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ESTIMATING THE TECHNICAL EFFICIENCY OF A FIRM BY ELIMINATING THE EFFECTS OF EXOGENOUS FACTORS

URMAS SEPP*

Institute of Economics, 200101 Tallinn, Estonia

The paper discusses the problem of estimating technical efficiency in a way objective from the standpoint of firms. Such estimation is possible if the effect of factors uncontrollable by the firm is eliminated. To carry out the corresponding analysis a system of factors of technical efficiency and possibilities for solving this system are presented. An approach based on a deterministic model of the effect of factors noncontrollable by the firm is recommended. The application of this technique is possible in case of limited information when only time-series data can be used. In connection with this also the way technical efficiency is to be estimated is referred to. For practical application of this approach an optimal programming problem is formulated. A numerical example is given.

1. Introduction

Before starting the discussion of the problem let us define the terms needed.

By technical efficiency we refer to the characteristic of the production activities of a given production unit (e.g. a firm) showing the level of the realization of the production possibilities. Production efficiency is calculated as the ratio of the actual production to its possible quantity. The nearer the actual production to the possible quantity, i.e. the nearer the ratio to one, the higher the production efficiency (and vice versa).

The term »production possibilities» denotes here the quantity of output of the given firm under the complete realization of its production potential. Production potential is the maximum capacity of the firm to produce products of a certain assortment during a definite period of time.

Production efficiency depends both on the factors which are controllable and which are noncontrollable by the firm. Let us call the factors of the former group endogenous, and those of the latter group, exogenous.

The essence of the problem treated below is how to eliminate exogenous factors in estimating production efficiency. In many occasions it is very effective to determine to what extent the realization of production possibilities is due to the factors noncontrollable by the firm, and to what extent it is caused by the effect of endogenous factors. Such estimation is of great practical importance, especially when trustworthy characterization of the firm's operation is required.

Mathematically the problem of distinguishing endogenous and exogenous effects lies in

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decomposing the deviations of the actual efficiency from the maximum efficiency (which is always equal to one) into two parts. One part of the deviations are caused by exogenous factors, another by endogenous ones.

The deviation of actual efficiency from the maximum efficiency can be defined as »inefficiency», and the problem could be reformulated accordingly. In this case our goal would be to find a way of decomposing inefficiency into the effects of exogenous and endogenous factors.

The formulation of such decomposition problem is not new. First discussions of the topic were published by Aigner, Lovell and Schmidt (1977), Schmidt and Lovell (1979) already in the late 1970s. Those two as well as the more recent reports analyzing the decomposition of inefficiency dealt with the problem within the framework of stochastic frontier functions. In such a case the inefficiency is presented by two components. For example, in Aigner Lovell and Schmidt (1977), Schmidt and Lovell (1979) the inefficiency is treated by the sum $v_t + u_t$, where v_t is the stochastic part of underused production possibilities caused by exogenous factors; and u_t is the share of inefficiency due to the firm's actions. Without going into details we may say that a stochastic frontier production function is used to estimate components of inefficiency by mean values characterizing the whole set analyzed (see Aigner, Lovell and Schmidt (1977), Schmidt and Lovell (1979), Meeusen and van den Broeck (1977)), or by individual values describing the activity of a certain firm belonging to the set analyzed (Materov (1981), Afanasiev and Sokolov (1985), Danilin, Materov, Rosenfield and Lovell (1985)). The application of the stochastic production function in the role considered calls for a large number of statistical data in order to obtain adequate results. The only suitable informational base is cross-section data of a sector/branch or of the whole national economy. In case of such varied initial information the use of a stochastic function, reflecting an abstract technology of all firms belonging to a set, becomes highly questionable in the analysis of the inefficiency of a concrete firm. The realization of the hypothesis of the behavior of u or v (see Aigner, Lovell and Schmidt (1977), Schmidt and Lovell (1979), Meeusen and van den Broeck (1977)) with respect to each firm is also dubious (irrespective of the fact that these hypotheses are not dubious for the set as a whole).

Therefore it seems that a stochastic frontier function applying cross-section data can actually be used only for finding out the average values of the inefficiency components.

