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A NOTE ON DEPRECIATION ALLOWANCES, TAXATION AND RISK-TAKING

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Incentive effects of depreciation rules for risk-taking are considered. If true economic depreciation is stochastic, profit taxes with expected-value depreciation allowances decrease risk-taking. However, if real capital is used only in the risky sector, more generous depreciation allowances increase risk-taking. It is also shown that the incentive effect of risk-taking-revenue taxes to increase the risky fraction of investment is stronger in the case of more generous depreciation allowances.

1. Introduction

The impact of depreciation and depreciation allowances on the investment behavior of firms under conditions of certainty is well known (cf., e.g. Sandmo (1974)). If depreciation allowances exceed true economic depreciation, the optimal stock of capital is higher than it is with equal rates. Sinn (1985), (1988) shows that this effect carries over to an intertemporal general equilibrium setting. Do these results carry over to the case of uncertain profits?

Since the seminal paper by Domar and Musgrave (1944), there have been several treatments of the effect of profit taxation on the portfolio choice of a risk averse expected-utility maximizing investor. (cf. Allingham (1972), Koskela (1984), Koskela and Kanniainen (1984), Mossin (1968), Sandmo (1969), (1977), Stiglitz (1969) and Atkinson and Stiglitz (1980) for some earlier mayor contributions and Sandmo (1985) and Buchholz (1987) for further references). Depreciation allowances are not treated in these models, although they have sometimes been interpreted as not only describing portfolio choice problems, but also choices between bonds and real investment. Obviously, portfolios consist of holdings of cash or bonds and holdings of real assets, e.g., shares of firms, and the profits of firms crucially depend on depreciation allowances.

This paper concentrates on the effect of capital depreciation and depreciation allowances on risk-taking under uncertainty.

2. The benchmark case

Consider the problem of an investor in the Domar – Musgrave model. Let (A – a) be the amount that the investor holds as cash, and a be the amount of risky investment. The risky investment yields a stochastic net return Za, i.e. Z per unit of investment, and this return depends on profit taxes and depreciation allowances. The investor chooses a to maximize expected utility of his wealth at the end of the

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holding period, $Y = A + aZ$. Maximization of expected utility $U(Y)$ with respect to the investor's decision variable a requires

(1) $E[U'(Y)Z] = 0$,

(2) $E[U''(Y)Z^2] < 0$,

if both assets are held. Equation (2) holds because of risk aversion. Assume that a is used to buy a capital good. Each unit of capital good may yield gross return X, which is generally stochastic. The rate of true economic depreciation of the capital good may be δ, that of depreciation allowances for tax purposes may be δ_t. The riskless asset, typically cash or bonds, does not depreciate. A proportional profit tax t is levied on profits.[1] Therefore, the rate of net return, Z, can be described by

(3) $Z = X - \delta - (X - \delta_t)t$.

If true economic depreciation and depreciation allowances are equal, (3) becomes

(4) $Z = (X - \delta)(1 - t)$.

The determination of $\partial a/\partial t$, the optimal reaction of the investor to a change of the profit tax, is a standard problem, yielding (cf. Mossin (1968), p. 75)

(5) $\partial a/\partial t = a/(1 - t)$.

An increase in profit taxes induces the investor to increase his relative share invested in the risky activity. In sections 3 and 4 this reaction will be compared with the reaction in the case where true economic depreciation and depreciation allowances differ[2].

3. *Stochastic depreciation and expected-value depreciation allowances*

The breakdown of machines and the unforeseeable devaluation of capital goods as a result of technological change or the invention of new technologies make depreciation a random variable but, in many countries, depreciation allowances are determined by ex-ante rules: allowances are equal to the expected value of true economic depreciation.

To derive the impact of such non-equivalence of stochastic true economic depreciation and depreciation allowances, consider the following case. True economic depreciation δ is a random variable. Government cannot monitor true economic depreciation, but it knows the expected rate $E\delta$ and sets depreciation allowances $\delta_t = E\delta$.

