Finnish Economic Papers – Volume 4 – Number 1 – Spring 1991

THE ECONOMICS OF CROWDING OUT*

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We consider fiscal policy in a non-monetary, general equilibrium framework with uncertainty, in which the financing of the government budget has no effects. We show that »crowding out» and »crowding in» depend on whether the fiscal policy requires a gestation period or whether its effect is contemporaneous with the taxation by which it is financed. The optimality conditions and the comparative statics of the equilibrium are entirely different for these two cases. In addition to this timing question, the optimality conditions depend on whether the public good has infrastructure effects and/or has intrinsic utility.

1. Introduction

In the popular press, »crowding out» refers to a phenomenon whereby excessive government spending diminishes private investment. The process by which this is usually said to occur is the following: The government plans fiscal policies which throw its budget into deficit. This means that the government must borrow money in the private markets. This government borrowing raises real interest rates, which makes private investment in productive processes less profitable.

The economics of crowding out is problematic and has received little attention from the profession.[1] In perfect markets, it is clear that the usual explanations for crowding out, which focus on the pressures that excessive government borrowing puts on the financial markets, cannot hold. This is so since (as demonstrated by Ricardo and subsequently by Barro (1974)) the method of financing of the government budget cannot matter in a perfect market. An explanation for crowding out (if it exists at all, another unknown) must proceed on two levels: First, is it possible in a perfect market economy that government fiscal policy affects private investment, and if so, how? Second, is it possible that in an imperfect market economy (where the Ricardian equivalence principle need not hold) the form of financing of the government deficit will affect private investment?

* *We would like to thank an anonymous referee for very helpful suggestions.*

[1] *A notable exception is Friedman (1978), whose paper deals largely with portfolio crowding out in a monetary model. There are also references to crowding out effects (though he doesn't use this term) in Friedman (1962, p. 79ff).*

In this paper, we focus on the first of these questions. We focus on a perfect market economy in which the Ricardian equivalence proposition holds. This allows us to assume that all government spending is financed with taxation, and allows us to abstract from the financing question which has received most of the attention in the popular press. We shall show that even in such a perfect market economy, government fiscal policy *can and does*, indeed, affect private investment. Moreover, the effects of fiscal policy on private investment depend very much on three different parameters:

The first of these parameters is whether the goods produced by the government affect the efficiency of private investment. We call this the *infrastructure effect* of fiscal policy. Many goods produced by the government clearly have infrastructure effects: roads, bridges, flood control, education, etc. Studies that have already explored the effect of public expenditures on the productivity of private production include Grossman and Lucas (1974) who build a theoretical model to demonstrate the relationship as well as Barro (1981) and Aschauer (1989) who study the empirical significance of these productivity effects.

The second parameter is whether the goods produced by the government have *intrinsic utility*, i.e., whether the government-produced goods are consumables in and of themselves. While some government goods clearly have no intrinsic utility (snow fences on highways, for example, would seem to have only infrastructure effects), others (roads, schools, garbage collection, medical care) would seem to possess both infrastructure and intrinsic utility properties, while still others (the Marine Corps band, parks and the statues in them) have only intrinsic utility. The intrinsic utility of public goods and the relationship of public and private goods, especially their degrees of substitutability, was analyzed early on by Friedman (1962) and Bailey (1971). More recently Possen and Slutsky (1980) study the importance of these effects in an intertemporal setting.

The third parameter is the *timing* of government investment relative to the provision of the public good which is created. Some goods produced by the government require a lead time (gestation period) similar to that required for many goods provided by private investment. Thus, the provision of educational facilities tomorrow requires an investment by the government today for building schools. Similarly, levees for flood control require a period of time to be built. Government investment of this type goes through a process similar to the process by which the private sector plans capital investments: At a first date the provision for the investment is made, and at a later date the investment becomes productive. In the case of lags, the taxation with which the government goods are financed *precedes* their provision.

An important subset of the goods provided by the government, however, requires few or no timing lags to fall in place. Thus, for example, Marine Band concerts can be provided almost simultaneously with the budgetary provision for their financing. Snow fences on highways are not much different, and neither is the provision of medical care for the elderly (provided, of course, that the hospitals and clinics already exist). For this set of government goods, the taxation with which they are financed is *contemporaneous* with their provision.

Our results show that the timing question is crucial in determining whether government fiscal policy »crowds out» or »crowds in» private investment. In general, government goods which have a lead time tend to »crowd out» private investment, whereas goods which are provided contemporaneously with the taxation which finances their production tend to »crowd in» private investment. The type of good provided is also important. Public goods which have no infrastructure effects tend to reduce private consumption.

In our framework, it is also possible to determine the effect of government fiscal policy on real interest rates, and again this effect is shown to depend on the parameters outlined above. Suppose that the production functions are Cobb-Douglas. Then we show that a countercyclical change in fiscal policy will *raise* the riskless, one-period, real rate of interest if the production of the government good requires a lead time, and that the same kind of change will *lower* the one-period riskless real rate if taxation for the government good is simultaneous with its production.

Finally, we show that the social welfare implications of »crowding out» and »crowding in» are not as simplistic as these terms would seem to imply. Either »crowding out» or »crowding in» may lead to increases or decreases in social welfare, depending on whether the policy moves the equilibrium closer to or further away from the social optimum. This optimum itself depends on the three factors cited above. Whereas there appears to be a popular bias in favor of countercyclical fiscal policy, our results show that in some cases the optimal policy is in fact procyclical.

The paper is constructed as follows: In section 2 we set out two simple models which incorporate private consumption and investment and the production of public goods. In both models there is a single privately produced consumption good (also used for private and public investment) and a single publicly produced good. In both models a representative consumer maximizes his lifetime utility of consumption in a two-period model (today and tomorrow). For simplicity, a consumption-leisure choice is omitted from the model. Its inclusion would complicate the proofs but not change the character of the results.[2] There is uncertainty about the state of the world tomorrow, and this uncertainty is spanned by an Arrow-Debreu production technology. In both models, the government can potentially provide both intrinsic utility and infrastructure effects. The difference between the two models lies in the timing of the taxation vis-a-vis the provision of the public good. In model I the government good requires a lead time of one period: Taxes collected today will produce government goods (whether in the form of infrastructure effects or consumption goods, or both) only in the next period. In Model II the infrastructure effects and public goods consumed tomorrow are produced from taxes collected tomorrow.

Section 3 explores the optimality conditions for Model I, and section 4 the partial statics results for this model. In sections 5 and 6 we derive optimality and partial statics results for Model II. A final section concludes the paper.