A stochastic frontier function can also be measured using panel data. Panel data are obtained as a result of combining cross-section data with time series data on individual firms. In this case, using a stochastic frontier function, it is possible to find out the level of efficiency of a firm belonging to the set investigated (see Schmidt and Sickles (1984)). This level is an average figure for a certain period. But if the aim is to find the dynamics of a firm's efficiency and to analyse it, the stochastic function obtained by the treatment of panel data will be of no use. The same holds in case the investigator is interested in the abovementioned problems but has at his disposal only time series data characterizing a concrete firm.

To evaluate the dynamics of technical efficiency and to determine the effects of endogenous and exogenous factors for a single firm a different approach is needed. Thus, to tackle these tasks we will use the approach based on the deterministic frontier function. Obviously, it would have been possible to proceed from an alternative approach, i.e. nonparametric analysis of efficiency. Banker and Maindiratta (1988) have shown the use of nonparametric analysis on the basis of time-series data to measure the technical and economic efficiency of a firm. Still, the approach based on a parametric frontier function is preferred. It enables to connect the efficiency analysis with the evaluation of frontier function. In addition, we point out the fact that a single product transformation process is examined.

2. *The factors of technical efficiency*

We have composed a factor system of technical efficiency analogically to M.J. Farrell's total or overall factor system of efficiency, where the resulting variable is equal to the product of factor efficiency indices (see Farrell (1957)). In our case we can write the following two-element system of technical efficiency indicators:

(1) $C_j = C_j^v \quad C_j^s$,

where

(2) $C_j = Q_j^T / Q_j^P$;

(3) $C_j^v = Q_j^v / Q_j^P$;

(4) $C_j^s = Q_j^T / Q_j^v$;

Q_j^T is the quantity of actual output formed under the influence of exogenous and endogenous factors; Q_j^P is the quantity of output corresponding to the production possibilities; Q_j^v is the quantity of output under the effect of exogenous factors. In other words, Q_j^v shows the quantity of potential output under the noncontrollable by the firm conditions.

The coefficient C_j^v shows the share of the quantity of the potential output produced under the external conditions prevailing in the period j in the quantity of potential output produced under the most favorable external conditions.

Thus, C_j^v shows how much the production potential of the period j is smaller due to the influence of exogenous factors than it would be if these factors were lacking. Thus, C_j^v may be called exogenous production efficiency.

C_j^s is the indicator of the utilization of the production potential formed under the influence of factors controllable by the firm. As the effect of exogenous factors has been eliminated in this indicator, it would in a pure form describe the level of production efficiency achieved as a result of the firm's activities. Consequently, C_j^s is the parameter of endogenous production efficiency.

Mathematically, the indicator C_j is the product of C_j^s and C_j^v. Thus, C_j is a generalization of these measures showing the level of technical efficiency (technical efficiency is determined by exogenous and endogenous factors).

Technical efficiency can be treated as a function of the intensity of using inputs. At that intensity is determined by exogenous and endogenous factors.

The idea of efficiency analysis is to investigate the realization of the production possibilities. The realization of the production possibilities is a problem of short-run management. Consequently, according to the present paper, efficiency consideration consists in an analysis of the short-run production. The long-run aspect of efficiency, i.e. the results of the formation of the production possibilities themselves will be left out of consideration.

So, the aim of efficiency analysis is to obtain an estimate showing to what extent the efficiency of current production is determined by the activities of the firm itself, and to what extent other factors are involved.

3. *Measurement of efficiency indices*

It becomes evident from the above that for the analysis of technical efficiency three levels of efficiency — C_j, C_j^s and C_j^v — are to be known. These indices can be determined in two ways: (a) measuring directly the efficiency indiced, and (b) measuring the quantities of output Q_j^T, Q_j^V and Q_j^P necessary for the calculation of efficiency indices.

3.1 Direct measurement of efficiency indices

According to (1) technical efficiency is determined by the deviation of the actual quantity of output Q_j^T from the possible quantity Q_j^P. This deviation may be caused by an error of the frontier function (the mathematical form used does not correspond to the regularities of the growth of the object modeled) and by the effect of factors not taken into account. Let us assume that the mathematical model of the function used corresponds to the object. In this case the deviation is caused by factors not considered. Let us further assume that corresponding function includes all relevant inputs. Then only factors reflecting the intensity of the use of the inputs have been left out of consideration. Thus

(5) $C_j = g(K_j)$,

where $g(K_j)$ is a function describing the effect of the intensity of using the inputs of the production; K_j is the set of measures of the intensity of using inputs.

