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THE TAXATION OF HOUSEHOLD DURABLE GOODS

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Starting from issues raised in current debates about tax policy, this paper considers the optimal taxation of durable consumer goods in the context of a two-period overlapping generations model. The consumer model is one where individuals work in the first period, being retired in the second, and invest their savings either in financial assets or in durable goods. The optimum tax analysis considers several problems of tax policy under alternative assumptions about the policy instruments available to the government and about the nature of preferences. The role of the compensated cross-elasticities between labour supply, savings and durable goods is emphasized.

1. Introduction

The tax treatment of the return on household capital in the form of durable consumer goods has been a matter of concern in public policy discussions in many countries. No doubt the main reason for this is that the case of durable consumer goods — housing in particular — serves as a focus of attention for the concern with the distortionary effects of capital taxation in general. In particular this has been the case in Norway, which represents perhaps a rather extreme case of a problem which is common to many Western countries. The relevant features of the tax system may briefly be described as follows.

The services yielded by the stock of durable goods — which would in principle be taxable under a comprehensive income tax — are taxable only in the case of housing. The value of living in an owner-occupied house is assessed for tax purposes as 2.5 per cent of the tax valuation of the house. However, the tax valuation itself is far below the market value of the house; typically about 10—20 per cent of the likely sales price. The implication of this is of course that the real yield of the housing stock which enters into the computation of taxable income, is less than 0.5 per cent of its market value. Capital gains on housing — both real and nominal — are as a rule tax exempt.

There is a wealth tax in Norway, and it applies also to stocks of durable goods. In addition to housing the tax base includes items such as cars, sailboats, furniture etc. Again, the tax valuation is typically below market value, although the degree of undervaluation varies from one item to another. As a general characterization one must clearly conclude that wealth holdings in the form of durable goods are very gently treated by the Norwegian tax system.

The taxation of financial assets is based on nominal values. The rates of return on financial assets are taxable at full nominal values, and interest on debt is deductible without limitation. With high rates of inflation and

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high marginal tax rates, this has led to real after-tax rates of return which have been very low and at times even negative for large groups of taxpayers. The wealth tax also applies to financial assets, although there are some lump-sum deductions for items like bank deposits and insurance policies. There is also a capital gains tax, although at lower rates than the regular marginal tax rate faced by most taxpayers.

If one considers durable goods as a store of value it is clear that there are significant asymmetries in the tax treatment of durable goods and financial assets. Many critics have pointed out that the most profitable portfolio strategy for individual taxpayers has been to borrow heavily in order to »overinvest» in housing and other durable goods. This line of argument usually continues by pointing out that demand for credit by individual consumers crowds out private productive investment, and that this creates problems for the long-run development of the economy. Calls for reform have focused on the need to limit the deductibility of interest on loans and to achieve more realistic tax valuations of durables, in particular of housing. Neutrality criteria have been invoked to argue for a uniform treatment of all types of investment, or at least for a uniform tax rate on all portfolio investments made by consumers.

A different argument which has also been advanced in the debate, is the following. The Norwegian system of personal income taxation is highly progressive: in 1986 well over 20 per cent of all taxpayers had a marginal tax rate in excess of 50 per cent (including social security contributions). This leads to heavy distortions of work incentives and thus to what may be the most serious efficiency losses in the economy. The tax encouragement to invest in housing and other durable goods acts as a positive counterincentive to this type of distortion. The discouragement to work effort is not as large as it might have been, since people can always find a low-tax outlet for their earnings by buying a house, a summer cottage, or a sailboat.

Both these views are at best very partial, even if one accepts their neglect of the distributional issue. The first point of view concentrates on the tax distortions of consumers' portfolio choice and to some extent their saving decision. The second one concentrates on distortions of labour supply. Taking a broader view it is difficult to argue that one set of considerations is *a priori* of greater importance that the other, and this calls for an integrated treatment of the optimum tax issue in which both dimensions of the problem are given due attention.

There are few specific applications of optimum tax theory to the present issue. The article by Atkinson (1977) is specifically concerned with housing expenditure and housing allowances, and studies the connections between housing demand and labour supply. His model does not take savings decisions into account, and thus it cannot be used to analyse the first of the two views described above. An intertemporal extension of the analysis has been provided by Pines, Sadka and Sheshinski (1985), but they investigate the optimal tax structure only under rather special assumptions about separability. This leads them to some highly interesting conclusions, but it is clearly of some importance to see whether some results can be derived even under more general assumptions about the structure of demand.

