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# OPTIMUM TAXATION OF INTEREST AND IMPUTED RENT FROM CONSUMER DURABLE GOODS

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*A simple two-period model is set up to carry out a normative analysis of the taxation of interest income and imputed rent from consumer durable goods. It is demonstrated how the optimum structure of this capital income taxation depends on savings and investment elasticities and cross effects with the labour market when labour income is also taxed.*

## 1. Introduction

An economy can save in different forms in order to increase the opportunities for future consumption. By investing in production equipment and by acquiring financial assets, i.e. claims on other countries, the opportunities for consumption in general are being expanded. By investing in consumer durable goods (dwellings, cabins, leisure boats, etc.) the specific opportunities for future consumption of the services rendered by these goods are being increased. Let us assume that the return to savings in general is given by a fixed interest rate. The underlying assumption may be that the country can lend or borrow at an exogenous interest rate in an external capital market. The interest rate is then the opportunity cost of capital held as household durable goods.

Under income taxation the return to savings is being taxed. Interest is being taxed, with interest on debt being deductible. And the income equivalent of services from household durable goods, usually termed the imputed rent, is being taxed. But in practice these different kinds of return to savings are not always taxed at the same rate. In particular it is common to tax the imputed rent much more leniently than interest.

The taxation of household durable goods, and housing in particular, is an important issue in the tax debate. It is often claimed that the imputed rent should be taxed at the same rate as interest. The argument is usually that the trade-off between investing in durable goods and saving in other assets should not be distorted. But under second best constraints it is not clear that it is desirable to avoid this particular distortion when savings and labour supply are distorted in the first place. A second best analysis of optimum taxation of household durable goods is required. An analysis of this kind was presented in Sandmo (1988). In the present paper the same problem is analysed using a somewhat different formulation. The current analysis is coached in savings terms rather than consumption terms.

The model is presented in section 2. The optimal taxation of capital income is analysed in section 3. I make some brief concluding remarks in section 4.

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## 2. *The model*

There is assumed to be an initial resource endowment, Y, which can be used for consumption in the initial period, $C_0$, for investment in household durable goods, D, or for other kinds of savings, S. Hence there is a budget constraint

(1) $Y = C_0 + D + S.$

There is an exogenous interest rate, i, on savings, and there is a tax rate $t_i$ on interest. The resources available in the future are due to labour and savings. Let L denote labour, w denote the wage rate and $t_w$ the tax rate on labour earnings. In similar models analysed elsewhere in the literature labour has often been assumed to be supplied in the initial period. See for example Atkinson and Sandmo (1980) and Sandmo (1988). One should note the different assumption adopted here. There is also a tax being imposed on the imputed rent on durable goods, $t_D iD$, where $t_D$ is the tax rate and iD is the imputed rent. Future consumption is then equal to

(2) $C_1 = I + (1 + (1-t_i)\, i)\, S + w\,(1-t_w)\, L - t_D iD$

The exogenous income variable I serves an analytical purpose only and is set equal to zero when not needed for this purpose. By substituting for S, we obtain the intertemporal budget constraint

(3) $(1 + (1-t_i)\, i)\, C_0 + (1 + (1-t_i)\, i + t_D i)\, D + C_1 = (1 + (1-t_i)\, i)\, Y + I + w\,(1-t_w)\, L$

For convenience we rewrite (3) as

(3') $q_0 C_0 + q_D D + C_1 = (1 + (1-t_i)\, i)\, Y + I + q_L L$

where the q's are the respective coefficients appearing in (3). These parameters have obvious interpretations as prices.

The utility function of the representative consumer is

(4) $U\,(C_0, C_1, D, L).$

Since only efficiency aspects will be considered, the analysis is confined to the one-consumer case.