The intensity of using inputs depends both on controllable and noncontrollable factors. To put it differently, the intensity of using inputs is determined by exogenous and endogenous factors:

(6) $C_j = g^{s,v}(K_j^s, K_j^v)$,

where $g^{s,v}(K_j^s, K_j^v)$ is the function showing the effect of exogenous and endogenous factors on the intensity of using inputs; K_j^s and K_j^v are sets of measures reflecting the effect of exogenous and endogenous factors, respectively.

Knowing the essence of the factors of technical efficiency defined in (2) – (4) the following dependences can be formulated

(7) $C_j^v = g^v(K_j^v)$

and

(8) $C_j^s = g^s(K_j^s)$,

where $g^v(K_j^v)$ and $g^s(K_j^s)$ are the function showing the effect of exogenous and endogenous factors on the intensity of using inputs.

To analyze the efficiency, i.e. to determine the levels of exogenous and endogenous efficiencies, functions (5) or (6) and (7) or (8) must be measured. The measurement of either function (7) or (8) is justified by the fact that from expression (1) the following formulas can be derived:

(9) $C_j^v = C_j / C_j^s$

or

(10) $C_j^s = C_j / C_j^v$.

Therefore, instead of the direct measurement of either exogenous or endogenous efficiency an indirect way can be used, i.e. corresponding indicators can be measured with the help of formulas (9) or (10). To apply this way of measurement presumes knowing of the above-mentioned functions and the sets of measures K_j, K_j^s and K_j^v are to be known. However, such information is actually impossible to obtain in case of practical analysis.

3.2 Measurement of efficiency indices by the quantities of output

According to formulas (2)—(4) it is necessary to determine three quantities of output — Q_j^T, Q_j^V and Q_j^P — for measuring the indices of technical efficiency. No problems arise in estimating the real output Q_j^T.

The evaluation of Q_j^P is more complicated. The quantity of output corresponding to the production possibilities Q_j^P can be measured using the frontier production function. If the frontier is specified as Cobb-Douglas function,

(11) $Q_j^P = A \prod_i r_{ji}^{\alpha_i}$,

where A and α_i are the parameters of the function; r_{ji} is the quantity of the input i (for example, the number of workers, etc.) in the period j; $j \in [1,m]$; $i \in [1,n]$; m is the number of periods in the time interval analyzed; n is the number of inputs included in the frontier function.

Of course, for application of function (11) its parameters must be found out. The frontier function (11) can be measured applying the technique presented by Sepp (1986), which is based on the evaluation of parameters B and β_i of the mean production function $f_j = B \prod_i r_{ji}^{\beta_i}$ by OLS (after taking logarithms) from

$Q_j^T = f_j\, e^{\rho_j}$,

where ρ_j is an error of the function.

Knowing β_i it is possible to calculate $b_i' = \beta_i / \Sigma\beta_i$ and $M_j = \prod_i r_{ji}^{\beta_i'}$ needed for estimating the parameter w by OLS (after taking logarithms) from

$$\frac{Q_j^T}{f_j} = M_j^w\, e^{\varepsilon_j},$$

where $\varepsilon_{\hat{j}}$ is an error of the function and $\hat{j}$ denotes the observation where $Q_j^T > f_j$. Now we must find $\Delta B = \max_{\hat{j}} (\frac{Q_{\hat{j}}^T}{f_{\hat{j}}} : M_{\hat{j}}^w)$.

Finally parameters of the frontier function can be calculated in accordance with the following relationships:

$A = B\, \Delta B$ and $\alpha_i = \beta_i + w\, \beta_i'$.

To calculate the third quantity of output Q_j^v necessary for the application of the factor system of technical efficiency the following way of reasoning might be used. As Q_j^v is a potential quantity, it must be calculated using the frontier function. Furthermore, in addition to inputs also variables describing exogenous factors must be included into the frontier function (11). Let us assume that we know a set of exogenous factors K_j^v (it is not important whether this set consists of one or several indices). Hence

(12) $Q_j^v = f^v(r_{ji}, K_j^v)$,

where $f^v(r_{ji}, K_j^v)$ is the frontier function representing the quantity of potential output under the effect of exogenous factors.

