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SOME MODELS OF STOCHASTIC PLANNING MECHANISMS

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Alternative models of stochastic planning mechanisms of an abstract socialist economy are studied. The mechanisms are based on the decentralization of the initial optimal planning model. The main attention is paid to mixed stochastic coordination (prices and limits) of agents with Benassy's (1986) competitive mechanism. The implementation of optimal plans is demonstrated. The models of mechanisms studied are theoretically interesting, but they are oversimplified for practical results, and in order to come closer to reality, a more realistic set of assumptions should be made as a starting point.

1. Introduction

In the case of an indeterministic approach to an economy the mechanism version in economics is clearly stronger than the central planning-theoretical and equilibrium-theoretical ones. On the basis of uncertainty paradigms the overcentralistic central planning theory and the overdecentralized equilibrium theory are too limited to be able to describe real phenomena. In the former case every agent is directly pushed into optimality by the omnipotent and completely informed center. In the letter case there is nobody to steer stochastic prices and rations towards equilibrium.

The existing stochastic planning mechanisms theories provide inadequate explanations for many problems. As a matter of fact, various theories ignore more or less at least two issues. First, the treatment of mixed or combined stochastic coordination problems (prices and limits or rations) is lacking. Second, the alternative problems of working out equilibrium coordination rules, especially alternative pricing rules are overlooked. The paper tries to fill the two gaps, but because of the complications of the problem it manages to give only introductory explanations. Toward this end the paper has mainly attempted to classify the problems and sketch the solutions in a heuristic treatment for a simple planning model.

In Ennuste (1985) a fully centrally coordinated planning mechanism under uncertainty with mixed signals where both future prices and limits (rations) are state-dependent is presented. This approach is basic to the theoretical analysis of centrally planned economics under uncertainty. But it is unrealistic because it requires immense central state-dependent coordination.

This paper aims to provide coordination schemes that will reduce the burden of the center. Here we assume that the center deals only with the aggregated central planning problem and the central planning of the wealth or endowment and profit allocation. The model for the determination of the detailed planning indicators is dealt with separately, and constraints (central order) for this model come

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from the aggregated model. This detailed model is decomposed into a decentralized mechanism with Benassy's competitive planning markets. In these planning markets or offices mixed stochastic coordination with prices and quantities is used. So we shall draw a bridge between fully centrally coodinated economies and the recent non-Walrasian and Walrasian economies under uncertainty.

In other words, we assume that optimal state-dependent wealth rations and central orders for decentralized mechanisms are determined exogenously by the center, and only detailed prices and quantities are determined in the decentralized mechanism endogenously. The central allocation of initial endowments and profits enables us to avoid special problems of stock markets, etc. The analysis of a stock market would introduce considerable complications into the model, so we shall deal only with the case of production where the profits are allocated centrally (without stock markets). This enables us to focus on the essential idea of an equilibrium that includes production in as simple a context as possible.

It should also be noted that there is already quite a lot of literature on the problem of optimal pricing under uncertainty and some major results have been achieved, but, the evaluation of these results in combination with stochastic rationing is still a bottleneck, see e.g. Rees (1982) and Bennet (1984). One aim of this paper is to try to contribute to this evaluation. As a matter of fact in a stochastic environment it is practically impossible to work out perfectly adaptive or state-dependent future coordination parameters as the number of possible future conditions in enourmous and the situation changes very quickly. Therefore, in practice we are always forced to operate with somewhat infexible coordination. In this connection comparison of the application of the combinations of different kinds of coordination parameters is of special interest. Increased inflexibility of prices seems not to reduce efficiency too much though it brings about serious difficulties in balancing supply and demand. To achieve their balance complimentary rationing or limitation is required. Such a scheme is known in practice where mechanisms are often used for complimentary rationing.

On the other hand, with coordination of plans under inflexible limits or rations great losses of efficiency might occur since the scopes of the plans of the rationed economic units are now extremely limited under stochastic conditions, although no special difficulties are expected in balancing. Raising the efficiency coordination with limits should be complemented by coordination with prices with regard to certain resources if possible. Thus, combined coordination methods might yield better results under uncertainty than the so-called pure methods.