Maximizing (4) with respect to all the arguments subject to (3') leads to the supply and demand functions

(5)
$$\begin{aligned} &C_0\,(q_0, q_D, q_L, q_0\, Y + I),\\ &C_1\,(q_0, q_D, q_L, q_0\, Y + I),\\ &D\,\;(q_0, q_D, q_L, q_0\, Y + I),\\ &L\,\;(q_0, q_D, q_L, q_0\, Y + I), \end{aligned}$$

and the corresponding indirect utility function

(6) $V\,(q_0, q_D, q_L, q_0\, Y + I).$

In the following partial derivatives with respect to $q_0$ are defined as derivatives with respect to both entries of $q_0$ in the functions. The effects of the tax rates on the respective prices are easily found to be

(7) $$\frac{\partial q_0}{\partial t_i} = -i,\ \frac{\partial q_D}{\partial t_i} = -i,\ \frac{\partial q_L}{\partial t_w} = -w,\ \frac{\partial q_D}{\partial t_D} = i.$$

Making use of the well-known standard properties of the indirect utility function, and letting λ denote the marginal utility of income, we can derive the marginal effects of taxes on utility:

(8) $$\frac{\partial V}{\partial t_i} = \lambda C_0 i + \lambda iD - \lambda iY = -\lambda iS,$$

(9) $$\frac{\partial V}{\partial t_w} = -\lambda wL,$$

(10) $$\frac{\partial V}{\partial t_D} = -\lambda iD.$$

We shall focus on the tax policy to be applied in the future. The tax revenue is equal to

(11) $R = t_i iS + t_D iD + t_w wL = t_i i\,(Y - C_0 - D) + t_D iD + t_w wL.$

The government is assumed to have a fixed tax revenue requirement, $R_0$. Hence the tax policy is chosen under the constraint that

(12) $R\,(t_i, t_D, t_w) = R_0,$

where the tax revenue has been expressed as a function of the tax parameters.

## 3. *The optimal tax policy*

The optimal tax policy is the vector of tax parameters that maximizes the utility function (6) subject to the constraint that (12) is satisfied. First order conditions can be derived by making use of the Lagrange function

(13) $F = V(q_0, q_D, q_L, q_0Y) + \mu(R(t_i, t_D, t_w) - R_0)$.

The first order conditions characterizing the optimal tax policy are

(14) $$\frac{\partial F}{\partial t_i} = \frac{\partial V}{\partial t_i} + \mu \frac{\partial R}{\partial t_i} = 0,$$

(15) $$\frac{\partial F}{\partial t_D} = \frac{\partial V}{\partial t_D} + \mu \frac{\partial R}{\partial t_D} = 0,$$

(16) $$\frac{\partial F}{\partial t_w} = \frac{\partial V}{\partial t_w} + \mu \frac{\partial R}{\partial t_w} = 0.$$

Our present concern is with the taxation of capital income, and we shall focus on the two former conditions which are conditions for optimal taxation of capital income whether the taxation of labour income is optimal or not. Eliminating $\mu$ we can rewrite the first order conditions as

(17) $$\frac{\frac{\partial R}{\partial t_i}\lambda}{-\frac{\partial V}{\partial t_i}} = \frac{\frac{\partial R}{\partial t_D}\lambda}{-\frac{\partial V}{\partial t_D}} = \frac{\frac{\partial R}{\partial t_w}\lambda}{-\frac{\partial V}{\partial t_w}}$$

where the last equation is satisfied only if the taxation of labour income is optimal.

The effects of tax rate changes on revenue are easily derived from (11) taking into account the relations in (5) and the tax effects on prices presented in (7). We find that

(18) $$\frac{\partial R}{\partial t_i} = iS - t_i i \frac{\partial C_0}{\partial q_0}(-i) - t_i i \frac{\partial C_0}{\partial q_D}(-i) - t_i i \frac{\partial D}{\partial q_0}(-i) - t_i i \frac{\partial D}{\partial q_D}(-i) + t_D i \frac{\partial D}{\partial q_0}(-i) + t_D i \frac{\partial D}{\partial q_D}(-i) + t_w w \frac{\partial L}{\partial q_0}(-i) + t_w w \frac{\partial L}{\partial q_D}(-i),$$