Unfortunately, the application of function (12) involves a number of rather complicated problems of specification; for example, should $f^v(r_{ji}, K_j^v)$ represent the same model as the frontier function (11), or not, how should Q^v depend on K^v, etc. After a separate measurement of functions (11) and (12) contradictions of estimation may turn out to be a problem, etc.

So, for an analysis of technical efficiency the measurement of efficiency indices by means of the quantities of output is not rational.

3.3 Measurement of efficiency indices in a combined way

In arranging practical analysis it is advisable to combine the calculation methods of both approaches. It would be useful to calculate the level of technical efficiency as a ratio Q_j^V / Q_j^P, with Q_j^P determined by the frontier function estimated in the above-presented way. However, the indices of endogenous and exogenous efficiencies must be measured in a different way.[1]

The exogenous efficiency can be computed on the basis of function (12). The measurement of the function $f^v(r_{ji}, K_j^v)$ will be essentially simplified if we assume that

$$f^v(r_{ji}, K_j^v) = (A \prod r_{ji}^{\alpha_i})\, g^v(K_j^v).$$

In the case of the presented specification only the component of the exogenous efficiency $g^v(K_j^v)$ must be found to measure $f^v(r_{ji}, K_j^v)$. No contradiction with frontier function (11) will arise if $f^v(r_{ji}, K_j^v)$ is calculated in the above-mentioned way. What is even more important — the measurement of $g^v(K_j^v)$ will be a practically soluble problem which we will deal with in the next section. Thus, having calculated the technical efficiency C_j (from formula (2) and with the help of the frontier function), and having measured separately $g^v(K_j^v)$, i.e. the exogenous efficiency, we can use for measuring the endogenous efficiency C_j^s an indirect approach requiring no supplementary information. According to formula (10) the figure sought can be expressed as

(13) $C_j^s = C_j / g^v(K_j^v)$.

Acting in the way described we joined two approaches to the measurement of efficiency indices. The technical efficiency C_j is calculated on the basis of the quantities of output, and the exogenous efficiency C_j^v is found on the basis of direct estimations.

4. Formulation of a problem of estimating exogenous efficiency[2]

For the practical realization of the above presented treatment it is necessary to estimate $g^v(K_j^v)$. Since $Q_j^v \geq Q_j^T$[3], then in accordance with (2) and (3)

(14) $C_j^v \geq C_j$.

Inequality (14) reflects the hypothesis according to which the exogenous efficiency is always higher than the technical efficiency. This hypothesis is true because of two circumstances: (1) the range of factors whose effects are involved in technical efficiency is wider than that of exogenous efficiency (the former is determined both by endogenous and exogenous factors while the latter is determined by exogenous factors alone); (ii) the effect of efficiency factors can be only negative.[4]

With the help of (13) and (14) the following inequality is formed

[1] *The presented variant would, in principle, be analogous to the case where the level and function of the endogenous efficiency could be ascertained. Then the exogenous efficiency could be found as the ratio of technical efficiency to endogenous efficiency. To spare the space we will not examine this variant more closely.*

[2] *The problem presented is based on the assumption that the technical efficiency C_j is known before analyzing production efficiency.*

[3] *This is caused by the fact that Q_j^v is the potential output and Q_j^T is the actual output, and the potential output is always bigger than the actual.*

[4] *The effect of a factor is usually regarded as either unfavorable, neutral, or favorable. This scale could be changed so that instead of estimates »unfavorable», »neutral» and »favorable», the estimates »extremely unfavorable», »unfavorable» and »neutral» are used. This is the scale applied in estimating efficiency factors.*

(15) $g^v(K_j^y) \geq C_j$.

Let us recall here that the maximum value of $g^v(K_j^y)$ is equal to one. This results from the inequality $Q_j^y \leq Q_j^p$. This means that if the effect of the exogenous factors is highly favorable (there exist no external factors hindering the firm from using its production possibilities), Q_j^y will be equal to Q_j^p. As generally firms operate under less favorable external conditions, the potential output will also be smaller. Therefore

(16) $C_j \leq g^v(K_j^y) \leq 1$.