In this case the rate of return is

(6) $Z = X - (X - E\delta)t - \delta$.

Inserting (6) into (1) and using the implicit function theorem, one gets

(7) $\partial a/\partial t = E[U'(Y)(X - E\delta)]/E[U''(Y)Z^2] + E[U''(Y)Za(X - E\delta)]/E[U''(Y)Z^2]$,

which cannot be signed without further restrictions. If, e.g., depreciation is non-random, (7) reduces to the Domar-Musgrave result (5). True economic depreciation and depreciation allowances are equal in this case. To see that even the opposite reaction is plausible, consider the particular case where true depreciation is the only source of randomness. In this case δ is a random variable and X is not stochastic, i.e. $X \equiv x \in \mathbb{R}$. Equation (7) can be transformed to

(8) $\partial a/\partial t = -a(x - E\delta)\,\partial a/\partial A + (x - E\delta)E[U'(Y)]/E[U''(Y)Z^2]$

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[1] *As is well-known from the results of Stiglitz (1972) and Atkinson and Stiglitz (1980), the effect of profit taxes on risk-taking depends crucially on the stochastic properties of tax revenues, on the use of these revenues, and how this use enters the utility function of investors. Here, as is standard in the recent Domar-Musgrave literature (cf. e.g., Ahsan (1988), (1989)), it is assumed that tax proceeds are used in a way that does not influence the investment behavior of households, e.g. by supplying public goods that influence the investors' utility in an additively separable way.*

[2] *Also Bulow and Summers (1984) address this issue. Similarly to the results derived here, they show that, given stochastic true depreciation, the disinvestment incentives of profit taxes with expected-value depreciation allowances can be offset by »additional depreciation allowances» which are considered in section 4 here. Their model is very different, as they exclude by assumption tax-induced changes in the willingness to assume risk. The analysis is more in the spirit of the Domar-Musgrave literature and takes the income effects of taxation on this willingness to assume risk into account. In a framework with only two possible states of nature, some manipulations can show that the effects of depreciation rules which have been considered in this paper are equivalent to combinations of state dependent taxes and subsidies. State dependent taxes have been analysed by Buchholz (1987, pp. 102–120).*

(for details cf. appendix). As $E[U''(Y)Z^2] < 0$ and $E[U'(Y)] > 0$, the second term in (8) has a negative sign. Moreover, $\partial a/\partial A \geq 0$, if U exhibits constant or decreasing absolute risk aversion (cf. Arrow 1970). This permits the conclusion:

Proposition 1 If depreciation is the only source of uncertainty and U exhibits constant or decreasing absolute risk aversion, and if depreciation allowances equal the expected value of true economic depreciation, profit taxes induce a decrease in the proportion of capital invested in the risky activity.

This result is the *inverse* of the Domar-Musgrave result, where profit taxes induce investors to increase risk taking. The intuition of this result is that, with risky depreciation, proportional profit taxation with expected -value depreciation allowances is similar to a proportional tax on *expected* profits. Government appropriates some revenues, but it does not participate in risk bearing, as it does in the benchmark case.

4. *Investment tax credits*

Now turn to the question of excessive depreciation allowances. Firms have to pay a profit tax on »taxable profit» with a proportional tax rate. Taxable profits are defined as gross profits aX minus true economic depreciation δ and some additional depreciation allowances $a\delta_c$ with $\delta_c \in \mathbb{R}_+$. These allowances are proportional to the amount a of risky capital. Allowances are higher than true economic depreciation.[3]

Notice that only the additional allowances are non random. Gross profits and true economic depreciation, X and δ, are both assumed to be random variables here. Depreciation allowances are similar to the benchmark case, i.e. they reflect the randomness of true economic depreciation, but there is an additional non random depreciation allowance. The rate of return becomes

(9) $Z = (X-\delta)(1-t) + t\delta_c$.