This falls fortunately into the theoretical framework of J. Benassy's (1986) competitive market mechanisms theory. In this approach deterministic prices and rations are determined in parallel and if we generalize this approach to the stochastic case all the immense work of determining state-dependent stochastic prices and quantities is done here by the decentralized agents in the state-dependent planning offices. This approach is interesting theoretically but not on a practical level because it is still impossible to organize a complete set of current state-dependent planning offices.

To move closer to realistic coordination mechanisms under uncertainty, we shall study models in the spirit of J. Green (1973) and J. Grandmont (1982). In such an approach the central role is played by an agent's correct expectations about the state-dependent future spot prices in his decision-making. In the framework of this simple model the sufficient conditions for the existence of a competitive equilibrium are met on some assumptions. But here agents are assumed to be able to associate correctly future prices with states (correct expectations) to the limited numbers of goods they are dealing with. Clearly, if they cannot do it, the equilibrium will not be optimal (efficient).

To move still closer to realistic coordination mechanisms we discuss interval planning problems. They have always been practically tempting, but the mathematical clumsiness of the interval models allows us to make only the first steps in this field.

To start with, we still need some introductory remarks, then we shall describe the initial central social choice model and transform it into a decentralized setting where the problems of alternative pricing and rationing mechanisms will be discussed.

2. *Introductory remarks on abstract stochastic planning mechanisms*

According to Hurwicz (1986) there are two aspects of investigations into mechanisms. The traditional equilibrium theory takes a mechanism or modus operandi (e.g. perfect competition) as given, and studies its properties (e.g. Pareto-optimality) and performance correspondence. In the theory of centralized economies the reverse problem has come to be investigated. Given a correspondence regarded as a social desideratum (e.g. optimization correspondence), are there mechanisms that implement it? In the following we treat the latter problem in a stochastic setting.

Let I_j, $j = 0, 1, \ldots, n$, denote the a priori admissible information set for unit j, (j = 0 is the center). Here we abstract away the time dependence of I_j, and with that we abstract away also the explicit study of rolling planning. The admissible full communication information set is $I = I_0 x, \ldots x I_n$. Information $i_j \in I_j$, $i \in I$ determines the probability spaces for the future (S_j, C_j, P_j) and (S, C, P), where $S = S_0 x \ldots x S_n$. Here $s_j \in S_j$ is an elementary state of the world of j, and it determines the agents' preferences, technology, initial endowments, etc. C_j is σ-field and P_j is propability measure. An elementary state of the economy is $s \in S$. The social choice rule D(s) is an optimization correspondence to a set Z(s) of feasible plans or outcomes, $Z(s) \subseteq J(B)$, $B \subseteq C$, where B is sub σ-field and J(B) is measurability set for plans. The task is to find for D(s) an implementation of the decentralized mechanism[1] $N(s) = D(s)$, $i \in I$.

Here we assume that model D has a certain structure. It has a detailed setting at the beginning of the planning period or in the threshold period and an aggregated part at the end of the period or in the horizonal period.

Now we assume that the center j = 0 has all aggregated information and the agents j = 1, . . . ,n have detailed information. Accordingly it is reasonably to decompose D into two subproblems: the detailed threshold problem F and horizonal problem H. There are many alternative decomposition and coordination principles for this decomposition. To discuss these problems is not our task here. Here we assume that the horizonal problem is dealt with by the center j = 0 with the help of some kind of system of models. And the solution of this problem gives constraints and some aggregative indicative indicators for the threshold problem. As far as the center has all the aggregated information the mentioned system of models is cooperative, and economically it is a central planning problem of allocation with constrained resources.

In the case of the large detailed threshold problem F we must pay special attention to the following. In the case of incomplete information the important problem is whether there are any decentralized (privacy-preserving) mechanisms pushing the decentralized economy to the optimum state. Namely, privacy-preserving is especially important in the case of incomplete information (mainly in the form of subjective probabilities), and it is next to impossible to get this incomplete private information $i_j \in I_j$, $j = 1, \ldots, n$ into one center without any manipulation or distortion, i.e. to enforce truth-telling. The same argument tells that it is reasonable to organize the choice of divisional strategies in a game-theoretic setting played non-cooperatively (Nash equilibrium).

Let us now define a model of an abstract decentralized stochastic planning submechanism for the threshold problem F. Since our main interest concerns this submechanism, we shall call it simply a mechanism in the following.