Within the interval $[K_{min}^v, K_{max}^v]$ (where K_{min}^v and K_{max}^v are respectively the minimum and maximum values of K_j^y) inequality (16) is realized by an infinite number of the curves $g^v(K_j^y)$. Therefore the question arises which of the possible specifications is the right one for reflecting C_j^y.

The curve $g^v(K_j^y)$ accompanied by the minimum sum of the squariances of $g^v(K_j^y)$ from the values of C_j should be preferred. Thus the criterion of preference is

(17) $\min \Sigma [C_j - g^v(K_j^y)]^2$.

According to (16) such a $g^v(K_j^y)$ is considered correct which is close to C_j. This preference is motivated by the fact that in such a case $g^v(K_j^y)$ will coincide (or nearly coincide) with the extreme values of C_j. These C_j-s stand for the technical efficiency in case of which the endogenous efficiency is the highest.

In formulating the problem of estimating $g^v(K_j^y)$ it is also important to take into consideration the dependence of the exogenous efficiency on K_j^y. In this connection it is logical to assume that if external conditions improve, i.e. K_j^y increases, the exogenous efficiency will grow. This means that the better the external conditions, the higher the potential possibilities determined by these circumstances, consequently, the smaller the difference between these possibilities and the maximum possibilities developing under the most favourable external conditions. This is why the exogenous efficiency is the highest under the most favourable external conditions.

It follows from the above-said that $g^v(K_j^y)$ reflecting exogenous efficiency should increase monotonically in accordance with also monotonically increasing K_j^y in the interval $[K_{min}^v, K_{max}^v]$. Therefore

(18) $\delta g^v(K_j^y)/\delta K_j^y > 0$,

where $K_j^y \in [K_{min}^v, K_{max}^v]$.

Let us finally assume that in the point where technical efficiency is equal to one, i.e. in the point where the production possibilities are completely realized, the exogenous and endogenous efficiencies are both equal to one. Such an assumption is logically well founded. Obviously there is no need to prove that the exogenous efficiency C^v is maximum in this case (under the most favourable external conditions described by the maximum value of K_j^y, the production possibilities coincide with the maximum possibilities). The maximum value of the endogenous efficiency C^s ensues from (10), whence $C_j^s = C_j / C_j^v = 1/1$.

Let us introduce this assumption into the problem of estimating $g^v(K_j^y)$ in the form of the constraint

(19) $g^v(K_{max}^v) = 1$.

Summing up, we can say that the problem of estimating $g^v(K_j^y)$ or exogenous efficiency is a problem of optimal programming with an objective function (17) and the constraints (15), (16), (18) and (19).

5. A numerical example

To measure the exogenous and endogenous efficiencies a set of the values of C_j and K_j^y characterizing the production process of an Estonian textile enterprise in various subperiods is known. C_j is calculated using the method presented in section 3. K_j^y as interpreted above is the indicator describing the effect of exogenous factors. K_j^y has been specified as the ratio showing the share of maximum possible working time of the labor used. To find this ratio the used working time is to be divided to the maximum working time. In the USSR there it is compulsory to report both these indicators to the department of statistics. The used working time is the time measure involving all kinds of labor services used (incl. also overtime). The maximum possible working time is calculated as the sum of the used working time and the losses of working time (excl. the losses due to annual leaves of absence).

In connection with the K_j^y specification the question will arise whether this specification is appropriate for expressing the effects of exogenous factors. It would be natural to assume that the use of working time depends on both exogenous and endogenous factors. Unfortunately, the economic situation under which the enterprise observed worked was such where only the exogenous factors affected the level of this index. The situation was extremely difficult from the point of view of the enterprise as it had to operate under labor shortage. So, the reasons for a lowering the share of used working time were only of the kind not depending on the activities of the enterprise. Incomplete use of the maximum working time was due to maternity leave, study leave, sickness leave and absences allowed by the law. Of course, an enterprise could have hired temporary labor force to fill the place of those on leave. Unfortunately, spare labor was nowhere to be found.

