; the low-interest-rate regime, with $r = i^*$, and the high-interest-rate regime, with $r = i^*/(1-\tau i^*)$. Below, I show that both regimes are feasible but that the internal dynamics of the model pushes the economy to the low-interest-rate regime.

For notational simplicity, I keep as a benchmark case the case where $d_g(t) = d_p(t) = F_g(t) = 0$. This implies that $m(t) = R(t)$ and $a(t) = R(t) + (1+\beta)F_p(t)$ with $\beta = -\tau i^*$, if $F_p < 0$, and $\beta = 0$ otherwise. Assume also that $e = 0$. Now the public sector flow budget identity (42) implies that the lump-sum transfers $g(t) = i^* R(t)$.

Assume that the prevailing interest rate regime is $r = i^*$, implying the steady state solution. The definition of the initial stock of wealth (36) now gives:

$$\text{(A1)} \quad W(0) = 2\ R(0) + F_p(0) + y/i^*$$

We see that the net foreign reserves appear on the right-hand side of (A1) multiplied by two; the first time as a component of the initial money stock and the second time as the present value of the lump-sum transfers.

As $R = m$, the demand for money equation (40) gives in turn:

(A2) $W(0) = [(1+\gamma)/\gamma] R(0)$

Solve (A1) and (A2) for $R(0)$ to obtain:

(A3) $R(0) = [\gamma/(1+\gamma)] (F_p(0) + y/i^*)$

Using the definition $a = R + F_p$, equation (A3) can be solved for $F_p(0)$:

(A4) $F_p(0) = (1-\gamma) a(0) - (\gamma/i^*) y$

As (A4) was derived under the assumption $r = i^*$, the household sector must be a net investor in the foreign financial markets, i.e. $F_p(0) > 0$. This is, however, possible only if $a(0) > [\gamma/(1-\gamma)] y/i^*$. If this condition is not fulfilled, the economy must be in the high-interest-rate regime $r = i^*/(1-\tau i^*)$ where $F_p(0) < 0$.

I next show that, although initially the economy would be in the high-interest-rate regime, the internal dynamics of the model pushes the economy to the low-interest-rate regime.

The initial wealth is now:

(A5) $W(0) = a(0) + y/r + \int_0^\infty e^{-rt}\, i^* R(t)\, dt$

where $R(t) = m(t) = \{\gamma i^*/[r(1+\gamma)]\} W(0)\, e^{(r-i^*)t}$. After substituting this relation into (A5) and integrating the right-hand side, we obtain:

(A6) $b_2 W(0) = a(0) + y/r$

where $b_2 = \{1 - (i^*\gamma)/[r(1+\gamma)]\} > 0$. Next shift the initial point of time onwards from 0 to t_1. As wealth grows at the rate $r - i^*$, we obtain:

(A7) $b_2 W(0)\, e^{(r-i^*)t_1} = a(t_1) + y/r$

Subtract (A6) from (A7) to obtain:

(A8) $a(t_1) - a(0) = \{e^{(r-i^*)t_1} - 1\} b_2 W(0)$

which shows that $a(t)$ grows with the growth of wealth and, hence, $a(t)/y$ grows avove the critical level set by equation (A4).

The behaviour of the model can now be summarized as follows: If initially the share of financial wealth in total wealth is below a critical level, the household sector is a net borrower in the foreign financial markets. Because of the cash-in-advance constraint in debt service, $r = i^*/(1-\tau i^*)$, which is above the subjective time preference rate. This results in positive net saving, which is allocated between the money balances and foreign assets. The foreign debt of the household sector diminishes and at some point of time the debt is paid back and the household sector tends to become a net investor in the foreign financial markets. At that moment, the interest rate drops so that $r = i^*$ and the economy settles on its steady state path where the stock of wealth, consumption and the stock of money demanded stay constant. The stock of foreign reserves also remains at a fixed level if there are no changes in the supply of domestic credit.