[2] *In a model that includes leisure in the utility function one can show that capital inputs and leisure are positively correlated so that leisure acts to partially smooth the fluctuations in consumption caused by the productivity shocks. Thus, while the inclusion of leisure would affect the magnitude of the results obtained in the paper, it would not change their direction.*

2. The Models

We consider two families of models which differ in their interpretations of government policy. These differences are expressed through the budget constraints; in both families of models, a representative consumer maximizes his expected lifetime utility of consumption of a privately produced good (denoted with c) and of a public good (denoted by G). The consumption good serves as input into the private production process, and this process takes one period. The consumption good is also an input into the government production process. Inputs are paid for by lump sum taxes denoted by t.

In both models the representative consumer has available a *complete* production technology with which to plan his lifetime private consumption stream over two periods (Date 0 and Date 1). Date 0 may be thought of as representing »today». At Date 1 (»tomorrow») one of two possible states of the world may occur — State 1 or State 2. At Date 0 the consumer invests inputs z_1 and z_2 to yield production in States 1 and 2. The output of the private good in State 1 is given by $\alpha f(z_1)$ and output in State 2 is given $\beta f(z_2)$. Throughout we shall assume that the production function f is increasing and concave in inputs. Our modelling of production uncertainty is similar to that of Diamond's (1967) multiplicative uncertainty. We allow for the possibility that the government's fiscal policy may effect the coefficients of productive efficiency α and β; however, in the absence of such effects we shall assume that $\alpha < \beta$, so that State 1 is (from the consumer's point of view) a »bad» state of the world, and State 2 is a »good» state.[3]

[3] *A seemingly more general modelling of uncertainty would be to have a number of »complex» technologies, each of which produces output in both states of the world in the future. However, it is well-known that this structure can be reduced to the model which we employ, in the following sense: Suppose that in equilibrium the consumer uses at least as many »complex» technologies as there are states of the world and that these technologies are independent (i.e., the output vectors span a Euclidean space whose dimension is that of the number of states of the world). Then the resulting equilibrium is equivalent to one in which the consumer invests in orthogonal »primitive» technologies, such as the ones considered here (see Benninga and Protopapadakis 1986).*

The consumer maximizes an intertemporally additive utility function whose arguments are the private and the public good in each state of the world:[4]

(1) $\max U(c_0, G_0) + U(c_1, G_1) + U(c_2, G_2)$.

We consider two classes of models. In the first class, the production of the government good is subject to a one-period delay, paralleling delays in the private production process:

(I)

$$c_0 = \bar{c} - z_1 - z_2 - t_1 - t_2$$
$$c_1 = \alpha(t_1) f(z_1)$$
$$c_2 = \beta(t_2) f(z_2)$$
$$G_1 = G(t_1),\ G_2 = G(t_2).$$

In Model II, the production of the government good is contemporaneous with its consumption:[5]

(II)

$$c_0 = \bar{c} - z_1 - z_2$$
$$c_1 = \alpha(t_1) f(z_1) - t_1$$
$$c_2 = \beta(t_2) f(z_2) - t_2$$
$$G_1 = G(t_1),\ G_2 = G(t_2).$$

In both Models I and II we allow for the possible effect of the government good on productive efficiency. We call this the *infrastructure effect*. An important set of special cases involves models in which the infrastructure effect is missing, and in which the government good is desired only for its own utility. In this case $d\alpha/dt = d\beta/dt = 0$.

In both models we shall make the following assumptions:

A.1 $f' > 0$, $f'' < 0$, $G' > 0$, $G'' < 0$.

A.2 $\partial U/\partial c > 0$, $\partial U/\partial G > 0$, $\partial^2 U/\partial c^2 < 0$, $\partial^2 U/\partial G^2 < 0$. These are the standard concavity conditions for the utility function.

A.3 $\partial U/\partial c \to \infty$ as $c \to 0$. This is the Inada condition and it guarantees that the z's will always be positive.

A.4 $0 < \alpha(t) < \beta(t)$ for all $t \geqq 0$; $\alpha'(t)$, $\beta'(t) \geqq 0$. This assumption means that State 1 is a »worse» state (in terms of production efficiency) than State 2. Furthermore, productive efficiency in both states is a non-decreasing function of taxation; if productive efficiency is an increasing function of taxation, then — as indicated above — government spending will be said to have an infrastructure effect. Note that even in the absence of government spending, both α and β are assumed to be positive.

A.5 If $\alpha(t_1) < \beta(t_2)$, then $\alpha'(t_1) \geqq \beta'(t_2)$. This assumption means that government spending is relatively more effective in the worse of the two states.[6]

It is well-known that the complete markets structure of our two models is sufficiently rich to price financial assets and to determine optimal inputs for more complicated production functions than those we consider. This is done through the *Arrow-Debreu state prices*, determined by the consumer's marginal rates of substitution between today and states of the world tomorrow (see Arrow 1964, Debreu 1959). We denote these prices by q_1 and q_2, where

(2) $$q_1 = \frac{\partial U/\partial c_1}{\partial U/\partial c_0}, \quad q_2 = \frac{\partial U/\partial c_2}{\partial U/\partial c_0}$$

The state prices q_1 and q_2 are the equilibrium

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[4] *Expression (1) ignores subjective state probabilities and assumes that the consumer does not discount future consumption. Both of these features could be added to the model with no substantive changes in the results, though at the cost of more notational complexity.*

[5] *In contrast to the models studied here one could instead have assumed incomplete production technologies. In that case increases in investment or taxation would increase consumption in both the »bad» as well as the »good» state. For example, the constraints in model I would then become:*

$$c_0 = c - z - t$$
$$c_1 = \alpha(t) f(z)$$
$$c_2 = \beta(t) f(z)$$
$$G = G(t)$$

That framework is more in line with the way fiscal policy is normally studied in a world of uncertainty (see, for example, Abel (1988)). If one cannot target policy to a particular state (as, for example, the »bad» state), however, it becomes more difficult to discuss procyclical or countercyclical policy. Nonetheless, the type of crowding out or crowding in discussed in this paper will continue to hold in that type of a framework and will still depend on whether one has model I or model II type of timing.

[6] *Assumption A.5 is used in the paper only in the proofs of Theorems 1 and 9. None of the partial statics results (which give conclusions about »crowding out») require this assumption.*

market prices at Date 0 for a State 1 or State 2 payoff (respectively) of one unit of private consumption. A particular application of the state-pricing approach we shall be interested in concerns the *real risk-free rate of interest*. A one-period, riskless bond sold at Date 0 which pays off one unit of the consumption good in Date 1 irrespective of whether State 1 or 2 occurs will be sold at Date 0 for

$$\text{(3)} \qquad \frac{1}{1+r} = q_1 + q_2,$$

where r is the riskless rate of interest.