In connection with the specification of K_j^v it must, as a matter of fact, be mentioned that it would have been more correct to use intensity indices of not one but all major inputs (first of all for the index describing the use of capital). They could have been included either as independent variables or in a generalized form (mean, etc.) in the function of the exogenous intensity. Unfortunately, insufficient records are kept regarding the use of capital in enterprises of the USSR and also Estonia. Therefore, adequate information is not available. However, this does not cause essential errors since under shortage it is the labor and the intensity of its use that are the constraints determining the limits of intensity of using the rest of inputs. So, the intensity of the labor use depending on exogenous factors should be, as viewed from that aspect, representative also with respect to other inputs.

The initial data are presented in the Table 1.

First we studied the dependence of technical efficiency on exogenous factors. Applying correlation analysis it appeared that the correlation coefficient was equal to 0.81. The determination coefficient was accordingly 0.65. It means that 65 % of the technical efficiency variation resulted from the influence of exogenous factors. Thus, the remaining 35 % of variation is the effect of endogenous factors.

To formulate the measurement problem of exogenous efficiency a specification of $g^v(K_j^v)$ is necessary. To obtain an adequate specification $g^v(K_j^v)$ must be defined by as large a number of models as possible his will give us a chance to minimize the errors of function due to the model's inadequacy. In our example we established only one specification of $g^v(K_j^v)$ as the empirical calculation showed the dependence of C_j on K_j^v to be approximately linear. Thus

$$g^v(K_j^v) = a_0 + a_1K_j^v + a_2(K_j^v)^2. \tag{20}$$

For solving case (20) of problem (15)–(19)

Table 1. Data and results of efficiency analysis.

Measure of exogenous factors	Technical efficiency	Exogenous efficiency	Endogenous efficiency
K_j^v	C_j	$C_j^v = g^v(K_j^v)$	C_j^s
.8493	.9694	.9603	.9906
.8654	.9895	.9747	.9850
.8530	.9741	.9582	.9837
.8676	.9923	.9763	.9838
.8610	.9840	.9749	.9907
.8667	.9911	.9692	.9779
.8667	.9912	.9761	.9848
.8610	.9840	.9837	.9997
.8483	.9682	.9456	.9767
.8672	.9917	.9777	.9859
.8607	.9837	.9746	.9908
.8610	.9840	.9828	.9987
.8540	.9753	.9747	.9994
.8472	.9669	.9495	.9820
.8506	.9710	.9521	.9804
.8562	.9780	.9633	.9849
.8475	.9672	.9549	.9873
.8664	.9908	.9905	.9997
.8473	.9670	.9609	.9937
.8576	.9798	.9540	.9737
.8626	.9860	.9648	.9785
.8709	.9964	.9907	.9944
.8567	.9787	.9566	.9774
.8522	.9731	.9684	.9952
.8458	.9651	.9545	.9890
.8517	.9724	.9673	.9947
.8643	.9882	.9786	.9903
.8653	.9894	.9734	.9839
.8718	.9976	.9924	.9948
.8519	.9727	.9556	.9824
.8550	.9766	.9643	.9874
.8448	.9639	.9638	1.0000
.8562	.9781	.9580	.9795
.8647	.9887	.9633	.9744
.8677	.9924	.9807	.9882
.8683	.9932	.9674	.9740
.8721	.9978	.9874	.9895
.8738	1.0000	1.0000	1.0000