The task is to find a non-cooperative stochastic game G whose outcome implements F. Here we assume that the agent $j = 1, \ldots, n$ participates on the basis of i_j in a non-cooperative game with $\hat{Z}_j$ as his strategy domain[2], $\hat{z}_j$ as a strategy, $\hat{Z} \in \hat{Z}_1 x \ldots x \hat{Z}_n$, e_j as the expected utility function, h_j as the outcome function, $h_j : \hat{Z} \times S_j \to Z_j$, $e_j : Z_j \times S_j \to R$ where R is a set of real numbers, and Z_j is the outcome set for j.

The strategies $\hat{z}_j^* \in \hat{Z}_j$ are called Nash equilibriums for the game if for each $j \in N$:

$$\hat{z}_j^* = \arg\max e_j (h_j (\hat{z}_{-j}^*, \hat{z}_j, s_j)),$$

[1] *The term implementation is used in the same sense as e.g. in Williams (1986), i.e. the decentralized mechanism concerns the issue of the incentives of the agents.*

[2] *Here we omit the states s from the strategies and plans to keep notation clear. We also omit the subtle problem that each player j is uncertain about the strategies actually being used by all other players, see Aumann (1987).*

where $(\hat{z}^*_{-j}, \hat{z}_j) = (\hat{z}^*_1,\ldots,\hat{z}^*_{j-1}, \hat{z}_j, \hat{z}^*_{j+1},\ldots,\hat{z}^*_n)$ and $h(\hat{z}^*) = (h_1(\hat{z}^*, s_1),\ldots,h_n(\hat{z}^*, s_n))$ is the Nash equilibrium outcome. We say that a mechanism implements D if $h(\hat{z}^*) = F$.

As suggested by Hurwicz (1986) and Marchak (1986) the abstraction of the mechanism just given can serve for describing any resource-allocation scheme along a rich variety of schemes possible in principle. There are two polarized concepts of these schemes. The classical claim is that the information repeatedly iteratively exchanged (strategies) among members consists of price and excess demand (price or competitive mechanisms). The alternative is a centralized revelation-command mechanism in which information is gathered in the center ($j = 0$), where individual consumptions and productions are computed and then issued as commands to the agents.

As we have already claimed, because of the stochastic and subjective nature of the information it seems unthinkable to organize revelation of private information in all details to the center for the threshold model. Moreover, this information would exceed the computational capacity of the center, and in the end, the agents would not be interested in following commands.

On the other hand, the ideal price mechanisms in the stochastic case are also unthinkable. In this case the ideal prices and net-trades are state-dependent and impractical (as there are infinitely many possible states in reality). So, in reality some mechanism should implement approximate prices, and thus cannot guarantee the realization of optimum and stability. It means that a sound stochastic mechanism is a mixed one. In this mechanism parallel to prices coordination with quantities is applied limiting or rationizing the agents' actions, and achieving better stability than prices alone could do in a stochastic situation. So a mixed mechanism might achieve a better approximation of the equilibrium. It is interesting to note that in the theory of mathematical hierarchical systems mixed coordination is well-known as the Mesarovitc interaction prediction principle.

In this paper it is assumed that in the center state-dependent endowment and profit allocations for the agent of the threshold problem are computed. They are computed on the basis of an aggregated central model. These endowments are issued to the agent as coordinating constraints in which they can implement the price-limit equilibrium. The central allocation of endowment allows us to avoid complicated problems of the stock market and private ownership of fixed assets.

Next we describe one simple version of the D model, and decompose it first into H and F models and then we decompose the F model with mixed coordination into Walrasian agents' problems.

3. *The stochastic initial planning model and decentralized equilibrium*

3.1. *The initial model*

Let us have an economy with n units or activities indexed by $j = 1,\ldots,n$ and m goods indexed by $i = 1,\ldots,m$. Let $s \in S$ be an elementary countable state of the world, C is a σ-field of the measurable subsets of S, and P is a probability measure assumed about (S, C). So (S, C, P) is an abstract probability triple.