Our approach in the succeeding sections is to examine separately the optimality conditions and the comparative statics in each model. As we shall show, each of these models yields different insights into the interaction between private investment and fiscal policy.

3. *Model I: Optimality Conditions*

The first order conditions for optimality in Model I are:

$$\text{(4)} \qquad \begin{aligned} &-U_1(0) + \alpha(t_1) f'(z_1) U_1(1) = 0 \\ &-U_1(0) + \beta(t_2) f'(z_2) U_1(2) = 0 \end{aligned}$$

for production

$$\text{(5)} \qquad \begin{aligned} &-U_1(0) + U_2(1) G'(t_1) \\ &+ \alpha'(t_1) f(z_1) U_1(1) = 0 \\ &-U_1(0) + U_2(2) G'(t_2) \\ &+ \beta'(t_2) f(z_2) U_1(2) = 0 \end{aligned}$$

for taxation

where

$$\text{(6)} \qquad \begin{aligned} &U_1(m) = \partial U/\partial c_m, \; U_2(m) = \partial U/\partial G(t_m), \\ &U_{12}(m) = \partial^2 U/\partial c_m \partial G(t_m), \\ &U_{11}(m) = \partial^2 U/\partial c_m^2. \end{aligned}$$

Define the consumer's *coefficient of relative risk aversion* (RRA) as

$$\text{(7)} \qquad RRA(c) = -cU''(c)/U'(c).$$

When the utility function is separable in the government and the private goods, the following proposition shows that the relative sizes of the investment in the private technology in the two states is determined by the consumer's relative risk aversion. The proposition also shows that when the utility function is separable, the consumer will *always* consume less of the private good in the bad state (State 1) than in the good state (State 2). This proposition will be used to establish the relation between optimal taxation and investment when the utility function is separable.

Proposition 1: Suppose that U is separable in c and G, i.e., $U_{12} = 0$. Then in the maximization problem (I), if $\alpha(t_1) < \beta(t_2)$, then

1.1 $c_1^* < c_2^*$,

1.2 $z_1^* \gtreqless z_2^*$ if and only if $RRA \gtreqless 1$.

Proof:

1.1 Rewriting the first-order conditions for production (4) we get

$$\text{(8)} \qquad \frac{\partial U(c_1, G_1)/\partial c_1}{\partial U(c_2, G_2)/\partial c_2} = \frac{\beta(t_2) f'(z_2)}{\alpha(t_1) f'(z_1)}.$$

Now suppose that $z_1 < z_2$. Then it follows from the budget constraints for (I) that as long as $\alpha(t_1) < \beta(t_2)$, $c_1 < c_2$. On the other hand, suppose that $\alpha(t_1) < \beta(t_2)$, but that $z_1 > z_2$. Then the right hand side above is greater than 1, which means that the LHS must also be greater than one. Separability of the function guarantees that $c_1 < c_2$.

1.2 Rewrite (4) as:

$$\text{(9)} \qquad \frac{\alpha(t_1) f'(z_1) U_1(1)}{\beta(t_2) f'(z_2) U_1(2)} = 1,$$

Figure 1 graphs the left-hand side of this equa-

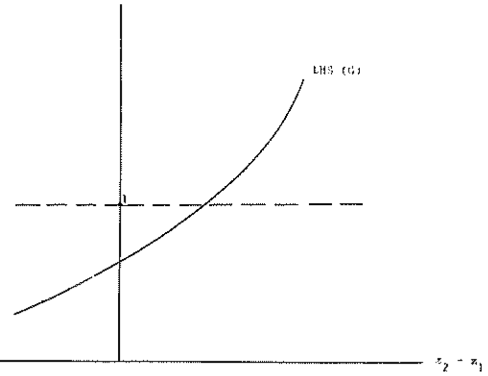

Figure 1.

tion when $U_{12}=0$ as a function of z_2-z_1.[7] It is readily observed from this figure that $z_1 \leqq z_2$ if and only if the intercept of the curve is less than or equal to 1. Every intercept of this curve is of the form

$$(10) \quad \frac{\alpha U'(\alpha f(z))}{\beta U'(\beta f(z))}=\frac{\alpha U'(\alpha d)}{\beta U'(\beta d)},$$

where $z=z_1=z_2$ and $d=f(z)$.

When $\alpha=\beta$, (10) is equal to 1. Now let $\alpha=\beta-\varepsilon$. Expanding (10) with a first-order Taylor expansion, we get

$$\frac{\alpha U'(\alpha d)}{\beta U'(\beta d)}=\frac{(\beta-\varepsilon)U'[(\beta-\varepsilon)d]}{\beta U'(\beta d)}$$

$$(11) \quad =\frac{(\beta-\varepsilon)\{U'(\beta d)-\varepsilon d U''(\beta d)\}}{\beta U'(\beta d)}$$

$$=\frac{\beta U'(\beta d)-\varepsilon U'(\beta d)-\varepsilon\beta d U''(\beta d)+\varepsilon^2 d U''(\beta d)}{\beta U'(\beta d)}$$

$$\simeq 1+\frac{\varepsilon}{\beta}[RRA(\beta d)-1],$$

where we have eliminated terms of order less than 1. This proves the proposition.

Q.E.D.

We now wish to establish the relation between optimal taxation and investment in Model I. We shall establish separate propositions for the cases of separability ($U_{12}=0$) and complementarity ($U_{12}>0$) or substitutability ($U_{12}<0$) between the government and the private good.

In order to prove the following two propositions, we rewrite the first-order conditions (4) and (5) as

$$(12) \quad \frac{U_1(c_1, G_1)}{U_1(c_2, G_2)}=\frac{\beta(t_2)f'(z_2)}{\alpha(t_1)f'(z_1)},$$

$$(13) \quad \frac{U_2(c_1, G_1)G'(t_1)}{U_2(c_2, G_2)G'(t_2)}$$

$$=\frac{U_1(0)-\alpha'(t_1)f(z_1)U_1(c_1, G_1)}{U_1(0)-\beta'(t_2)f(z_2)U_1(c_2, G_2)}$$

[7] *The assumption that the utility function is separable in the government and the private good is necessary, since we assume that the curve in Figure 1 is a function only of z_1 and z_2.*

Theorem 1: Suppose that $U_{12}=0$, $\alpha(t_1^*)<\beta(t_2^*)$, and α', $\beta'\neq 0$. Then $t_1^*>t_2^*$ if $z_1^*\geqq z_2^*$ (this happens if and only if $RRA\geqq 1$).