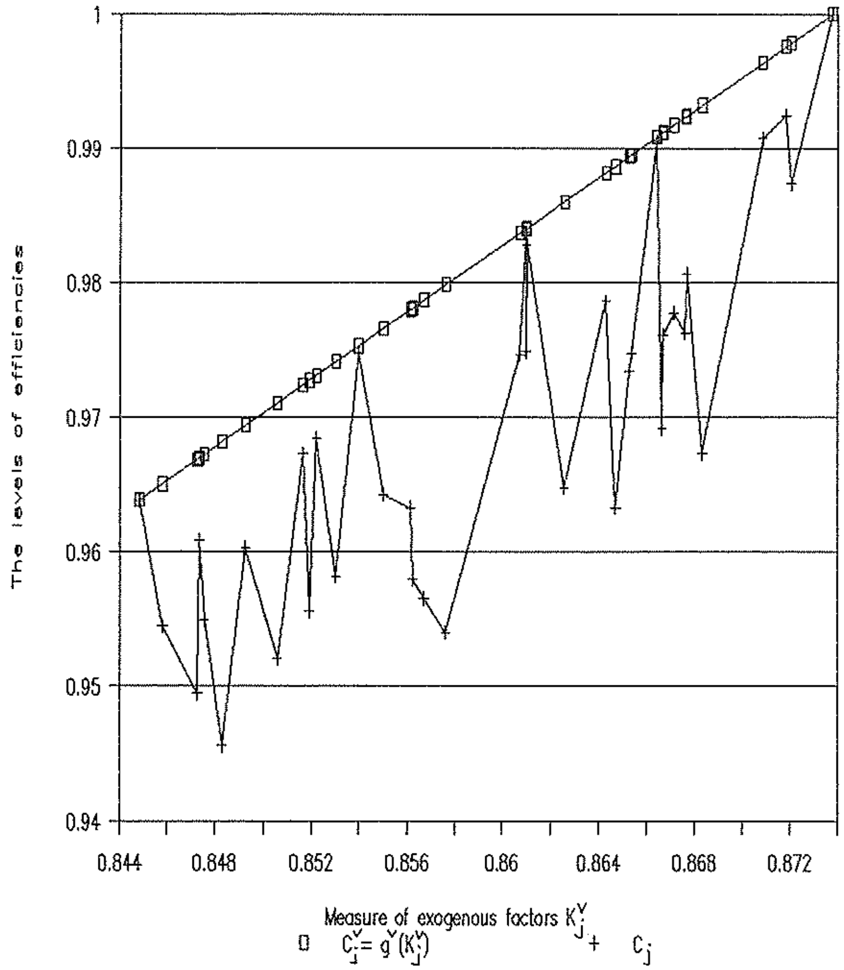

Figure 1. Graphic presentatin of the estimated values of $g^v(K_j^v)$.

a modernized form of Nelder-Mead method (see D. Himmelblau (1975) p. 163 – 173) was used. The results were the following: a_0 = –0.090884; a_1 = 1.248422; a_2 = 0.000053.

Then we were able to calculate the theoretical values of the above-mentioned function (see Table 1 and Fig. 1). Using (10) we can measure the endogenous efficiency. The indices of endogenous efficiency are shown in the table and Figure 2, as well. So, not to bother the reader with details we did not aim at analyzing the variation and dynamics, etc. of the endogenous and exogenous efficiencies. For example, Figure 2 shows rather well that the exogenous efficiency exceeds the endogenous efficiency starting from a certain level of external conditions ($K_j^v \in (0.864 \ldots 0.868)$). Before that level, in case of worse external conditions, the situation is just the opposite. As technical efficiency is growing, it follows that, in case of better external conditions, exogenous intensity is a relatively more essential factor of technical efficiency than under worst conditions.

This set of data allows various other essential observations, but at the moment something else is important. The question of main interest is the reliability of the estimated values of the exogenous efficiency.

Using the method by Humal (1961) to consider resulting variable change by factors the effects of exogenous factors in the technical