The policy arguments which have been outlined above are pure efficiency considerations. Because my concern here is with the efficiency issue as such, I shall feel justified in neglecting distributional problems and analyse the consumer side of the economy as if it consisted of perfectly homogeneous individuals. This is in contrast to Atkinson (1977), who treats the problem of redistribution within a fixed population of consumers, and to Pines, Sadka and Sheshinski (1985), who discuss the problem of intergenerational redistribution as it arises in the housing context. The present procedure means that the analysis becomes more sharply focused on the policy issue at hand, but it also means that the theory can only give partial insight into the problem of tax policy as it faces political decision-makers. But this point is well known from the general theory of optimum taxation, and there is no need to treat every single problem of tax policy under the most general set of assumptions.

Another problem neglected in the analysis is the view of housing as a »merit good» in the Musgrave (1959) sense. It may be that in the view of policy makers housing is a commodity which is »too important» to be left to the free interplay of market forces and to the principle of consumer sovereignty. Special tax incentives for house purchasers may be needed in order to stimulate them to consume the socially desired amount. Public finance theorists have always found this concept a difficult one to make precise, and I shall not attempt to do so here.

The plan of the paper is as follows. Section 2 presents the basic model of consumer behaviour on which the analysis is based. In section 3 the assumptions about the tax structure are introduced, and the general equilibrium framework is introduced in section 4. The conditions for optimal taxation are presented in general form in section 5. Typically, tax debates take place under varying assumptions as to the range of policy instruments available to the government, and section 6 analyses some cases where the government can only change a subset of the tax rates; some taxes are unavailable or pre-assigned. Debates about policy also frequently make implicit assumptions about the structure of consumer preferences, and the implications of some special cases of this kind are studied in section 7. The final section contains some concluding remarks.

2. *Consumer behaviour: the basic model*

A popular model in much recent work on problems of tax policy is that of a consumer who allocates consumption over the two periods of his life — as in the classical formulation of Irving Fisher (1930) — working in the first period and being retired in the second. In addition to first and second period consumption, time worked in the first period is an argument in the utility function. Work in the second period is exogenously fixed at zero and need not be taken into account explicitly.

This model is extended here to take account of durable goods. The treatment is highly stylised and is intended to account for two features of durable goods demand with a minimum of analytical complexity. The first feature is naturally that the possession of durable goods enters into the utility function of the consumer by yielding consumption services. The other feature is that durable goods are also a store of value; the choice between financial saving and investment in durable goods is also a portfolio choice from the consumer's point of view. In the original Fisher formulation real capital fulfilled only the latter purpose, so that it is in the feature that durable goods yield direct utility to the consumer that the novelty of the present formulation lies.

The restriction to a two-period framework naturally means that important features of durable goods demand must be neglected. Thus, the empirical fact that durable goods demand varies systematically over the life cycle can only very crudely be represented by the model; see Englund (1985) for a more satisfactory treatment of this in the context of housing.

Let c_i be the consumption of non-durable goods in period i $(i = 1, 2)$. Further, let h be labour supply in period 1 and z the stock of durable goods. The preferences of the consumer can be represented by the utility function

(1) $u = u(c_1, c_2, z, h).$

The utility function is increasing in non-durable and durable goods consumption and decreasing in labour. It is assumed to be strictly quasi-concave and differentiable.

We first derive the budget constraint of the consumer in the absence of taxes or — alternatively — on the assumption that there is only lump sum taxation. Income in the first period consists of wages and exogenous income, the latter including any lump sum transfer. Income is used to buy durable and non-durable goods and to invest in financial assets. The first period budget constraint then becomes

(2) $c_1 + z + s = wh + a.$

z and s are the amounts bought of the real (durable goods) and financial asset, respectively, and a is the amount of lump sum income. Basically, non-durable consumption is taken as the numéraire good, but since we shall not worry about changes in relative producer prices, we may as well fix the relative producer price of durable and non-durable goods at unity.[1]

[1] *This assumption implies, when we come to the analysis of taxation, that there can be no explicit treatment of the taxation of capital gains on consumer durables.*

In the second period the consumer sells his financial assets and consumes the value of the principal with interest added. Moreover, he sells his stock of durable goods; for simplicity, we neglect depreciation of this stock. We can then write the second period budget constraint as

$$(3) \quad c_2 = s\ (1+r) + z.$$

Combining the two, we obtain

$$(4) \quad c_1 + \frac{1}{1+r}\, c_2 + \frac{r}{1+r}\, z = wh + a.$$

Maximizing (1) subject to the intertemporal budget constraint (4) yields the first-order conditions