Consider now the investor's decision. Inserting equation (9) in conditions (1) and (2) yields the following conditions

(10) $E[U'(Y)((X-\delta)(1-t) + t\delta_c)] = 0$,

(11) $E[U''(Y)[(X-\delta)(1-t) + t\delta_c]^2] < 0$.

Again, (10) is the first order condition, and (11) is the second order condition for a maximum. (11) is met as the household is assumed to be risk averse. By the implicit function theorem it can be shown that a slight change of profit taxes implies

(12) $\partial a/\partial t = [a/(1-t)][1 + \Phi_1] + \Phi_2$,

with $\Phi_1 = \delta_c\, \partial a/\partial A$ and $\Phi_2 = E[U'(Y)((X-\delta)-\delta_c)]/E[U''(Y)[(X-\delta)(1-t) + t\delta_c]^2]$. The term $\Phi_1 > 0$ if U exhibits constant or decreasing absolute risk aversion, and $\Phi_2 > 0$ (for details cf. appendix).

Proposition 2 If U(Y) exhibits constant or decreasing absolute risk aversion, if profits are uncertain and depreciation allowances exceed true economic depreciation by an amount that is proportional to the amount of risky capital stock, then profit taxes increase the proportion of risky assets by more than in the case with depreciation allowances that equal true economic depreciation.

To put it differently, a subsidy on risky real investment ($\delta_c > 0$) increases the incentives for risk taking from a profit tax. Under plausible assumptions the increase of δ_c itself also induces more risk taking:

Proposition 3 If U(Y) exhibits constant or decreasing absolute risk aversion, for $t \in (0,1)$ an increase in depreciation allowances increases the optimal share of risky investment ($\partial a/\partial \delta_c > 0$).

To prove the result, differentiate (10) with respect to δ_c to get

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[3] *This kind of »additional depreciation allowances» is a frequently used approximation (see, e.g., Bulow and Summers (1984) and Dammon and Senbeth (1988)) to accelerated depreciation allowances. For a discussion of these two concepts see, e.g., Boadway and Bruce (1979, pp. 99n.).*

(13) $\partial a/\partial \delta_c = at\ \partial a/\partial A + \{-E[U'(Y)t]/E[U''(Y)((X-\delta)(1-t)+t\delta_c)^2]\}$

(for details cf. appendix). Using (11) and $U'(Y) > 0$ for all Y reveals that the term in curved brackets in (13) is positive. The term $[a\ t\ \partial a/\partial A]$ proves to be non-negative if $\partial a/\partial A \geq 0$, i.e. in the case of constant or decreasing absolute risk aversion.

5. Conclusions

This paper considers the effect of depreciation allowances on the investment behaviour under uncertainty. The main conclusion is that government has to consider carefully whether true economic depreciation is stochastic or not. If depreciation is the main source of randomness, and if government wants to increase risky real investment via increasing profit taxes, then depreciation allowances should reflect the randomness of true economic depreciation. If they do not, increasing profit taxes might induce investors to reduce risk-taking, contrary to the ordinary Domar-Musgrave case. The effects of increased depreciation allowances were also analysed. It was shown that increased depreciation allowances induce firms to choose higher levels of risky real investment and increase the effect of profit taxes.

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Appendix

Derivation of (7) and (8): differentiation of (1) with respect to t using $Y = A + aZ$ and equation (6) yields

(C1) $E[U''(Y)Z^2]\,\partial a/\partial t - E[U''(Y)Za(X - E\delta)] - E[U'(Y)(X - E\delta)] = 0$

$\Leftrightarrow$

(7) $\partial a/\partial t = E[U''(Y)Za(X - E\delta)]/E[U''(Y)Z^2] + E[U'(Y)(X - E\delta)]/E[U''(Y)Z^2]$.

Differentiation of (1) with respect to a and A yields

(C2) $\partial a/\partial A = -E[U''(Y)Z]/E[U''(Y)Z^2]$.