Proof:
If $U_{12}=0$, then Proposition 1 holds, and it thus follows that $z_1>z_2$ if and only if $RRA>1$. Furthermore, by assumption A.5, it follows that if $\alpha<\beta$, then $\alpha'>\beta'$. In this case, the right-hand side and in turn the left-hand side of (12) >1, which means that the right-hand side of (13) must be <1. Since the utility function is assumed to be separable, it follows that the left-hand side of (13) may be rewritten as

$$\frac{U'(G_1)G'(t_1)}{U'(G_2)G'(t_2)}<1,$$

which implies that $t_1>t_2$.

Q.E.D.

Based on empirical surveys of consumers' portfolio allocations, we may conclude that the typical magnitude of relative risk aversion is around 2 (Friend and Blume (1975), Morin and Suarez (1983)). Thus, if the utility function is separable, Theorem 1 indicates that optimal taxation is *countercyclical,* in the sense that $t_1^*>t_2^*$ and $c_1^*<c_2^*$.

The relation between optimal investment and taxation for the non-separable case ($U_{12}\neq 0$) is not as clear-cut as in the separable case. This is because, when the utility function is non-separable, taxation and private investment interact. When the private and public goods are complements, individual tend to shift private consumption into those states in which the government is providing more public goods. When the private and public goods are substitutes, the opposite tendency holds. For these cases we have the following proposition.

Theorem 2:

2.1 Suppose $U_{12}<0$ and that α', $\beta'\neq 0$. If $\alpha(t_1^*)<\beta(t_2^*)$ and $z_1^*\geqq z_2^*$ then $c_1^*<c_2^*$ implies that $t_1^*>t_2^*$ and $c_1^*>c_2^*$ implies that $t_1^*<t_2^*$.

2.2 Suppose that $U_{12}>0$ and α', $\beta'\neq 0$. If $\alpha(t_1^*)<\beta(t_1^*)$ and $z_1^*>z_2^*$, then $c_1^*>c_2^*$ implies $t_1^*>t_2^*$. Furthermore, $t_1^*\leqq t_2^*$ will only be optimal if $c_1^*<c_2^*$.

Proof:

2.1 If $z_1>z_2$, then the right-hand side of (12) is >1. Given assumption A.5, this means that the right-hand side of (13) <1. Given $U_{12}<0$, if $c_1<c_2$ and $t_1 \leqq t_2$, the left-hand side of (13) will be >1, which is a contradiction. Similarly, if $c_1>c_2$ and $t_1 \geqq t_2$, the left-hand side of (12) is <1, which is also a contradiction.

2.2 When the government and the private good are complements, the proof parallels that of case (i) with suitable changes made for the assumption that $U_{12}>0$.

Q.E.D.

Remark: We cannot establish Proposition 1 unequivocally for the case of nonseparability. However, if the complementarity effect is relatively small, it still follows from Proposition 1 that we would expect $c_1<c_2$ and that $z_1>z_2$ if and only if $RRA>1$. In that case, Theorem 2 showed that $t_1>t_2$. While the case of $c_1>c_2$ is slightly less plausible than that of $c_1<c_2$, this case cannot be ruled out when the utility function is non-separable. In particular, if the government and the private goods are close to perfect substitutes, this case may occur. Theorem 2 showed in that situation that $t_1<t_2$.

Theorems 1 and 2 consider the case where fiscal policy has both a consumption and an infrastructure effect. We now consider the case where fiscal policy has only consumption effects, i.e., $\alpha' = \beta' = 0$.

Theorem 3: If $\alpha' = \beta' = 0$ then:

3.1 If $U_{12}=0$, then $t_1^* = t_2^*$

3.2 If $U_{12}<0$, $c_1^* \gtreqless c_2^*$ if and only if $t_1^* \lesseqgtr t_2^*$

3.3 If $U_{12}>0$, $c_1^* \gtreqless c_2^*$ if and only if $t_1^* \gtreqless t_2^*$

Proof:

Write the first-order conditions (5) as

$$(14) \quad \frac{U_2(c_1, G_1)G'(t_1)}{U_2(c_2, G_2)G'(t_2)} = 1.$$

The proof of 3.1 follows directly from (14). If $c_1>c_2$ and $t_1>t_2$, then the left-hand side of (14) is less than one, given that $U_{12}<0$. Thus for an optimum, t_1 must be lowered relative to t_2. The remaining statements may be proved in a similar manner.

Q.E.D.

The intuition behind statement 3.1 in Theorem 3 is contained in Proposition 1. When fiscal policy has no infrastructure effects and when the utility function is separable in the public and private goods, both goods are on a similar structural footing (both require one-period gestations). Proposition 1 shows that were $\alpha=\beta$, private investment would be equal for both states. In the absence of infrastructure effects, this is essentially the case for the production of the government good, and the result of statement 3.1 follows naturally.

The intuition behind statement 3.3 is that when the government and the private good are complements and when infrastructure effects are absent, individuals shift private consumption into those states where there is more of the government good. When the government and private goods are substitutes as in 3.2, on the other hand, the consumption shift is in the opposite direction.

4. *Model I: Comparative Statics*

Total differentiation of the first order conditions for production gives

$$(15) \quad \begin{bmatrix} U_{11}(0)+A(1) & U_{11}(0) \\ U_{11}(0) & U_{11}(0)+A(2) \end{bmatrix} \begin{bmatrix} dz_1 \\ dz_2 \end{bmatrix} = \begin{bmatrix} \{-U_{11}(0)+B(1)\}dt_1 - U_{11}(0)dt_2 \\ \{-U_{11}(0)+B(2)\}dt_2 - U_{11}(0)dt_1 \end{bmatrix}$$

where

$$(16) \quad \begin{aligned} A(1) &= \alpha(t_1)f''(z_1)U_1(1) \\ &\quad + [\alpha(t_1)f'(z_1)]^2 U_{11}(1) \\ A(2) &= \beta(t_2)f''(z_2)U_1(2) \\ &\quad + [\beta(t_2)f'(z_2)]^2 U_{11}(2) \end{aligned}$$

$$(17) \quad \begin{aligned} B(1) &= -\alpha(t_1)f'(z_1)U_{12}(1)G'(t_1) \\ &\quad - \alpha'(t_1)f'(z_1)U_1(1)[1-RRA(c_1)] \\ B(2) &= -\beta(t_2)f'(z_2)U_{12}(2)G'(t_2) \\ &\quad - \beta'(t_2)f'(z_2)U_1(2)[1-RRA(c_2)] \end{aligned}$$

This system has solutions:

$$\begin{aligned} dz_1 = 1/\Delta\{&(B(1)A(2)+U_{11}(0)[B(1) \\ &-A(2)])dt_1 - U_{11}(0)[A(2) \\ &+B(2)]dt_2\}, \end{aligned}$$

(18)
$$dz_2 = 1/\Delta\{(B(2)A(1) + U_{11}(0)[B(2) - A(1)])dt_2 - U_{11}(0)[A(1) + B(1)]dt_1\},$$

where $\Delta > 0$ is the determinant of the matrix on the left-hand side of (15).