$$u_1 - \lambda = 0,$$

$$u_2 - \lambda\, \frac{1}{1+r} = 0,$$

$$u_z - \lambda\, \frac{r}{1+r} = 0,$$

$$u_h + \lambda w = 0.$$

These can be rewritten as

$$(5) \quad \frac{u_2}{u_1} = \frac{1}{1+r},$$

$$(6) \quad \frac{u_z}{u_1} = \frac{r}{1+r},$$

$$(7) \quad -\frac{u_h}{u_1} = w.$$

Conditions (5) and (7) are familiar from the theory of saving and labour supply while (6) may be interpreted as saying that the marginal value product of the stock of durables — in terms of the marginal rate of substitution — should be equal to the present value of the return on financial assets. It is possible to model the problem in such a way as to bring out more explicitly that the marginal rate of substitution involves both pure preferences and the technology of producing services from the stock of durable goods. However, there is really no need to make this kind of separation explicit in the present context.[2] It is worth noting that a change in the rate of interest alters the relative price of present and future consumption in general, and also the relative price of durable and non-durable goods. For future reference we define the pre-tax prices of non-durable and durable goods as $p_2 = 1/(1+r)$ and $p_z = r/(1+r)$.

The solution to the optimization problem implies demand functions for consumption goods as well as a supply function for labour. One can of course combine the demand functions to obtain a demand function for saving in the form of financial assets. It is worth pointing out that while in the usual version of the two-period model of labour supply and saving (as in Atkinson and Sandmo (1980) or King (1980)) financial saving is always positive, it could well be negative in the present model. This is naturally because there are two assets, and total saving can be positive even if financial saving is negative. This case is actually of particular interest since it captures the situation where the consumer borrows in the first period in order to finance durable goods purchases. The loan is then paid back in the second period with revenue from the sale of durables. This is of course a highly stylized version of the usual story of the life cycle of real and financial investment, and its only defence lies in the simplicity with which it captures both the consumption and portfolio aspects of durable goods demand.

3. *Modelling the tax structure*

There are various ways in which one could model a tax system which would be representative of the policy context discussed in the introduction. One possibility would be to have a tax on labour income and one on capital income, possibly at different rates to capture the fact that although nominal rates of tax on the two sources of income are often the same, the effective rates typically are not. This could be combined with a tax on the imputed value on durable goods services. The three tax rates would be sufficient to model all possible relative price distortions in the economy, and such a model is easy to formulate — at least as long as one is only interested in commodity demand and labour supply. A formulation along these lines does, however, create problems for an analysis of optimum taxation,

[2] *But see section 7 below for a discussion of a special case involving separation of this kind.*

and it is convenient to work with a somewhat different tax structure which is exactly equivalent in terms of the distortions that it creates.

In this formulation there is a proportional tax rate t_w on labour income. In addition there are taxes on non-durable consumption in the second period t_2, as well as a tax on the stock of durables, t_z, also to be paid in the second period. The latter can clearly be thought of as a tax on the services of the stock, if these are proportional to the stock itself. The tax on future non-durable consumption is an artificial creation, since it is hard to imagine consumption taxes which are differentiated by generation. But as regards effects on relative prices, this is equivalent to a tax on interest income, and it is easier to work with; a similar formulation was used in Atkinson and Sandmo (1980).

The budget constraint for the first period now becomes

$$(8) \quad c_1 + z + s = w\,(1-t_w)\,h + a,$$

and for the second period

$$(9) \quad (1+t_2)\,c_2 = s\,(1+r) + (1-t_z)\,z.$$

Combining these by eliminating s, we obtain

$$(10) \quad c_1 + \frac{1+t_2}{1+r}\,c_2 + \frac{r+t_z}{1+r}\,z = w\,(1-t_w)\,h + a.$$

The first-order conditions are now

$$u_1 - \lambda = 0$$

$$u_2 - \lambda\,\frac{1+t_2}{1+r} = 0,$$

$$u_z - \lambda\,\frac{1+t_z}{1+r} = 0,$$

$$u_h + \lambda w\,(1-t_w) = 0,$$

or, eliminating λ,

$$(11) \quad \frac{u_2}{u_1} = \frac{1+t_2}{1+r},$$

$$(12) \quad \frac{u_z}{u_1} = \frac{r+t_z}{1+r},$$

$$(13) \quad -\frac{u_h}{u_1} = w\,(1-t_w).$$

The right-hand sides of (11)—(13) define the consumer prices in the model; it will be convenient to denote these as P_2, P_z and W.

It may be useful to point out that if the model had been formulated in terms of a tax on interest income rather than the tax on future non-durable consumption, the changes in the first-order conditions can be seen by setting $t_2 = 0$ and writing $r(1-t_r)$ instead of r in equations (11)—(13). The nature of relative price distortions is exactly the same, but the price of durable goods is no longer linear in the tax rates. This creates problems for the usual kind of characterization results familiar from the optimum tax literature, which is the reason why the present formulation has been chosen.