Using $X \equiv x$ and factoring out the constant $(x - E\delta)$ and then inserting (C2) in (7) yields equation (8):

(8) $\partial a/\partial t = -a(x - E\delta)\,\partial a/\partial A + (x - E\delta)\,E[U'(Y)]/E[U''(Y)Z^2]$.

Derivation of (12): differentiation of (10) with respect to t yields

(C3) $E[U''(Y)((X-\delta)(1-t) + t\delta_c)^2]\,\partial a/\partial t - E[U''(Y)((X-\delta)(1-t) + t\delta_c)\,a((X-\delta)-\delta_c)] - E[U'(Y)((X-\delta)-\delta_c)] = 0$.

solving for $\partial a/\partial t$ and decomposing yields

(C4) $\partial a/\partial t = [a/(1-t)]\,E[U''(Y)((X-\delta)(1-t) + t\delta_c)^2]/E[U''(Y)((X-\delta)(1-t) + t\delta_c)^2] - [a/(1-t)]\,\delta_c E[U''(Y)((X-\delta)(1-t) + t\delta_c)]/E[U''(Y)((X-\delta)(1-t) + t\delta_c)^2] + E[U'(Y)((X-\delta)-\delta_c)]/E[U''(Y)((X-\delta)(1-t) + t\delta_c)^2]$.

differentiation of (10) with respect to A yield

(C5) $\partial a/\partial A = -E[U''(Y)((X-\delta)(1-t) + t\delta_c)]/E[U''(Y)((X-\delta)(1-t) + t\delta_c)^2]$.

Cancelling down (C4) and using (C5) yields

(C6) $\partial a/\partial t = a/(1-t) + [a/(1-t)]\,\delta_c\,\partial a/\partial A + E[U'(Y)((X-\delta)-\delta_c)]/E[U''(Y)((X-\delta)(1-t) + t\delta_c)^2]$.

Derivation of the signs of Φ_1 and Φ_2: As δ_c is non-negative by assumption, Φ_1 has the same sign as $\delta a/\partial A$. A sufficient condition for $\partial a/\partial A \geq 0$ is constant or decreasing absolute risk aversion. Φ_2 is equal with the third term of (C4). It is positive if the numerator is negative (the denominator is negative from (11)). $E[U'(Y)((X-\delta)-\delta_c)]$ can be shown to be negative by following manipulations:

$E[U'(Y)((X-\delta)(1-t) + t\delta_c)] = 0$ (from (10)).
$\Leftrightarrow$ $E[U'(Y)(X-\delta)(1-t)] + E[U'(Y)\,t\delta_c] = 0$.
$\Rightarrow$ $E[U'(Y)(X-\delta)(1-t)] < 0$ as $t\delta_c > 0$.
$\Rightarrow$ $E[U'(Y)(X-\delta)] < 0$, as $t \in [0,1)$.
$\Rightarrow$ $E[U'(Y)(X-\delta)] - \delta_c E[U'(Y)] < 0$.
$\Rightarrow$ $E[U'(Y)((X-\delta)-\delta_c)] < 0$.

Derivation of (13): differentiation of (10) with respect to δ_c yields

$E[U''(Y)((X-\delta)(1-t) + t\delta_c)^2]\,\partial a/\partial\delta_c + E[U''(Y)((X-\delta)(1-t) + t\delta_c)(at)] + E[U'(Y)t] = 0$,

or, solving for $\partial a/\partial\delta_c$:

$\partial a/\partial\delta_c = -atE[U''(Y)((X-\delta)(1-t) + t\delta_c)]/E[U''(Y)((X-\delta)(1-t) + t\delta_c)^2] - E[U'(Y)t]/E[U''(Y)((X-\delta)(1-t) + t\delta_c)^2]$.

using $\partial a/\partial A$ as given by (C5) yields (13).