Theorem 4:

4.1 If RRA $\geqq 1$ and if U_{12} is not too positive, then $B(1)$, $B(2) \geqq 0$.

4.2 If $B(1)$ and $B(2) \geqq 0$, then

(19)
$$dz_1/dt_1 < 0,\ dz_2/dt_1?,\ dz_1/dt_1 + dz_2/dt_1 < 0$$
$$dz_1/dt_2?,\ dz_2/dt_2 < 0,\ dz_1/dt_2 + dz_2/dt_2 < 0$$

Proof: Statement 4.1 follows directly from (17) and 4.2 from the solutions in (18).

Q.E.D.

In the absence of both infrastructure effects and intrinsic value $B(1) = B(2) = 0$ and the only effects on z_1 and z_2 come from the wealth effects that stem from the tax changes. These wealth effects reduce both z_1 and z_2. When infrastructure effects are present and public and private goods are substitutes, i.e., $U_{12} < 0$, the amount of investment required in the own period, i.e., dz_1/dt_1 or dz_2/dt_2, will be even less than in the absence of these effects. The infrastructure effects mean that less private investment is required to produce the same amount of output and substitutability between public and private goods means that an increase in public goods lowers the demand for private goods. On the other hand, the impact on dz_1/dt_2 and dz_2/dt_1 becomes ambiguous. When there are infrastructure effects present more private resources can be shifted to the state in which the government is not providing public output. Similarly, substitutability between public and private goods means that individuals will want to shift more of the private output to the state in which public goods are not being provided. Therefore, since wealth effects and infrastructure and intrinsic value effects move in opposite directions, the signs of dz_2/dt_1 and dz_1/dt_2 are ambiguous.

A major concern of the public discussions on crowding out has been the effect on private investment and output of government fiscal policy. Theorem 4 shows that in Model I, the general effect of an increase in taxation in either state is to cause a *decrease* in total private investment. While we may describe this effect as the »crowding out» of private investment, the pejorative overtones of this term may not be justified, since — as amply demonstrated in Section 3 — either crowding out or crowding in may move us closer to or farther away from the optimum, with the corresponding changes in welfare.

In general we cannot predict output effects, since these depend on α' and β'. However, as the following theorem shows, when there are no infrastructure effects in Model I, the own output effects of a change in taxation are predictable:

Theorem 5: When there are no infrastructure effects, output falls in States 1 and 2 when taxes are increased in either of States 1 or 2, as long as $B(1)$ and $B(2)$ are non-negative. That is,

$$dc_1/dt_1 < 0 \text{ and } dc_2/dt_2 < 0$$

Proof:

We shall prove the claim for State 1 (the other statement is proved in a similar fashion). Write

$$\frac{dc_1}{dt_1} = \alpha'(t_1)f(z_1) + \alpha(t_1)f'(z_1)\frac{dz_1}{dt_1}.$$

If $\alpha' = 0$, this expression is unambiguously negative by (19).

Q.E.D.

In the absence of infrastructure effects, Theorem 5 shows that a countercyclical increase in fiscal policy (i.e., an increase in t_1) *decreases* both output and consumption of the private good in State 1.

Corollary: If there are no infrastructure effects and $U_{12} = 0$, $dc_1/dt_1 < 0$, $dc_2/dt_1 < 0$, $dc_1/dt_2 < 0$, and $dc_2/dt_2 < 0$.

A final result of this section concerns the effect of fiscal policy on the riskless interest rate. Combining the state prices (2) with first-order production conditions gives[8]

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[8] *In an Arrow-Debreu complete markets economy (of which our Models I and II are examples) the expressions in (20) result from the fact that firms maximize their state-dependent net-present value (see Arrow (1964), Debreu (1959)).*

(20) $q_1 = \frac{1}{\alpha(t_1) f'(z_1)}$ and $q_2 = \frac{1}{\beta(t_2) f'(z_2)}$.

Thus the state prices (and the riskless rate of interest (3)) depend on the optimal choice of inputs and taxation. In the following theorem we show that if the production functions are Cobb-Douglas, countercyclical changes in taxation will cause the riskless interest rate to rise.

Theorem 6: If the production functions are Cobb-Douglas, then a pure countercyclical change in taxation ($dt_1 > 0$) raises the riskless interest rate r, when $\alpha(t_1) < \beta(t_2)$.

Proof:

We shall show that as long as $\alpha(t_1) < \beta(t_2)$ and the production function is Cobb-Douglas, that

(21) $$\frac{-\alpha f''(z_1)}{[\alpha f'(z_1)]^2} > \frac{-\beta f''(z_2)}{[\beta f'(z_2)]^2}.$$

To see this, note first that a Cobb-Douglas production function is of the form

$$f(z) = \xi z^k, \; 0 < k < 1,$$

where ξ is some positive constant.

It follows that

$$f'(z) = k\xi z^{k-1}, \; -f''(z) = (1-k)k\xi z^{k-2}.$$

From this, it follows that

$$\frac{-f''(z)}{[f'(z)]^2} = \frac{(1-k)kz^{k-2}}{\xi k^2 (z^{k-1})^2} = \frac{(1-k)}{\xi k z^k}.$$

Expression (21) for the Cobb-Douglas production function thus becomes

$$\frac{(1-k)}{\alpha \xi k z_1^k} > \frac{(1-k)}{\beta \xi k z_2^k}$$

if and only if $c_2 = \xi \beta z_2^k > \xi \alpha z_1^k = c_1$.

If $c_2 > c_1$ and $\alpha(t_1) < \beta(t_2)$, then with a Cobb-Douglas production, equation (21) must hold.

To prove the theorem, recall that the one-period interest rate is determined by

$$1/(1+r) = q_1 + q_2,$$

where

$$q_1 = 1/(\alpha f'(z_1)) \text{ and } q_2 = 1/(\beta f'(z_2)).$$

Taking the derivative of $1/(1+r)$ with respect to t_1, we get

$$\frac{d}{dt_1}\left(\frac{1}{1+r}\right) = \frac{dq_1}{dz_1}\frac{dz_1}{dt_1} + \frac{dq_2}{dz_2}\frac{dz_2}{dt_1}.$$

From (21) it follows that $dq_1/dz_1 > dq_2/dz_2$. From (19) it follows that $d[1/(1+r)]/dt_1 < 0$, which implies that r rises.

Q.E.D.

Corollary: When there are no infrastructure effects and $U_{12} \geqq 0$, an increase in taxation (and in turn government expenditures) raises the riskless interest rate.