(19) $$\frac{\partial R}{\partial t_D} = iD - t_i i \frac{\partial C_0}{\partial q_D} i - t_i i \frac{\partial D}{\partial q_D} i + t_D i \frac{\partial D}{\partial q_D} i + t_w w \frac{\partial L}{\partial q_D} i,$$

(20) $$\frac{\partial R}{\partial t_w} = wL - t_i i \frac{\partial C_0}{\partial q_L}(-w) - t_i i \frac{\partial D}{\partial q_L}(-w) + t_D i \frac{\partial D}{\partial q_L}(-w) + t_w w \frac{\partial L}{\partial q_L}(-w).$$

Let us now introduce the following notation. Let $s_{ij}$ be the compensated derivative of quantity i with respect to price j, where i = 0 (for $C_0$), 1 (for $C_1$), D, L, and j = 0, 1, D, L indicating the prices $q_0$, $q_1$, $q_D$ and $q_L$. Note that since labour is supplied $s_{Lj} = -s_{jL}$ for $j \neq L$. Let also $S_j$ be the compensated derivative of S with respect to price j. Then making use of the Slutsky equation we can rewrite (18) – (20) as

(18') $$\frac{\partial R}{\partial t_i} = iS - t_i i \frac{\partial S}{\partial I} iS - t_D i \frac{\partial D}{\partial I} iS - t_w w \frac{\partial L}{\partial I} iS + t_i i(-s_{00} - s_{D0})(-i) + (t_D i - t_i i)(-s_{0D} - s_{DD})\, i + t_w w (s_{0L} + s_{DL})\, i,$$

(19') $$\frac{\partial R}{\partial t_D} = iD - t_i i \frac{\partial S}{\partial I} iD - t_D i \frac{\partial D}{\partial I} iD - t_w w \frac{\partial L}{\partial I} iD - t_i i s_{0D} i + (t_D i - t_i i) s_{DD} i - t_w w s_{DL} i,$$

(20') $$\frac{\partial R}{\partial t_w} = wL - t_i i \frac{\partial S}{\partial I} wL - t_D i \frac{\partial D}{\partial I} wL - t_w w \frac{\partial L}{\partial I} wL - t_i i s_{L0} w + (t_D i - t_i i) s_{LD} w - t_w w s_{LL} w.$$

Since $S = Y - C_0 - D$, it follows that

(21) $S_j = -s_{0j} - s_{Dj}$.

Making use of this equation we can rewrite (18') as

(18'') $$\frac{\partial R}{\partial t_i} = iS - t_i i \frac{\partial S}{\partial I} iS - t_D i \frac{\partial D}{\partial I} iS - t_w w \frac{\partial L}{\partial I} iS - t_i i S_{0i} + (t_D i - t_i i) S_D i - t_w w S_L i.$$

Having established the marginal effects of the tax rates on revenue and utility, we can derive the following optimality conditions from (17):

(22) $$-t_i i \frac{S_0}{S} + (t_D i - t_i i)\frac{S_D}{S} - t_w w \frac{S_L}{S} = -t_i i \frac{s_{D0}}{D} + (t_D i - t_i i)\frac{s_{DD}}{D} - t_w w \frac{s_{DL}}{D} = -t_i i \frac{s_{L0}}{L} + (t_D i - t_i i)\frac{s_{LD}}{L} - t_w w \frac{s_{LL}}{L},$$

where the first equation is the condition for optimal taxation of capital income, and the last equation is satisfied if one strikes the optimal balance between taxation of capital and labour income. Introducing elasticities we can write these conditions as

$$-\Theta_0\sigma_{S0} + \Theta_D\sigma_{SD} - \Theta_L\sigma_{SL}$$
$$(23) \quad = -\Theta_0\sigma_{D0} + \Theta_D\sigma_{DD} - \Theta_L\sigma_{DL}$$
$$= -\Theta_0\sigma_{L0} + \Theta_D\sigma_{LD} - \Theta_L\sigma_{LL},$$

where $\sigma_{ij}$ is the Slutsky elasticity of quantity i with respect to price j, and

$$\Theta_0 = \frac{t_i i}{q_0},$$
$$(24) \quad \Theta_D = \frac{t_D i - t_i i}{q_D},$$
$$\Theta_L = \frac{t_w w}{q_L}.$$