The initial central resource constraint of the economy is $b(s) \in R^m$, where $b_i \geqslant 0$ is input or »consumption» of the economy (initial central endowment), and $b_i < 0$ is output. The plan or outcome of activity j is described by the vector $z_j(s) = (z_{ij}(s))$, $i = 1,\ldots,m$, which must belong to a feasible set $Z_j(s)$. Here $z_{ij}(s) \geqslant 0$ is input and $z_{ij}(s) < 0$ is output. We assume that $Z_j(s)$ is closed, convex, bounded and has nonempty interior with the probability one. We also make the standard assumption that $z_j(s) \in J(B_j)$, where B_j is sub-σ-field of the activity j, $B_j \subseteq C$ and $J(B_j)$ is a measurability set for the activity j. For example, if $B_j = C$ then the activity j is totally conditional on s, and if $B_j = \{S\}$ then the activity j is totally unconditional of s (deterministic).

The utility of the activity j is presented by the function $u_j(z_j(s),s)$ defined on $Z_j(s)$, continuous and stictly monotone and strictly concave with the probability one in its arguments.

We also make the standard assumption about the measurability of the constraints $i = 1,\ldots,m$ on sub-σ-fields $B_i \subseteq C$. For example, if $B_i = C$ then the constraint i must be considered for every $s \in S$, if $B_i = \{S\}$ then only the mathematical expectation E of the constraint is considered.

We assume that the objective function of the economy is the mathematical expectation of the sum of the activities' utilities. Now the initial optimal problem is the following. Maximize on the basis of the plan $\hat{z}(s) = (\hat{z}_j(s))$, $j = 1,\ldots,n$ the objective function

(1a) $E \Sigma u_j (z_j(s), s)$

subject to

(1b) $E [\sum_j z_{ij}(s) - b_i(s) \mid B_i] \leqslant 0, i = 1,\ldots,m,$

(1c) $z_j(s) \in Z_j(s), j = 1,\ldots,n,$

(1d) $z_j(s) \in J(B_j), j = 1,\ldots,n,$

where all the stochastic constraints here and below hold with the probability one.

Here we should note that (1) may describe also complicated dynamic problems.

To design a mechanism for problem (1) we make a set of »natural» assumptions which will lead to the decomposition of problem (1) into two submodels: an aggregated horizonal H and a detailed threshold F submodel.

Assumption A1: The mechanism elaborates rolling plans. So it is »natural» to elaborate detailed planning indicators only for the very beginning or threshold of the planning period, and further consider only aggregated indicators.

Assumption A2: Only the center has the information and capability to deal with the aggregated or horizonal part of the problem, and only the activities managers have the information about their detailed indicators.

Assumption A3: No center is capable of getting reliable, detailed stochastic information from the activities managers and coping with it.

The reasonable conclusion from these assumptions is to design two submechanisms: one for centrally elaborating the aggregated indicators, and the other, the decentral submechanism for elaborating the detailed indicators. Here we consider the first submechanism trivial, so our interest is concerned with the latter problem. Now we shall describe and comment on the decentralized subproblems of the activities in the framework of the threshold model F.

3.2. *Equilibrium of decentralized activities*

We shall now assume that the detailed threshold submodel has the same structure as (1). To decentralize model (1) we use the Lagrangian relaxation. For this purpose an optimal solution $z(s)^o$ to problem (1) is assumed to exist, and the regularity conditions of (1) are met.

Now the following Lagrangian problem is obtained.

(2a) $\min \max L(z(s), p(s)) = \min \max E \{\sum_{j=1}^{n} u_j(z_j(s), s) - \sum_{i=1}^{m} p_i(s) E(\sum_{j=1}^{n} z_{ij}(s) - b_i(s) \mid B_i)\},$

subject to

(2b) $z_j(s) \in Z_j(s), z_j(s) \in J(B_j),$

(2c) $p_i(s) \geqslant 0, p_i(s) \in J(B_i),$

where $p(s) = (p_i(s)), i = 1,\ldots,m$, is the Lagrangian price.

Let the saddle point (optimal) price $p^o(s)$ be given. Then (2) breaks into activity subproblems or agent problems, $j = 1,\ldots,n$. These are the following:

(3a) $\max E\{u_j (z_j(s), s) - \sum_{i=1}^{m} p_i^o(s) E(z_{ij}(s) \mid B_i)\},$

subject to

(3b) $z_j(s) \in Z_j(s), z_j(s) \in J(B_j).$

Let the optimal solution of (3) be $z_j^o(s)$, and now $z^o(s) = (z_j^o(s)), j = 1,\ldots,n$ is the optimal solution of (1).