4. *A general equilibrium framework*

It is perfectly possible to construct a general equilibrium model for optimum tax analysis by simply interpreting the model of the consumer as a picture of the private sector in a one-consumer economy. Such a model would, however, have some artificial features. The budget constraint of the government would have to be formulated as a restriction on the present value of taxes which can be raised from the representative consumer, and — at least at first glance — it is difficult to see the justification for such an assumption. A more natural formulation is obtained by following the lead of Atkinson and Sandmo (1980) and King (1980) and incorporate the analysis in an overlapping generations model; see also Pines, Sadka and Sheshinski (1985). In this framework two generations coexist in the economy in any one period. The working generation pays taxes on wage income, while the retired generation pays taxes on non-durable consumption and on the stock of durable goods. Population grows at the rate n, so that there are $1/(1+n)$ as many individuals in the retired as in the working generation. Otherwise, all individuals are exactly alike, so that the government's budget constraint can be written as

$$(14) \quad t_w\,w\,h + \frac{1}{1+n}\,[t_2\,c_2 + t_z\,z] - a = R$$

per member of the working generation. We can rewrite this in terms of consumer and producer prices as

$$(w-W)\ h+\frac{1+r}{1+n}\ [(P_2-p_2)\ c_2+(P_z-p_z)\ z]-a=R.$$

The revenue requirement, R, is exogenously given. Connecting this with the production and supply of public goods is a simple extension of the analysis, but yields little additional insight.

We close the production side of the model by assuming that producer prices are given, which is the equivalent to assuming the constancy of r and w. Moreover, we assume that the rate of interest is equal to the rate of population growth, i.e. $r=n$. This implies that the economy is moving over time on the golden rule path: The marginal productivity of capital in private firms is equal to the natural rate of growth of the economy. The simplification that this entails can be seen by noting, comparing the two budget constraints (10) and (14), that the government and private consumers are discounting future income and tax payments at the same rate. One can imagine the equality of the two rates to come about through the use of government debt policy or other policy instruments which affect the intergenerational distribution of income. This point has been extensively discussed in the literature, e.g. by Atkinson and Sandmo (1980), and it has been demonstrated that it is only in the golden rule case that optimum tax analysis in the overlapping generations model becomes formally similar to the standard formulation. Limiting attention to this special case is justified by the attention which the more general case has already received in the literature. The government's budget constraint now becomes[3]

$$(15)\quad (w-W)\ h+(P_2-p_2)\ c_2+(P_z-p_z)\ z-a=R.$$

A fully satisfactory formulation of the social welfare function would require us to write this as depending on the utility levels of all future generations. We take here the simpler approach of King (1980) and Sandmo (1985) by assuming that the social welfare function can be written as the utility of a representative generation, which is simply the function (1). With the model of taxation in section 3 we can then write the indirect utility function as

$$(16)\quad v=v\ (P_2,\ P_z,\ W,\ a).$$

The government's problem is then to choose consumer prices to maximize (16) subject to the budget constraint (15).

5. *The optimum tax model*

The constrained optimization problem can be solved by writing the Lagrange function as

$$(17)\quad L=v\ (P_2,\ P_z,\ W,\ a)+\mu\ [(w-W)\ h+(P_2-p_2)\ c_2+(P_z-p_z)\ z-a-R]$$

and setting its partial derivatives equal to zero. Using Roy's theorem, we have that

$$(18)\quad -\lambda c_2+\mu\ [(w-W)\ \frac{\partial h}{\partial P_2}+c_2+(P_2-p_2)\ \frac{\partial c_2}{\partial P_2}+(P_z-p_z)\ \frac{\partial z}{\partial P_2}]=0,$$

$$(19)\quad -\lambda z+\mu\ [(w-W)\ \frac{\partial h}{\partial P_z}+(P_2-p_2)\ \frac{\partial c_2}{\partial P_z}+z+(P_z-p_z)\ \frac{\partial z}{\partial P_z}]=0,$$

$$(20)\quad \lambda h+\mu\ [(w-W)\ \frac{\partial h}{\partial W}-h+(P_2-p_2)\ \frac{\partial c_2}{\partial W}+(P_z-p_z)\ \frac{\partial z}{\partial W}]=0,$$

$$(21)\quad \lambda+\mu\ [(w-W)\ \frac{\partial h}{\partial a}+(P_2-p_2)\ \frac{\partial c_2}{\partial a}+(P_z-p_z)\ \frac{\partial z}{\partial a}-1]=0.$$

In this formulation we have included lump sum transfers as a possible policy instrument.

[3] *It may be useful to check that the budget constraints of consumers and the government together imply the satisfaction of the economy's overall production constraint. Deducting (15) from (10), and remembering the definition of consumer prices, we obtain*

$$c_1+p_2\ c_2+p_z\ c_z+R=wh,$$

which is the linear production possibility locus of the economy.