The partial statics results of Model I look like the popular characterization of the effects of fiscal policy on private investment. Under plausible assumptions, more government spending »crowds out» private investment and causes total output of the private sector to fall. In addition, countercyclical changes in fiscal policy can raise interest rates. To these results we need, however, to add two caveats: First, we can derive no information about welfare gains and losses from these results, since they are independent of the optimal distribution of private and public investment. Second, these results are based on a model in which there is a one-period delay between taxation and the production of the government good. As we shall show in the two succeeding sections, both the optimality and the partial statics results are extremely sensitive to this structural assumption.

5. *Model II: Optimality Conditions*

First-order conditions for Model II are given by:

(22)
$$-U_1(0) + \alpha(t_1) f'(z_1) U_1(1) = 0$$
$$-U_1(0) + \beta(t_2) f'(z_2) U_1(2) = 0$$

for production

$$[\alpha'(t_1) f(z_1) - 1] U_1(1) + U_2(1) G'(t_1) = 0$$

(23) $[\beta'(t_2) f(z_2) - 1] U_1(2) + U_2(2) G'(t_2) = 0$

for taxation

The general case for Model II is much more complicated than that of Model I. In this section we consider extensively the instance where both α and β are linear in taxation. This example well illustrates the complexities of Model II's optimality conditions. We assume that:[9]

$$\alpha(t_1) = \gamma t_1 + a, \ \gamma > 0, \qquad (24)$$
$$\beta(t_2) = \delta t_2 + b, \ \delta > 0.$$

We further assume that $0 < a < b$, so that in the absence of taxation there is positive productivity in each state of the world, and so that State 1 may be said in some sense to be »worse» than State 2.

Case 1: Suppose that the government good has *no* intrinsic utility. The first-order conditions for taxation now give

$$f(z_1) = 1/\alpha'(t_1) = 1/\gamma, \qquad (25)$$
$$f(z_2) = 1/\beta'(t_2) = 1/\delta.$$

Consumption in State 1 is given by

$$c_1 = \alpha(t_1) f(z_1) - t_1 = (\gamma t_1 + a) f(z_1) - t_1 = t_1 [\gamma f(z_1) - 1] + a f(z_1) = a f(z_1), \qquad (26)$$

where the first-order conditions for taxation (25) give the last equality. Similarly,

$$c_2 = b f(z_2) \qquad (27)$$

It thus follows that

$$c_1 - c_2 = a/\gamma - b/\delta \qquad (28)$$

Now examine the first-order conditions for production (22); rearranged, these give

$$\frac{U_1(c_1)}{U_1(c_2)} = \frac{\beta(t_2) f'(z_2)}{\alpha(t_1) f'(z_1)}. \qquad (29)$$

If $\gamma \geqq \delta$, then by (25) $f(z_1) < f(z_2)$, so that $f'(z_1) > f'(z_2)$. We have also shown that if $\gamma > \delta$ and $a < b$, then $c_1 < c_2$, which implies that the LHS of the last expression > 1. Thus the RHS must be > 1, which can only be if $\beta(t_2) > \alpha(t_1)$.

This establishes:

Theorem 7: Suppose that α and β are linear in taxation and have the form specified by (24) and suppose that the government good has no intrinsic utility, but only an infrastructure effect, then if the slope of α is greater than or equal to the slope of β,

$$c_1^* < c_2^* \text{ and } \beta(t_2^*) > \alpha(t_1^*).$$

The relative sizes of t_1^* and t_2^* depend on the slopes of α and β (in general anything is possible).

Remarks:

1. We cannot rule out the following seemingly perverse case: Suppose that α and β are linear as above and the government good has no intrinsic utility. If $\gamma < \delta$ (i.e., $\alpha' < \beta'$) then by the first-order conditions for taxation $f(z_1^*) > f(z_2^*)$, which implies that $f'(z_1^*) < f'(z_2^*)$. Now if $a/\gamma - b/\delta > 0$, then $c_1^* > c_2^*$ and it now follows from (29) that $\alpha(t_1^*) > \beta(t_2^*)$.
In this case, both the intercept and the slope of α are less than those of β. But in equilibrium a large-enough investment t_1 is made in State 1 so that $\alpha > \beta$ and the result is that $c_1 > c_2$.[10]
2. The above results assume that the government good has no intrinsic utility. If we assume that the government has both intrinsic utility and an infrastructure effect, then the paradoxical nature of the above results may still hold.

Case 2. The government good has no infrastructure effects, but only intrinsic utility. In this case we may prove the following theorem:

Theorem 8: Suppose that the government good has no infrastructure effects but does have intrinsic utility. Then:

8.1 $c_1^* < c_2^*$ and $t_1^* < t_2^*$ if $U_{12} = 0$.
8.2 $t_1^* < t_2^*$ if $U_{12} > 0$ and $c_1^* < c_2^*$.

[9] *Theorem 7 and the discussion which precedes and follows it explicitly assume that α and β have the functional form given in (24). The intuition behind these results can be generalized for other functional assumptions. The linear case, however, is both simple and gives most insight into the nature of the results.*

[10] *This example violates Assumption A.5. However, the same phenomenon may be shown to hold in more complex examples where A.5 is satisfied.*

Proof:

First note from the first-order conditions for taxation and production that

$$\text{(30)} \quad \frac{U_1(1)}{U_1(2)} = \frac{U_2(1)G'(t_1)}{U_2(2)G'(t_2)} = \frac{\beta f'(z_2)}{\alpha f'(z_1)}.$$

Suppose that $U_{12}=0$. Then if $z_1>z_2$, it follows from the first-order conditions for production that $c_1<c_2$, and it thus follows from (30) that $t_1<t_2$. On the other hand, if $z_1 \leqq z_2$, then either $c_1<c_2$, in which case the previous argument goes through, or $c_1 \geqq c_2$, which — again from (30) — would lead us to conclude that $t_1>t_2$. But this is a contradiction of the feasibility conditions embodied in the Model II budget equations, since it cannot be that $\alpha<\beta$, $z_1 \leqq z_2$, $t_1 \geqq t_2$, and $c_1 \geqq c_2$.

Suppose $U_{12}>0$ (private and government consumption are complementary). Then if $c_1<c_2$ and $t_1=t_2$, it follow that $U_1(1)/U_1(2)>1$, which by (30) shows that $[U_2(1)G'(t_1)/U_2(2)G'(t_2)]>1$. However, from complementarity it follows that at $t_1=t_2$ and $c_1<c_2$, this last expression is <1. By lowering t_1 and raising t_2, we lower the denominator and raise the numerator of $[U_2(1)G'(t_1)/U_2(2)G'(t_2)]$, at the same time raising the denominator and lowering the numerator of $U_1(1)/U_1(2)$.