We transform the problems (3) of the agents $j = 1,\ldots,n$ into equivalent Walrasian form:

(4a) $\max EU_j (z_j(s), s, y_j)$

subject to

(4b) $E \Sigma p_j^o(s) E(z_{ij}(s) \mid B_i) + y_j = 0,$

(4c) $(z_j(s), y_j) \in X_j(s), z_j(s) \in J(B_j),$

where $U_j(z_j(s), s, y) = u_j(z_j(s), s) + y_j$, and $X_j(s) = Z_j(s) \times Y_j$ and $Y_j = (y_j \mid y_j \geqslant y_j^o),$

where y_j is the agents' j expected income (numeraire good).

The new constraint (4b) is the budget constraint of the agent, and y_j^o is the agents' j expected optimal income determined by the center (according to the aggregated model H)[3]. It follows from the strict concavity of (4a) that the solution of (4) is $z_j^o(s)$.

Program (4) has the same structure as the agents' program in Benassy (1986). The only difference is that in case of (4) we must consider not just one determined event of the world but all the events.

Under the assumptions made above the Walrasian equilibrium of the agents with programs (4) exists, and the prices are strictly positive, and the results of Benassy can be further used for programs (4).

4. *Benassy's stochastic mechanism*

Below the planning mechanism will be described on the example of programs (4) of the agents $j = 1,\ldots,n$ in a game where the stochastic outcome functions satisfy Benassy's conditions[4]. This game leads the Nash equilibrium to Walrasian or optimal coordination outcomes where coordination is combined (prices and quantities).

In this game agent $j = 1,\ldots,n$ sends future price and quantity (net-trade) messages to $i = 1,\ldots,m$ planning offices. Let $\hat{p}_j(s)$ and $\hat{z}_j(s)$ be the vectors of agent j's price and quantity messages. We call $\hat{p}(s) = \{\hat{p}_j(s) \mid j = 1,\ldots,n\}$ and $\hat{z}(s) = \{\hat{z}_j(s) \mid j = 1,\ldots,n\}$.

The plans (contracts) of exchange $z_{ij}(s)$ and prices $p_{ij}(s)$ actually achieved by the agent j in the office i are described by the strategic outcome functions:

$$(5a) \quad z_{ij}(s) = M_{ij}(\hat{p}(s), \hat{z}(s)),$$

$$(5b) \quad p_{ij}(s) = N_{ij}(\hat{p}(s), \hat{z}(s)).$$

We shall assume that these functions satisfy Benassy' assumptions for every s. First, we assume voluntary exchange. This means that no agent can be forced in any event to make more contracts than he plans in his quantity message, and trade at prices less favourable than the ones he has quoted. Secondly, a frictionless planning mechanism is assumed, i.e. agents do not miss opportunties for plans. And the third assumption is that of price priority. It says that in the planning office the demanders will give preference to the suppliers announcing the lowest prices, and conversely (this assumption automatically means that the mechanism is competitive). A consequence of this assumption is that suppliers who quote higher planning prices will get rationed plans, and conversely demanders who quote lower prices will be rationed.

Under these assumptions it has been demonstrated by Benassy (1986) that the Nash equilibrium of the game is also the Walrasian equilibrium or an optimal solution. So everybody announcing Walrasian prices and quantities for every event will also get Walrasian outcomes for that event. And no agent can improve his situation by changing his strategy while the other agents maintain their Walrasian strategies. Thus agents participate in the price and quantity setting in the planning mechanism, and they are interested in setting optimal plans.

Although this mechanism is sound for the theoretical analysis of decentralized planning, unrealistic assumptions are made. First, it is assumed that there are planning offices $i = 1,\ldots,m$ for every $s \in B_i$. And, secondly, every agent is assumed to be able to determine all the Walrasian prices $i = 1,\ldots,m$, and quantities $i = 1,\ldots,m$, for every event $s \in B_i$, B_j. In the next section we shall eliminate some unrealistic assumptions.

There is a multitude of approaches and their combinations to make coordination processes simpler. The ultimate simplification is to base planning indicators on as coarse a σ-field as possible, that is to come down to the one-state world or a deterministic problem. Another line of simplification is to leave some parameters out of direct coordination, i.e. to coordinate them indicatively with some kind of aggregated parameters, etc. So, there is a broad field of all kinds of alternative heuristic combinations of simplification approaches, out of which only two will be described in the next sections.