It is of course well known that if there were no constraints on policy instruments, a full optimum would be attained by exclusive use of this instrument with no use of distortionary taxation. However, even if we do not take this possibility seriously — and we clearly should not — it may be usefully included as a possible policy instrument in thought experiments involving partial optimization of the tax system.

We now use the Slutsky equations (which have been written out in full in the Appendix) to rewrite the first-order conditions as

$$(22)\quad -\frac{t_w}{1-t_w}\sigma_{2W}+\frac{t_2}{1+t_2}\sigma_{22}+\frac{t_z}{r+t_z}\sigma_{2z}=\alpha,$$

$$(23)\quad -\frac{t_w}{1-t_w}\sigma_{zW}+\frac{t_2}{1+t_2}\sigma_{z2}+\frac{t_z}{r+t_z}\sigma_{zz}=\alpha,$$

$$(24)\quad -\frac{t_w}{1-t_w}\sigma_{hW}+\frac{t_2}{1+t_2}\sigma_{h2}+\frac{t_z}{r+t_z}\sigma_{hz}=\alpha,$$

$$(25)\quad \alpha=0,$$

Here σ_{2W} etc. are the compensated elasticities, and α is the net marginal social value of the lump sum transfer of revenue from the public to the private sectors, i.e.

$$(26)\quad \alpha=\frac{\lambda-\mu}{\mu}+[(w-W)\frac{\partial h}{\partial a}+(P_2-p_2)\frac{\partial c_2}{\partial a}+(P_z-p_z)\frac{\partial z}{\partial a}].$$

Equations (22)—(24) are one version of the Ramsey conditions; the relative reductions of demand along the compensated demand curves should be the same for all commodities. Equation (25) implies that when lump sum taxes can be used the factor of proportionality should be zero. In that case the solution to equations (22)—(24) is to set all distortionary tax rates equal to zero, thus providing a proof of the point made above.

If lump sum taxes are not available the optimal second best set of taxes are characterized by equations (22)—(24), i.e. each tax rate depends on the whole set of compensated elasticities. This general insight is clearly of interest by showing what empirical information would in principle be desirable to implement an optimal tax structure.[4] But the empirical information that we do have, is partial and imperfect. It is therefore useful to examine various special cases which are of policy interest and have a structure which is simple enough to admit more meaningful statements about policy issues. Two classes of such cases suggest themselves. One is where some tax rates are given, the other is characterized by particular assumptions about preferences. Consideration of such cases may contribute to a better understanding of the more promising directions for tax reform in this area.

6. *Partial tax design with some pre-assigned taxes*

Suppose first that we are in a situation where the rates of tax on labour and capital income are both given. We then ask whether, assuming that we can choose the tax rate on durables as well as the lump sum transfer freely, it would be desirable to tax durables. This is a typical problem in second best theory; the tax rate t_z is clearly distortionary, but it could well be desirable, given the other distortions that already exist.

To study this problem we disregard equations (22) and (24) and combine (23) and (25) to obtain

$$(27)\quad \frac{t_z}{r+t_z}=\frac{1}{\sigma_{zz}}\cdot\left(\frac{t_w}{1-t_w}\sigma_{zW}-\frac{t_2}{1+t_2}\sigma_{z2}\right)$$

A tax on durable goods is desirable, even if it yields no net tax revenue, if an increase in the price of durable goods tends to increase labour supply and the demand for future nondurable consumption, with supply and demand being evaluated in compensated terms. This example clearly demonstrates that in order to discuss the efficient taxation of durable goods it is in principle as important to look at the interaction of the durable goods market with the labour market as with the capital market. Neutrality as commonly interpreted — equal rates of tax on real and financial saving — as well as the argument that the tax rate on

[4] *Equations (22)—(24) form a system similar to the one discussed in the classic contribution by Corlett and Hague (1953—54), except that they have two taxed goods instead of three. The extension to three taxed goods makes it difficult to derive the Corlett-Hague type of characterization, as shown in unpublished work by Vidar Christiansen.*

durables should be low in order to alleviate the high taxation of labour, should both the subordinated to the more general principle that what counts in the final instance is the distortion of quantities, not of prices.

However, it is of some interest in the present context to see whether there are cases in which these two more intuitive policy arguments can be defended on more theoretical grounds. To take the neutrality argument first, it is clear from (27) that if labour supply is independent of the price of durables ($\sigma_{zw} = 0$) and if durable and non-durable future consumption are perfect substitutes ($\sigma_{zz} = -\sigma_{z2}$), then (27) becomes simply $t_z/(r + t_z) = t_2/(1 + t_2)$, which is the neutrality principle.[5] Thus one may regard the argument for neutrality as being derived from a Fisher-type model with exogenous labour supply and a perfect rental market for durable goods. This is a highly special set of assumptions, and it is one of the benefits of a more general model that it can identify them.