Q.E.D.

We note that when $U_{12}>0$ we cannot rule out the (unlikely) case that $z_1^*>z_2^*$, $c_1^*>c_2^*$, and $t_1^*>t_2^*$. As long as the government and the private good are gross substitutes, for example, this case is impossible. Finally, note that Theorem 8 need not hold when the government and the private good are substitutes ($U_{12}<0$).

On the other hand, when the government good has only infrastructure effects, taxation will in general be positively correlated with private investment:

Theorem 9: Suppose the government good has no intrinsic utility and that at the optimum $z_1^*>z_2^*$. Then $t_1^*>t_2^*$.

Proof:

By equation (25), $\alpha'(t_1^*)<\beta'(t_2^*)$. The result now follows from assumption A.5.

Q.E.D.

Theorems 8 and 9 indicate why it is difficult to make general statements about the relationship between t_1 and t_2 when one allows for both intrinsic and infrastructure effects. In the absence of infrastructure effects, Theorem 8 shows that t_1^* tends to be less than t_2^*, whereas Theorem 9 indicates that for some infrastructure examples, t_1^* can be greater than t_2^*.

6. Model II: Comparative Statics

Whereas the optimality conditions for Model II are more complex than those of Model I, the comparative statics for this model are of the same order of complexity as those of Model I. An interesting difference between the two cases, however, is that some of the comparative statics results for Model II depend on the relation of the points considered to the optimal solution. To derive the comparative statics, we again totally differentiate the first-order conditions for production to get

$$\text{(31)} \quad \begin{bmatrix} U_{11}(0)+A(1) & U_{11}(0) \\ U_{11}(0) & U_{11}(0)+A(2) \end{bmatrix} \begin{bmatrix} dz_1 \\ dz_2 \end{bmatrix} = -\begin{bmatrix} C(1)dt_1 \\ C(2)dt_2 \end{bmatrix}$$

where the A's are the same as those in Model I's comparative statics:

$$\text{(16)} \quad \begin{aligned} A(1) &= \alpha(t_1)f''(z_1)U_1(1) \\ &\quad + [\alpha(t_1)f'(z_1)]^2U_{11}(1) \\ A(2) &= \beta(t_2)f''(z_2)U_1(2) \\ &\quad + [\beta(t_2)f'(z_2)]^2U_{11}(2) \end{aligned}$$

and

$$\text{(32)} \quad \begin{aligned} C(1) &= \{\alpha'(t_1)f'(z_1)U_1(1) \\ &\quad + \alpha(t_1)f'(z_1)U_{11}(1)[\alpha'(t_1)f(z_1)-1] \\ &\quad + \alpha(t_1)f'(z_1)U_{12}(1)G'(t_1)\} \\ C(2) &= \{\beta'(t_2)f'(z_2)U_1(2) \\ &\quad + \beta(t_2)f'(z_2)U_{11}(2)[\beta'(t_2)f(z_2)-1] \\ &\quad + \beta(t_2)f'(z_2)U_{12}(2)G'(t_2)\} \end{aligned}$$

This system has the following solutions:

$$(33)\quad \begin{aligned} dz_1 &= 1/\Delta\{-C(1)[U_{11}(0)+A(2)]dt_1 + C(2)U_{11}(0)dt_2\},\\ dz_2 &= 1/\Delta\{-[U_{11}(0)+A(1)]C(2)dt_2 + U_{11}(0)C(1)dt_1\},\end{aligned}$$

where $\Delta > 0$ is the determinant of the matrix on the left-hand side of (31).

The following theorem gives sufficient conditions for »crowding in» to exist in Model II:

Theorem 10: Provided either conditions 10.1 or 10.2 below are fulfilled, the partial statics of Model II (given by equations 33) have the following properties:

$$(34)\quad \begin{aligned} &dz_1/dt_1 > 0,\ dz_2/dt_1 < 0,\\ &dz_1/dt_1 + dz_2/dt_1 > 0.\\ &dz_1/dt_2 < 0,\ dz_2/dt_2 > 0,\\ &dz_1/dt_2 + dz_2/dt_2 > 0.\end{aligned}$$

The relevant conditions are:

10.1 $U_{12} \geqq 0$ and taxation is at or above its optimum level.

10.2 The government good has no infrastructure effects, taxation is optimal, and the public and private goods are not perfect substitutes.

Proof:

At the optimum or for values of t above the optimum level, the second term on the right-hand side of C(1) and C(2) is unambiguously positive (this follows from the first-order conditions for taxation (23)). When $U_{12} \geqq 0$, it follows that both C(1) and C(2) are positive, which gives the results.

When there are no infrastructure effects, C(1) becomes

$$C(1) = -\alpha(t_1)f'(z_1)U_{11}(1) + \alpha(t_1)f'(z_1)U_{12}(1)G'(t_1).$$

At the optimal taxation, this may be written (using (23)) as:

$$C(1) = \alpha(t_1)f'(z_1)[U_{12}(1)G'(t_1) - U_{11}(1)].$$

This expression is always non-negative. To see this, note that when the government and the private goods are perfect substitutes (i.e., when the utility function may be written U(c+G)), then $U_{12}G'(t) = U_{11}$. For less than perfect substitutability, $U_{12}G'(t) \geqq U_{11}$. If for this case the public and private goods are perfect substitutes, all of the terms in (34) are equal to zero.

Q.E.D.

Note that these results are diametrically opposite the results for Model I (equations (19)). Note also that these results are dependent on their relation to the optimum, whereas parallel results for Model I (given in Theorem 4) do not depend on their relation to the optimum. Finally, we observe that the conditions given in Theorem 10 are sufficient but not necessary; to prove that (34) holds, one need only show that C(1) and C(2) are positive.[11]

It follows directly from Theorem 10 that output rises in the state where taxation is increased and falls in the other state. This is not surprising, since in Model II, taxation is paid for from output in the *same* state as it has its effect. Thus, raising taxes in State 1, for example, means that more z_1 has to be invested at Date 0 in order to provide the resources at State 1 to pay for the increased taxes. Although *output* rises, whether or not *consumption* rises or falls is a more complicated question, as the next two theorems show. When the government good has no infrastructure effects and the government and the private good are substitutes, an increase in taxation makes private consumption fall (Theorem 11). On the other hand, if the public good has no intrinsic utility, then an increase in taxation tends to make private consumption rise (Theorem 12).