The Θ's are interpreted as relative tax rates. $\Theta_L$ is the tax rate on labour relative to the net wage rate. $\Theta_D$ is the tax rate on durable goods relative to their price. We observe that the net tax on the imputed rent is the gross tax rate $t_D$ minus the interest tax which in fact works as a subsidy. The interest is the opportunity cost of holding capital in the form of durable goods, and the tax on interest reduces this cost by reducing the after-tax interest. $\Theta_0$ is the tax on savings relative to the discount factor $1 + (1-t_i)i$. The effect of $t_i$ is to reduce the price of $C_0$. Hence $\Theta_0$ may also be interpreted as the relative subsidy to $C_0$.

Using the first equation of (23) we can solve for the relative tax rates on interest and imputed rent:

$$(25) \quad \frac{\Theta_D}{\Theta_0} = \frac{\sigma_{S0} - \sigma_{D0}}{\sigma_{SD} - \sigma_{DD}} - \frac{\Theta_L}{\Theta_0}\frac{\sigma_{DL} - \sigma_{SL}}{\sigma_{SD} - \sigma_{DD}}.$$

The formula demonstrates how the optimal balance between $\Theta_D$ and $\Theta_0$ is related to the elasticities and the relative taxation of labour and interest income. Alternatively, we can write (25) as

$$(26) \quad \frac{\Theta_D}{\Theta_0} = \frac{el_{q_0}(S/D)}{el_{q_D}(S/D)} + \frac{\Theta_L}{\Theta_0}\frac{el_{q_L}(S/D)}{el_{q_D}(S/D)}.$$

Formula (25) (or (26)) offers a guide to the optimal *structure* of capital income taxation while the *level* of capital taxation is determined by the tax revenue requirement and the level of labour income taxation which may or may not be optimal. Let us now consider a number of (increasingly complex) cases. In the presentation it is implicitly understood that effects, elasticities, etc. refer to the compensated entities. Let us first assume that the wage rate has no effect on savings and investment in durable goods, $\sigma_{DL} = \sigma_{SL} = 0$. The assumption will be relaxed below.

*Case 1*

Assume that $\sigma_{DL} = \sigma_{SL} = 0$. Let us also assume that the price of initial period consumption has no effect on the accumulation of durables, $\sigma_{D0} = 0$, and the price of durable goods has no effect on savings, $\sigma_{SD} = 0$. Then (25) takes the very simple form

$$(27) \quad \frac{\Theta_D}{\Theta_0} = \frac{\sigma_{S0}}{-\sigma_{DD}}.$$

This is simply the conventional inverse elasticity rule. The ratio between $\Theta_D$ and $\Theta_0$ is equal to the inverse ratio of $\sigma_{S_0}$ to the absolute value of $\sigma_{DD}$. The elasticity $\sigma_{S0}$ is the elasticity of savings with respect to the discount factor $1 + (1-t_i)i$ and thus reflects the interest elasticity of savings. If savings are relatively more elastic than investment in durable goods, interest should be taxed at a lower rate than the imputed rent on durable goods.