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[3] *Note, the condition $y_j \geq y_j^o$ is necessary only for the consumers to limit their expenditures. And the center has the right to enact these limits, because the initial endowments of the economy b(s) belong to the center.*

[4] *Recall the heuristic type of our treatment.*

5. *A modification of the mechanism in J. Green's and J. Grandmont's style*

The problem considered in this section arises from the fact that in reality complete current forward planning offices in state-dependent claims cannot exist because the number of the states is enormous. So we must reduce this ideal mechanism to a more practical one. For that purpose in this section we use the model studied by J. Green (1973) and J. Grandmont (1982). In this model an agent makes exchanges according to fixed plans at fixed prices on a futures market. That implies the non-contingent or non-state-dependent delivery of commodities at later dates. Also future spot markets are assumed to be active on these dates (the dates of the delivery or reciept of futures). So the possibility of future spot trades on forward markets is open to the agent. The prices on these markets are understandably unknown but the agents have expectations concerning future spot prices.

To simplify matters still more, assume that there are only two periods (implicitly we assume that the initial problem is a two-stage stochastic planning problem). The first period is deterministic. So in period 1 each knows his parameters, but does not know what will happen in period 2. Each trader knows that in period 2 there will be a spot market for goods available on that date. In period 1 there are planning offices for spot and futures or non-contingent future plans. But there is no possibility to make state-dependent contracts for the second period and spot prices for period 2 are not negotiated in period 1. So the agents must also have correct expectations concerning the future.

Consider a representative agent in period 1. His Benassy strategy is $(\hat{p}_{1j}, \hat{z}_{1j}, \hat{p}^2_{1j}, \hat{z}^2_{1j})$. It represents the prices $\hat{p}_{1j}$ of the current goods (spot) and the prices $\hat{p}^2_{1j}$ of planned fixed purchases of goods to be delivered in the second period. It also represents the respective planned quantities of goods $\hat{z}_{1j}$ and $\hat{z}^2_{1j}$ for sure delivery or receipt.

In period 2 the agent will receive the signal of the state s and give signal $(\hat{p}_{2j}(s), \hat{z}_{2j}(s))$, describing his spot prices and the respective quantities of goods in period 2. In period 1 the agent forms correct expectations $(p^o_{2j}(s), z^o_{2j}(s))$.

Now our agent's decision problem is similar to the one described by J. Grandmont (1982), and according to the assumptions made there the necessary and sufficient conditions for the existence of an equilibrium are present. Note that in this approach the assumption of correct expectations of future spot prices and deliveries are necessary. If this model satisfies. J. Benassy's conditions the Nash equilibrium will be Walrasian. So the agent's plans are not coordinated about forward state-dependent markets, but they are still compatible. This was achieved with the help of the correct foresight approach postulating that all the agent's expectations are correct. The correct or rational foresight approach is very convenient, but it is surely an improper tool for describing the reality, see e.g. Allen (1986) and Wittmann (1985). To alleviate this problem we assume that in elaborating expectations the agents are sustained by the indicative primal-dual solutions of the aggregated horizonal problem H.

6. *On interval planning mechanisms*

An interval planning mechanism would be the first step from deterministic point planning toward the indeterministic mechanisms. Although this step seems practically most tempting it has not been sufficiently elaborated upon theoretically. One reason for this seems to be the mathematical clumsiness of interval models on the one hand, and their theoretical awkwardness on the other.

Here we shall make an attempt to build an extremely simple interval planning model and to fit it into Benassy's mechanism. The choice of the possibly simple model helps us avoid some mathematical difficulties, but still enables us to demonstrate the fact that in order to get the freedom to work within the quantity interval the agent has to make payments according to the respective market prices. And, vice versa, for giving the others an opportunity to work within the intervals, the agent will get a revenue at the same prices. Thereby the Benassy's mechanism enforces the announcement of Walrasian interval demands and supplies and Walrasian prices for the intervals.

The economic content of the initial model is the following. The future net-trades $z_j(s)$ of the individual units are allowed to vary according to the future $s \in S$ in certain preplanned intervals. At that the averages of the intervals must meet the balance restrictions, and the overall extent of the intervals is also limited for the economy.