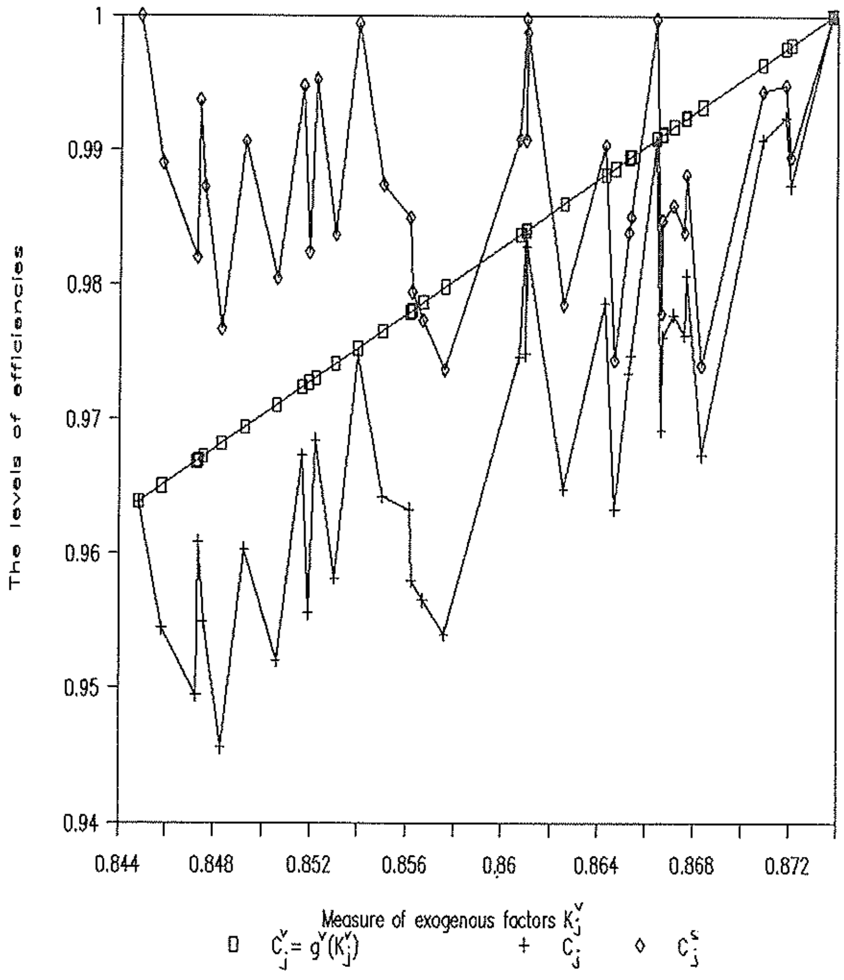

Figure 2. Graphic presentation of the levels of efficiencies.

efficiency variation were found. Then the mean value of the exogenous effects was calculated. It was equal to 58 %. This ratio means that 58 % of the inefficiency is caused by the influence of exogenous inefficiency (resp. factors). This ratio accords with the results of the correlation analysis. The confidence limits of the correlation coefficient measured on the basis of Fischer's z-value (with 95 % certainty) are 0.747 and 0.873. The corresponding limits of the determination coefficient are 0.558 and 0.762. So, applying Humal's method, the mean value of the exogenous effects estimated (0.58) lies within the limits mentioned. This means that the mean of exogenous efficiency, and therefore the individual values of exogenous efficiency can be regarded as reliable.

It would also have been interesting to check whether the measurement of the exogenous efficiency is affected when endogenous efficiency is also independently measured, or whether the levels of the exogenous intensity measured independently and using formula (9) are in accord. Unfortunately, the shortage of data referred to above did not allow us to find out the index K_j^s and to make corresponding researches.

6. *Conclusions*

1. The problem of assessing the level of production efficiency with eliminating external conditions arises when an objective estimation of a firm is needed.

2. In earlier investigations dealing with this topic the problem of decomposition of the inefficiency has been treated within a stochastic frontier function. As such a technique cannot be applied to analyze the dynamics of technical efficiency of one and only one firm, then to solve this problem a different way is needed.

3. The above-mentioned problem can be solved using a technique based on a deterministic frontier function. This function is complemented by a component reflecting the effect of exogenous factors of efficiency. After transformations it is obvious that the component of exogenous factors is identical to the indicator of exogenous efficiency of production, i.e. to the efficiency determined only by such factors which are noncontrollable by the firm. Knowing the values of the technical efficiency, i.e. the efficiency formed under the influence of both controllable and noncontrollable by the firm factors, a problem of estimating this component can be formulated.

4. Mathematically it is an optimal programming problem. In formulating the objective function the assumption according to which the exogenous efficiency coincides (or nearly coincides) with the relatively highest values of the technical efficiency is considered correct.

To fix the constraints two regularities can be relied on. First it is logical to assume that if exogenous conditions improve, the exogenous efficiency will grow. This means that the function specifying the exogenous component must be monotonically increasing within the interval analyzed. On the other hand, it is advisable to limit the values of the exogenous efficiency so that they would be higher than the level of the technical efficiency of production (the solution of this problem requires that the level of the technical efficiency should be known beforehand).

5. The validity of the results obtained using the suggested technique is tested by a numerical example. It was elucidated that the results can be regarded as reliable.

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