Theorem 11:

11.1 When C(1), C(2) > 0, $dc_1/dt_2 < 0$, $dc_2/dt_1 < 0$.

11.2 When $U_{12} \leqq 0$ and the government good has no infrastructure effects, $dc_1/dt_1 < 0$ and $dc_2/dt_2 < 0$.

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[11] Judd (1985) analyzes the impact of intertemporal shifts in taxation in a model where taxation and the production of the government are contemporaneous, as in our Model II. Judd assumes that the utility function is separable in the government and private good and that the government good has no infrastructure effects. Under these assumptions C(1) and C(2) are unambiguously positive, and Theorem 10 goes through. Although he is primarily interested in analyzing somewhat different questions than those considered here, Judd's results are consistent with ours.

Proof:
Differentiation of the budget conditions gives

$$(35)\quad \frac{dc_1}{dt_1} = [\alpha'(t_1)f(z_1) - 1] + \alpha(t_1)f'(z_1)\frac{dz_1}{dt_1}$$
$$\frac{dc_2}{dt_2} = [\beta'(t_2)f(z_2) - 1] + \beta(t_2)f'(z_2)\frac{dz_2}{dt_2}.$$

$$(36)\quad \frac{dc_1}{dt_2} = \alpha(t_1)f'(z_1)\frac{dz_1}{dt_2}.$$
$$\frac{dc_2}{dt_1} = \beta(t_2)f'(z_2)\frac{dz_2}{dt_1}.$$

Statement 11.1 is now immediate, since when both $C(1)$ and $C(2)$ are positive, then equations (34) hold.

To prove 11.2, note first that when there are no infrastructure effects it follows from (35) that

$$(37)\quad dc_1/dt_1 = -1 + \alpha(t_1)f'(z_1)dz_1/dt_1.$$

Furthermore, it follows from (33) that

$$(38)\quad dz_1/dt_1 = -1/\Delta\{C(1)[U_{11}(0) + A(2)]\},$$

where

$$(39)\quad C(1) = -\alpha(t_1)f'(z_1)U_{11}(1) + \alpha(t_1)f'(z_1)U_{12}(1)G'(t_1)$$

and

$$(40)\quad \Delta = A(1)A(2) + U_{11}(0)[A(1) + A(2)] = [U_{11}(0) + A(2)]A(1) + U_{11}(0)A(2).$$

Combining (37)–(40) gives

$$dc_1/dt_1 = -1 + \frac{[\alpha(t_1)f'(z_1)]^2}{\Delta}\{-U_{12}(1)G'(t_1) + U_{11}(1)\}\{U_{11}(0) + A(2)\}$$
$$= \frac{-\{[U_{11}(0) + A(2)]A(1) + U_{11}(0)A(2)\}}{\Delta} + \frac{\{U_{11}(0)+A(2)\}[\alpha(t_1)f'(z_1)]^2[U_{11}(1) - U_{12}(1)G'(t_1)]}{\Delta}.$$

When $U_{12} \leqq 0$, this whole expression is <0, since

$$|A(1)| > |U_{11}(1)|[\alpha(t_1)f'(z_1)]^2.$$

The proof for C_2 is similar.

Q.E.D.

Theorem 12: If the public good has no intrinsic value, then at or near the optimum, $dc_1/dt_1 > 0$ and $dc_2/dt_2 > 0$.

Proof:
It follows from the first-order conditions for taxation (23) that at the optimum

$$\alpha'(t_1)f(z_1) = \beta'(t_2)f(z_2) = 1.$$

Substituting this in (35) and using Theorem 10 now gives the result (note that if the government good has no intrinsic value, $U_{12} = 0$, so that Theorem 10 holds).

Q.E.D.

Theorems 10, 11, and 12 highlight important differences and similarities which result from the structural nature of fiscal policy. In Model II increases in government spending »crowd in» private investment, whereas in Model I, these increases »crowd out» private investment. In both models consumption of the private good drops if there are no infrastructure effects; this drop reflects the additional cost of financing the government good. In Model II the comparative statics depend on the optimal allocation of taxation versus private investment.

Finally, if we assume Cobb-Douglas production technologies, we have the following theorem:

Theorem 13: If $\alpha(t_1) < \beta(t_2)$ and if the production functions are Cobb-Douglas, then a pure countercyclical change in taxation ($dt_1 > 0$) lowers the riskless interest rate if $C(1) > 0$.

Proof:
From the proof of Theorem 6 it follows that if $\alpha(t_1) < \beta(t_2)$ and if $\alpha(t_1)f(z_1) < \beta(t_2)f(z_2)$, then $dq_1/dz_1 > dq_2/dz_2$ when the production function is Cobb-Douglas. From Theorem 8 it follows that if $U_{12} \geqq 0$, $c_1 < c_2$ and $t_1 < t_2$. By the budget equations for Model II, this means that

$$c_1 + t_1 = \alpha(t_1)f(z_1) < \beta(t_2)f(z_2) = c_2 + t_2.$$

Furthermore, by Theorem 10, $|dz_1/dt_1| > |dz_2/dt_1|$, and $dz_2/dt_1 < 0$. It thus follows that raising t_1 raises q_1+q_2, which means that the one-period riskless rate r (defined by $1/(1+r) = q_1+q_2$) falls.

Q.ED.

One should note that this effect is precisely the opposite of the effect on the one-period interest rate in Model I. In Model I, a countercyclical change in fiscal policy »crowds out» investment, by raising the required return to a riskless production technology. In Model II, a countercyclical change in fiscal policy lowers this required return, and »crowds in» investment. Furthermore, note that we can say little about the effect of procyclical changes in fiscal policy in Model II (as indeed we could say little about these changes in Model I).

A final difference between the two models relates to optimality. In Model I, we can make no statements about the Pareto effects of the »crowding out» of private investment which occurs as a result of more government spending. In this model, however, most of the cases for which we get unambiguous results imply that more government spending tends to reduce welfare. This is so because we can make such statements only if government policy is near or above its optimum.

7. Conclusion

We consider fiscal policy in a non-monetary, general equilibrium framework with uncertainty. In this framework the Ricardian equivalence proposition holds, and the *financing* of the government budget can have no effects. The partial statics of such a model allow us to determine whether fiscal policy »crowds out» or »crowds in» private investment. The model shows that »crowding out» or »crowding in» depend critically on the structural nature of the fiscal policy. The essential structural difference between the two types of fiscal policy we consider is whether the fiscal policy requires a gestation period or whether its effect is contemporaneous with the taxation by which it is financed. Both the optimality conditions and the comparative statics of the equilibrium are entirely different for these two cases. In addition to the timing of taxation as compared to the provision of the public good, the optimality conditions depend critically on whether the public good affects the efficiency of private production (infrastructure effects) and/or has intrinsic utility. Finally, we show that the usual normative connotations of »crowding out» and »crowding in» are misplaced, since either can lead to an increase or a decrease in social welfare, depending on whether the policy moves the equilibrium away from or closer to the global optimum.

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