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A NOTE ON ON-THE-JOB AMENITIES AND SUPPLY OF LABOR: The Cobb-Douglas case

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In an article published by Decker (1988) it is demonstrated that the effect of earnings taxation on the compensated labor supply is ambiguous. In this paper I demonstrate that with the Cobb-Douglas utility function an unambiguous result is attainable. In this case, the compensated tax effect on desired work hours is negative even if the on-the-job amenities are endogenous.

1. *Introduction*

The importance of job characteristic in labor supply analysis is receiving increasing attention. Atrostic (1982) provides an empirical investigation into labor supply elasticities where the consumption of job characteristics is considered explicitly. Decker (1988) builds a theoretical model in which he investigates the theoretical implications of income taxation on compensated labor supply. As far as I know, his work is the first in this direction, and his results are interesting: the compensated tax effect on labor supply is not necessarily negative. This contradicts the traditional models in which the compensated tax effect is always negative.

To look more closely on this result, I recalculate Decker's model using an explicit Cobb-Douglas utility function. It seems, however, that Decker's result cannot hold in the case of the Cobb-Douglas utility function. In this case the conventional result is true anyway: the utility compensated tax effect of labor supply is negative, even if there are on-the-job amenities.

2. *The model*

In Decker's model the worker derives utility from on-the-job amenities associated with his job. The wage rate is endogenous because there exists a systematic, compensated wage differential-based relationship between the work amenities in a given job and the wage the job pays. It is assumed that there is only a single dimension of job amenities, A. This is assumed to be consumed per work hour, H. Total consumption of work amenities K is equal to amenities consumed per hour multiplied by the hours worked by the individual, K = AH.

The special form of the utility function is

$$(1) \quad U(L, C, K) = L^{\alpha} C^{\beta} K^{\tau}; \alpha + \beta + \tau = 1, \quad 0 \leq \alpha \leq 1, 0 \leq \beta \leq 1, 0 \leq \tau \leq 1.$$

In Decker's model the budget constraint takes into account the relationship between the endogenous wage rate and on-the-job amenities. This relationship takes a linear form

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(2) $W = \alpha_0 + \alpha_1 A;\ \alpha_0 > 0,\ \alpha_1 < 0,$

where W is the wage rate. The theory of compensating wage differentials tells us that, in equilibrium, α_1 must be negative. α_0 is the worker's potential wage, i.e. wage the worker can earn if he consumes no amenities.

The consumption constraint is

(3) $C = (1-t)\ HW + Y,$

where Y is unearned income. The price of C is normalized to unity. The tax rate is t. The total time T is equal to hours of work H and leisure L, T = H + L. Combining these constraints we get

(4) $C = (\alpha_0 + \alpha_1 A)\ (1-t)\ H + Y.$

To simplify the interpretation it is useful to define Becker's (1965) »full income» $M = \alpha_0 (1-t)\ T + Y$. The full income is the income the worker earns if he pays no attention to amenities or leisure. The full income M is equal to budget constraint (4) in optimum if the utility weight of consumption is equal to one, $\beta = 1$.

The utility maximization (1) subject to budget constraint (4) results in Marshallian demand functions: $L(Y, w^0) = \alpha M/w^0$, w^0 is the net price of leisure, $w^0 = \alpha_0 (1-t)$, $C(Y, 1) = \beta M$, the net price of consumption is normalized to unity, $K(Y, w^1) = AH = \tau M/w^1$, w^1 is the net price of amenities, $w^1 = -\alpha_1 (1-t)$ (α_1 is negative). The demands of leisure, consumption and amenities are equal to their utility and price weighted budget shares.

The indirect utility function is

(5) $V(Y, w^0, w^1) = M\ (\alpha/w^0)^\alpha\ (\beta)^\beta\ (\tau/w^1)^\tau.$

For shortness I denote the three terms in brackets on the r.h.s. X. The expenditure function can then be written in terms of lump sum payment needed to attain the given level of utility V:

(6) $e(V, w^0, w^1) = VX^{-1} - Tw^0.$

This leads to the Hicksian demand for leisure:

(7) $$\frac{\partial e(V, w^0, w^1)}{\partial w^0} = l(V, w^0, w^1).$$

3. *The utility compensated tax effect*

To evaluate Decker's result in this connection the Slutsky decomposition for tax is needed. The Marshallian demand at lump sum payment $e(V, w_0, w_1)$ is the same as the Hicksian demand at utility V:

(8) $L(e(V, w^0, w^1), w^0) = l(V, w^0, w^1).$

The tax effect on the Hicksian demand for leisure is

(9) $$\frac{\partial l(V, w^0, w^1)}{\partial t} = \frac{\partial L(Y, w^0)}{\partial Y} \cdot \frac{\partial e(V, w^0, w^1)}{\partial t} + \frac{\partial L(Y, w^0)}{\partial w^0} \cdot \frac{\partial w^0}{\partial t}.$$

With the given functional form this becomes

(10) $$\frac{\partial l(V, w^0, w^1)}{\partial t} = \frac{-\alpha}{w^0} \cdot \frac{(1-\beta)}{(1-t)} VX^{-1} + \frac{\alpha}{w_0} \cdot \alpha_0 T + \frac{\alpha Y}{\alpha_0 (1-t)^2}.$$

At first glance it seems possible that the substitution effect is negative because the first term on the r.h.s. is negative, the others being positive, leaving the total effect undetermined. This is also what Decker states. But look closer. By setting (10) to be less than nought, and arranging, we obtain

(11) $$\frac{\partial l(V, w^0, w^1)}{\partial t} = \frac{-\alpha(1-\beta)}{(1-t)} . VX^{-1} + \alpha_0 T\alpha + \frac{\alpha Y}{(1-t)} < 0.$$

The first two parts on the r.h.s. is the income effect. It has a negative component: if the tax increases, the expenditure needed to achieve the given level of utility must decrease because the net price of leisure and amenities are lower after the tax increase than before it. The second part of the income effect is positive: if the tax increases, the expenditure needed to achieve the given level of utility must in-

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[1] *A CES utility function approach would reveal whether there are any values of substitution elasticities that justify Decker's results.*

crease because the potential net wage is now lower. The total effect can be easily evaluated. After manipulation, the equation (11) turns out to be

$$(12) \quad \frac{\partial l(V, w^0, w^1)}{\partial t} = -(1-\beta)\, VX^{-1} + M < 0 \text{ or } M < (1-\beta)\, VX^{-1}.$$

In other words if the decrease in the expenditure, because of lower prices of leisure and amenities, is bigger than the full income, the inequality will hold. But this cannot be true simply because the full income is at least equal to the expenditure that is needed to achieve the utility level dictated by the full income. Savings from the tax increase cannot be bigger than full income. So the traditional result, which states that the compensated tax effect on labor supply is negative, survives at least in this analysis that is based on the Cobb-Douglas utility function.

Clearly this result raises the question whether it holds with more general utility functions than Cobb-Douglas. To answer this question further research should be conducted in this field[1].

4. Conclusion

In this paper the Cobb-Douglas utility function is applied to the model derived by Decker (1988). In his model the sign of substitution effect on labor supply of an increase in the proportional rate of earnings taxation cannot be determined. However, if a Cobb-Douglas utility function is chosen, the indeterminacy is solved. The traditional result remains to be valid in this case: the sign of substitution effect is negative.

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