*Case 2*

Assume that $\sigma_{DL} = \sigma_{SL} = 0$, while other cross effects may be non-zero. (26) then takes the form

$$(28) \quad \frac{\Theta_D}{\Theta_0} = \frac{el_{q_0}(S/D)}{el_{q_D}(S/D)}.$$

This is also an inverse elasticity rule. The ratio between the relative tax rates $\Theta_D$ and $\Theta_0$ is inversely related to the ratio between the elasticity of S/D with respect to the price of durable goods and the elasticity of S/D with respect to the price of initial period consumption. It follows that if the relative quantities in question are more responsive to the discount factor than to the price of durable goods the imputed rent should be taxed at a higher rate than interest.

An interesting question is when there should be uniform taxation of interest and imputed rent, $t_i = t_D$. Uniform rates obviously imply that the effective tax on durable goods, $\Theta_D$, is zero. It follows immediately from (28) that this is the case if savings and investment in durable goods are equally elastic with respect to the discount factor $1 + (1-t_i)i$. We should note that the elasticities in question are partial elasticities implying that the price of durable goods is kept constant. Hence, neglecting cross effects to the labour market, uniform taxation at the optimum requires that savings and investment in durable goods increase by the same percentage if the interest rate increases while the effect on the rental price of durable goods is somehow offset. The effect of an increase in the interest rate is to make initial consumption relatively more expensive, which in turn releases substitution from initial consumption to savings and investment in durable goods. The crucial question is whether S and D show the same relative changes.

*Case 3*

Let us assume that all elasticities may be non-zero. We then have to take all the terms of (26) into account. Since the direct effects on S and D have been considered already, let us now focus on the latter term of (26) to reveal the significance of cross effects between the labour market and the capital market. Assuming that labour income and interest are being taxed both Θ's are positive. We also assume that the denominator is positive. We then see that if, other things equal, the impact of an increase in the wage rate is to increase (reduce) the saving-durable goods investment ratio (S/D), the relative emphasis on taxation of durables should be increased (decreased). To get an intuitive understanding of the result, let us for a moment neglect the effect on S. Then the ratio increases if D decreases. If a higher wage rate is accompanied by lower investment in durables it means that the labour supply and consumption of durables move in opposite directions. Then there is a case for taxing and reducing the consumption of durable goods in order to encourage the supply of labour which is distorted in the first place because of the income tax. If for instance having the opportunity to spend time in a nice house makes people less inclined to work, a tax on housing should be used to contain this effect.

If there is a positive (negative) cross effect, uniform taxation of interest and imputed rent requires that an increase in the interest rate for a constant rental price on durables reduces (increases) the saving-durables investment ratio, or, if uniform taxation is optimal in the absence of cross effects, it is not in the presence of interrelated markets.

## 4. Concluding remarks

The current study has presented the main determinants of the second-best optimal relative taxation of interest and imputed rent from household durable goods. The analysis has been carried out within a simple model that is stripped of all complications that are not essential for the main issue. The characteristics have been coached in savings terms rather than in the consumption terms that have been a rule in optimum taxation studies.

The analysis may be interpreted as taking a long-term perspective. It is concerned with the allocation of capital that emerges as a result of investment that is affected by the tax policy. But when a tax reform is considered it will in the short run affect a stock of capital that is the outcome of investment decisions of the past that are no longer sensitive to current or future tax policy. For some assets the rate of investment is very low compared to the stock of assets. In particular this is true for housing which is the most important household durable good. Since the stock does not respond very much in the short run, it is a good short-term tax base for the standard reason that it is desirable to choose tax bases that are relatively unresponsive in order to avoid tax-escaping substitutions that distort the allocation.

However, a policy which exploits the short-term rigidity will also be signalling that this is going to happen in the future. The reason is that the announcement of a tax policy that is different in the short run and in the long run, can hardly be credible, since the long-term horizon of today will gradually be transformed to the short-run perspective of tomorrow. Hence the long-term perspective seems to be the more interesting and important.

## References

**Atkinson, A.B. and Sandmo, A. (1980),** »Welfare implications of the taxation of savings,» *Economic Journal* 90, 93–105.

**Sandmo, A., 1988,** »The taxation of household durable goods,» *Finnish Economic Papers* 1, 13—24.