Consider now the argument that the rate of tax on durable goods should be kept low in order to counteract the disincentive to labour involved in the tax on labour income. Taking the elasticities as being approximately constant around the optimum, we see from (27) that since the own elasticity is negative, the rate of tax on durable goods should be decreasing in the rate of tax on labour income if compensated labour supply is a decreasing function of the price of durables. A particular interpretation of this is that if investment in housing implies planning for leisure, then an economy with a highly distorted labour market should design its tax policy such as to discourage investment in housing. This conforms well with the analysis in Christiansen (1985). On the other hand, assuming that future non-durable goods and durables are substitutes, leads to the conclusion that t_z should be higher, the higher the pre-assigned level of t_2.

This particular partial analysis is not the only case of interest. With reference to the Norwegian tax debate, the following case may be more relevant. Assume that the rate of tax on labour income is given; the political assumption behind this may be that since labour income constitutes the larger part of total income, the tax schedule must be taken as fixed for distributional reasons — reasons which are not explicitly accounted for in the present model. Assume moreover that for administrative reasons it is not possible to tax the holding of durable goods. How can one then characterize the optimum taxation of interest income, which is here represented by the tax on future consumption? Equations (23) and (24) can then be ignored, and (22) and (25) can be combined by yield

$$(28)\qquad \frac{t_2}{1+t_2} = \frac{\sigma_{2w}}{\sigma_{22}} \frac{t_w}{1-t_w}.$$

The tax on future non-durable consumption should be negative if it is complementary with labour and positive if the two are substitutes — again assuming that the tax has no effect on net revenue. The underlying economic intuition is the same as in the previous case; the tax on interest income should be designed with a view to its possible role in counteracting the distortionary effect of the tax on labour income.

These two examples illustrate the role of optimum tax theory in analyzing problems of partial tax design. The viewpoint of partial optimization of the tax system may often be a realistic one. Redesigning the overall system of taxation is politically and administratively very difficult and for that reason occurs infrequently. Reforms that take some parts of the tax system as given and attempt to redesign the remaining parts seem to occur more often, and tax theory clearly should have something to say on the efficiency aspects of such reforms. The present analysis demonstrates how partial optimization results can be derived by simply introducing additional constraints on the solution of a general optimum tax model.

7. *Special assumptions about preferences*

In policy debates arguments are often put forward which are based on implicit assumptions about preferences. Typically, in discussing problems where the immediate focus is on the portfolio decisions of consumers, one tends to forget the interaction with labour supply decisions and in fact to argue as if

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[5] *This could equivalently be expressed as $t_z/P_z = t_2/P_2$.*

labour supply is fixed. There is little reason to believe that this is a realistic assumption — particularly if one keeps in mind that the relevant elasticity is the compensated one. Still, it is of interest to examine the consequences for optimum taxation if labour were actually in completely inelastic supply. With this assumption, labour supply becomes a parameter rather than a variable in the utility function, and both the compensated and uncompensated labour supply functions are completely inelastic.

It is immediately clear that if this is the case, the optimum tax system taxes only labour income, since such a tax is equivalent to lump sum taxation. At least, this is true in an economy of identical consumers. With heterogeneous consumers and constraints on the form of the tax schedule, one might want to add other tax instruments to the optimum system. But focusing as we do here on the efficiency aspects, it is clear that this assumption would imply no distortionary taxation of consumption and portfolio decisions. Such a system might, however, encounter objections related to fairness and equity. We assume in addition, therefore, that the rate of tax on labour income is also fixed and ask how one can then characterize the optimum configuration of the remaining tax rates.

To study this case, we go back to the system of equations (22)—(25). Of these (24) and (25) are now irrelevant, and in the first two, the first terms on the left drop out because of the assumption of fixed labour supply. What remains of the optimum conditions can then be manipulated to yield.

$$(28) \quad \frac{t_z}{r+t_z} \Big/ \frac{t_2}{1+t_2} = \frac{\sigma_{22} - \sigma_{z2}}{\sigma_{zz} - \sigma_{2z}}.$$

If the cross elasticities are ignored, this is the familiar result that the ratio of the tax rates should be inversely related to the ratio of their compensated own elasticities. Since these have the same sign, the tax rates should either be both positive or both negative. One's natural inclination is to say that they must be positive. But this depends on whether the revenue from the labour income tax is greater or less than the amount required. If it is greater than this, the two distortionary tax rates should both be negative and (28) should be interpreted as a rule for optimum subsidies.