The mathematical form of our model is the following:

(6a) $\max \sum_{j=1}^{n} Eu_j(z_j(s_j))$

subject to

(6b) $z_j(s) \leqslant a_j(s)$,

(6c) $z_j(s) \in [\bar{z}_j \pm \Delta z_j]$,

(6d) $\sum_{j=1}^{n} \bar{z}_j \leqslant b$,

(6e) $\sum_{j=1}^{n} \Delta z_j \leqslant \Delta z$,

(6f) $\Delta z_j \geqslant 0,\ j = 1,\ldots,n$,

where $a_j(s)$ is a given stochastic limit, $\bar{z}_j$ is the planned average value of the net-trade interval of j, b is the given limit of the sum of the average of the net-trade intervals, and Δz is the given limit of the sum of the half intervals Δz_j of the net-trades. We shall assume that u_j is continuous and concave, the problem is regular, and all the stochastic conditions are met with the probability one.

Now we shall decompose program (6) into agent subproblems. For this we shall define the Lagrangian prices p and r for the conditions (6d) and (6e).

Now the program of the agent $j = 1,\ldots,n$ is

(7a) $\max Eu_j(z_j(s)) + y_j$

subject to

(7b) $z_j(s) \leqslant a_j(s)$,

(7c) $z_j(s) \in [\bar{z}_j \pm \Delta z_j]$,

(7d) $y_j + p\bar{z}_j + r\,\Delta z_j = 0$,

(7e) $\Delta z_j \geqslant 0$.

This problem satisfies the Benassy's agents' conditions, and we can use its mechanism. In it each agent sends price-quantity messages to the planning office. These messages are $\hat{p}_j$, $\hat{r}_j$, $\hat{z}_j$ and $\Delta\hat{z}_j$.

We should note that in model (6) we described only non-negative Δz_j. But it is easy to see that we can also model negative deviations $\Delta z_j < 0$. The latter models the flexibility of the agent's j net-trades, or in other words, the interval in which the net-trade or j should vary state-dependently, and the agent j will get a revenue for this kind of plan.

In summary, our simple interval planning mechanism model can support isolated rational individual behavoir of the agents in announcing interval demands and supplies.

7. *Final remarks*

The purpose of this paper was to clarify some problems of alternative decentralized planning mechanisms of an abstract socialist economy. In this economy the initial endowments belong to the center, and there is an initial global objective function. Planning as the elaboration of future value of activities is assumed to be intrinsic to this economy. A rational initial planning model is postulated as a stochastic optimization problem.

It is also postulated that the only reliable information the center has is aggregated. Detailed information is the private (differential) information of the agents, and its transfer to the center without distortion is impossible.

The paper recommends that the initial problem be decomposed into an aggregated horizonal subproblem and a detailed threshold subproblem. The former is dealt with by the center (possibly with the help of systems of models), and its results determine the constraints and some indicative indicators for the threshold problem. The latter one is immensely large, and its information is decentralized. It is recommended that this problem be tackled with the help of decentralized competitive mechanisms.

The paper examines these mechanisms on the basis of Benassy's competitive market game in the context of three optimal future price-limit coordination principles. In the case of the first mechanism the basic assumption is that there exists a complete set of current (pre-event), state-dependent, future goods planning offices. The second version says that the agents have current correct expectations about the optimal future (post-event) state-dependent prices and quantities. The third mechanism deals with interval planning. The elaboration of optimal price and quantity plans in all mechanisms is achieved.

However, the mechanisms studied are based on highly oversimplified assumptions. So the problem remains to be studied on more realistic assumptions and not oversimplified models. Significant open problems include the implementation of a correct expectation equilibrium (information acquisition, indicative planning by the center, etc.), imperfectly competitive markets, bidding for long-term contracts with relationship-specific investments under uncertainty, iterative coordination in the stochastic environment, diversification of the ownership of initial endowments, etc. And last but not least, Benassy's mechanism is a one-step process. It is based on the assumption that before this step some kind of tatonnement has implicitly taken place. So the problem remains to make this process explicit in the case of a stochastic world.

One more important open issue connected with the paper is the question of the submechanism of incentives. It is reasonable to start with the risk-neutral initial global model. In a decentralized setting this would mean that the normative objective functions of the agents are also risk-neutral. However, in reality the managers of the firms have risk-average utility functions. To encourage the risk-averse managers to make risk-neutral decisions, there must be some kind of additional risk-sharing enforcement submechanism enabling managers to transfer incomes between the states. According to Marshak (1986) such a submechanism can be designed and it can be studied separately.

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