The cross elasticities play an interesting role in condition (28). If durables and future consumption are substitutes — perhaps the most natural a priori assumption — this strengthens the conclusion that both tax rates should be of the same sign. If they are complements, however, this is no longer true; the exception would occur if one of the (negative) cross elasticities had a larger numerical value than the corresponding direct elasticity. This may be an empirically unlikely case to consider, but it is nevertheless of interest as a help to understand the support that theory can give to the intuitively more plausible case.

Another special case which it is interesting to consider is that of perfect substitutability between present non-durable and durable consumption. In many markets consumers have a choice between owning and renting while the consumption benefits — in so far as they can be »objectively» ascertained — are identical between the two alternatives. With such an assumption our model runs into some difficulties because it will typically involve a corner solution; the consumer will rent or own depending only on the price ratio of the two options, and we would no longer be able to work with a continuous demand function. In order to overcome this difficulty we introduce a small modification of the model. Let the benefits derived from the durable goods stock z be represented by the function f(z) which is concave and differentiable. We can then write the utility function as

$$(29) \quad u = u\,(c_1 + f\,(z),\, c_2,\, h).$$

Maximization of this subject to the budget constraint (10) yields the first order conditions (11) and (13) as before; however, instead of (12) we now have

$$(12') \quad f'\,(z) = \frac{r+t_z}{1+r} = P_z.$$

The interpretation of this condition is that the consumer sets the marginal productivity of the stock of durable goods — as measured in units of non-durable consumption — equal to its rental price. In the present version of the model this condition holds irrespective of the nature of the consumer's preferences. In other words, there is complete separation of portfolio decisions from decisions about consumption and labour supply.

This separation property has strong implications for the compensated demand and supply functions. From (12') it is clear that z depends solely on P_z, so that the income and cross elasticities of that demand function are all zero. But then condition (23) becomes simply

$$(23')\quad \frac{t_z}{r+t_z} = \frac{\alpha}{\sigma_{zz}}.$$

The rate of tax on durables should be inversely proportional to its compensated own elasticity.[6] One could of course have assumed directly that the cross elasticities are zero, but it is more instructive to derive this property from the separability assumption. Moreover, from (12') it follows that

$$(30)\quad \sigma_{zz} = [\frac{f''(z)\,z}{f'(z)}]^{-1}.$$

The more sharply the marginal productivity of the stock falls, the higher is the optimal tax rate. This is a »technological» interpretation of the compensated demand function which makes good economic sense in the case where there exist close non-durable substitutes for durable consumer goods.

8. *Concluding remarks*

The aim of this analysis has been to show how optimum tax theory can be applied to a problem in the current debate about economic policy. It is clear that no firm conclusions can be drawn without empirical estimates of the relevant parameters of the demand and supply functions. The theory is of interest first as a guide to which parameter estimates are the relevant ones, and second as a set of instructions for clear thinking about the policy issues even without the benefits of serious econometric estimates.[7]

The analysis brings out the fundamental point that rules for optimal tax design depend crucially on exactly which options are open to the policy maker. When some instruments are not available — e.g. when it is for some reason not practicable to tax durable goods — the rules governing the optimal use of the remaining instruments can disregard some parameters that would otherwise have been relevant for optimum taxation. It also illustrates the flexibility of the theoretical formulation; constraints on the use of tax instruments can be taken into account simply by striking out the relevant rows or columns of the first order conditions for optimum.

One may be in doubt about the fruitfulness of studying special cases of general preference orderings as long as we have little empirical knowledge of the realism of such cases. But one can also take a different view: In arguments about policy it is often easy to see that particular assumptions about preferences are in fact made; in the Norwegian debate the argument for »neutrality» has e.g. been put forward under the implicit assumption of separability between consumption and portfolio decisions. Theory can be of help to policy by taking this assumption seriously through the study of its implications; it is instructive to be able to see that more than separability is required for neutrality to result. It is in this spirit that the results for various special cases should be interpreted.

There are of course a number of interesting issues in the taxation of durable goods which have not been touched upon here. The assumption of constant producer prices limits the applicability of the theory to long run issues, and it would be interesting to study a more short run version of the model, in which changes in demand would not only be met by corresponding changes in supply but also by changes in producer prices. This is particularly relevant for those durable goods which are characterized by long production lags, housing in particular. Relaxing the assumption that consumers are identical not only with respect to income but also with respect to their access to housing and capital markets, could also add interesting new dimensions to the analysis.[8]

[6] *It can be shown that in a second best optimum we must have $\alpha < 0$. For a general discussion of elasticity formulae see Sandmo (1987).*

[7] *I do not know of any set of consistent estimates of the elasticities involved here. Summers (1987) claims that the inefficiencies involved in the taxation of housing and other durable goods in the United States are of empirically greater importance than the non-neutrality associated with the taxation of different capital assets at different rates; the latter is of course a problem which has received much more attention in the literature.*

[8] *For a survey of some issues in the taxation of housing with particular reference to the situation in the United States see Rosen (1985).*

Optimum tax theory is normative in the sense that it derives characterizations of optimum tax policy under the assumption of social welfare maximization. It does not pretend to be a positive theory, aiming to explain the structure of actual tax systems. To a certain extent, one could even argue that optimum tax theory might be particularly misleading in this respect. To come back again to the Norwegian situation, many economists have argued that there are likely to be efficiency gains from increasing the taxes on owner-occupied housing, basing this argument at least in part on the belief that the demand elasticity for housing is low. If the elasticity assumption were true, this would make taxes on housing difficult to avoid — but this would also be a good explanation of why there is strong political opposition to such tax increases. When a tax is difficult to avoid, it means that it is easy to define the group of taxpayers which are hit by the tax and correspondingly easy to form political pressure groups to lobby against the tax. This does not of course detract from the interest of optimum tax theory as such, but it may serve as a reminder both of the political obstacles to application of the theory and the insights which may be derived from positive theories of taxation and the public sector.

Appendix

Derivation of the Slutsky equations

The general principles underlying the derivation of the Slutsky equations are of course familiar, and there would be no need to write them out explicity except for the need to be precise about the notation.

To show the derivation of the Slutsky equations we take the labour supply function as an example. At the equilibrium position of the consumer the conpensated (h^*) and uncompensated (h) labour supply functions by definition take on the same value; hence we can write

(A1) $h\,(P_2, P_z, W, a) = h^*\,(P_2, P_z, W, u)$.

a must now be understood as the lump sum income required to attain the specified level of utility, i.e. a is a particular value of the expenditure function

(A2) $a = e\,(P_2, P_z, W, u)$.

Substituting into (A1) and differentiating with respect to the net wage rate W, we obtain

(A3) $$\frac{\partial h}{\partial W} + \frac{\partial h}{\partial a} \cdot \frac{\partial e}{\partial W} = \frac{\partial h^*}{\partial W}.$$

But e is the inverse of the indirect utility function v, and it must therefore hold that, using Roy's theorem,

(A4) $$\frac{\partial e}{\partial W} = -\frac{\partial v}{\partial W} \Big/ \frac{\partial v}{\partial a} = -h.$$

Substituting this into (A3) and rearranging terms, we can then write the Slutsky equation in this case as

(A5) $$\frac{\partial h}{\partial W} = h\,\frac{\partial h}{\partial a} + \left(\frac{\partial h}{\partial W}\right)_u.$$

The other equations are similarly derived, and we simply write them down for easy reference.

(A6) $$\frac{\partial c_2}{\partial P_2} = -c_2\,\frac{\partial c_2}{\partial a} + \left(\frac{\partial c_2}{\partial P_2}\right)_u,$$

(A7) $$\frac{\partial c_2}{\partial P_z} = -z\,\frac{\partial c_2}{\partial a} + \left(\frac{\partial c_2}{\partial P_z}\right)_u,$$

(A8) $$\frac{\partial c_2}{dW} = h\,\frac{\partial c_2}{\partial a} + \left(\frac{\partial c_2}{\partial W}\right)_u,$$

(A9) $$\frac{\partial z}{\partial P_2} = -c_2\,\frac{\partial z}{\partial a} + \left(\frac{\partial z}{\partial P_2}\right)_u,$$

(A10) $$\frac{\partial z}{\partial P_z} = -z\,\frac{\partial z}{\partial a} + \left(\frac{\partial z}{\partial P_z}\right)_u,$$

(A11) $$\frac{\partial z}{dW} = h\,\frac{\partial z}{\partial a} + \left(\frac{\partial z}{\partial W}\right)_u,$$

(A12) $$\frac{\partial h}{\partial P_2} = -c_2\,\frac{\partial h}{\partial a} + \left(\frac{\partial h}{\partial P_2}\right)_u,$$

(A13) $$\frac{\partial h}{\partial P_z} = -z\,\frac{\partial h}{\partial a} + \left(\frac{\partial h}{\partial P_z}\right)_u.$$

The symmetry conditions on the substitution effects imply that

$$(A14)\quad \left(\frac{\partial c_2}{\partial P_z}\right)_u = \left(\frac{\partial z}{\partial P_2}\right)_u ,$$

$$\left(\frac{\partial c_2}{\partial W}\right)_u = -\left(\frac{\partial h}{\partial P_2}\right)_u ,$$

$$\left(\frac{\partial z}{\partial W}\right)_u = -\left(\frac{\partial h}{\partial P_z}\right)_u .$$

The notation σ_{hW}, σ_{22} etc. will be used to denote the compensated *elasticities*.

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