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A MODEL OF IMPLICIT CONTRACTS WITHOUT PRECOMMITMENT ON EMPLOYMENT

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The purpose of this paper is to analyze an optimal one-period labor contract between one firm and one worker when there is no precommitment on employment. I assume, instead, that implementation is the responsibility of a third party (e.g. court of law) that observes only the wage rate and the state of nature. Under these informational assumptions the first-best contract is no longer attainable even though the state is publicly observed. It is demonstrated in this paper that (i) either the worker or the firm is rationed in every state of nature, except possibly in one state; (ii) the firm is rationed in low states; (iii) the contract stipulates the wage rate and the employment level to be increasing functions of the state whenever the firm is rationed.

1. Introduction

The earlier literature on implicit contracts (Baily (1974), Azariadis (1975)) did not pay much attention to the implementation of the contracts. Contracts were stipulated to be binding by assumption, or by leaving the enforcement issue completely to the hands of a powerful third party. If there were no asymmetric information between the inside parties of the contract (worker and firm) and the outside (third) party, implementation would not be a problem; the third party could fully enforce the contract. When the third party does not have full information then the contract must be constructed in such a way that enforcement is guaranteed, if we are ever going to observe the contract.

In this paper I study a special one period model of implicit contracts where I pay close attention to the implementation of the contract when the third party has less information than the inside parties. My informational assumptions will be rather specific, but I still hope that my model can shed more light on the general issue of enforcement in implicit contracts.

The enforcement issue has received some attention in the literature. Hart (1983) pointed out that if we want to appeal to the idea of an implicit contract, we have to show how it is enforced. Holmström (1981, 1983) noted that serious problems arise especially in a one-period model. In a multi-period context the

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inside parties alone can guarantee the implementation of the contract via reputation effects [Bull (1987)]. Bull (1983) argues that the Baily-Azariadis contracts will not be observed under certain informational assumptions, a very important one being that there is no third party enforcement.[1] It is exactly this enforcement and the third party observation issue that lie in the heart of this paper's explanation for the implementable contract.[2]

Younes (1984a, 1984b) studies among other things a simple general equilibrium model where labor hours are not observed by the outside party. That is why he introduces the ex post rationality or voluntary exchange constraints to the characterization of the contract. Also Eden (1984) studies a contract set-up where the firm will have to fullfil the ex post rationality constraint. However, Eden's rationalization for this constraint is different; it comes from the firm's optimal layoff decision. My model will build on the basic foundations laid in Younes (1984a, 1984b). My propositions give a more precise characterization of the contract. This is due to the fact that my model is a partial equilibrium model. However, the utility function of the worker is more general than in Younes's papers.

In this paper I will assume that the third party (e.g. courts) is able to observe only the wage rate and the state of nature, but not the hours worked. This seems to be a realistic assumption. Wage rates in different occupations are aggregate data, but the hours worked by any specific worker are idiosyncratic data. It is not claimed that in practice hours cannot be observed, but that it is easier to observe the wage rates than the hours worked by any worker. To approximate this fact I will study a polar case where the hours worked by any worker are not observed at all by the third party.

The third party is assumed to observe the state of nature which in my model will be a multiplicative random shock to the production function. The value of this stochastic shock tells something about the general business conditions of the firm. The relatively easy observability of general business conditions in particular industries and firms rationalizes the assumption that the state of the world is observed by the third party.

Suppose that one firm and one worker are contemplating a contract about the wage rate and the employment level in each state. What are the main characteristics of such a contract that will also guarantee enforcement? The worker should get some minimum level of expected utility, which reflects his outside opportunities. To guarantee voluntary enforcement in every state worker cannot be forced to supply more hours than is optimal for him at the given state and wage rate; firm cannot be stipulated to demand more hours than is optimal at the given state and wage rate. In other words, the contract cannot push parties outside their respective ex post supply and demand curves. But clearly they can be inside their respective curves in some states, and thus they can be rationed.[3]

Why should one introduce the ex post rationality constraints discussed above? Consider a situation after the state has been revealed. Suppose that contract stipulates worker to supply more hours at this contract-determined wage rate than is ex post optimal for him. What will worker do? Because, by assumption, the third party does not observe the hours worked there is no incentive for the worker to work more than is optimal for him at the prescibed wage. The same argument holds for the firm. Hence, intuitively it is plausible to expect that in high states of nature (i.e., states of high productivity) there is excess demand for labor, because the workers are not willing to work even with high wage rates more than is optimal for them. In low states one would expect to observe excess supply of labor, because there is not enough demand for labor, and the firm cannot be forced to demand more than is optimal for it. Indeed, these are the main findings of the paper, and they will be discussed in the sequel.

It is exactly those ex post rationality constraints that form the enforcement mechanism

[1] *The third party can only enforce those features of the contract that it observes.*

[2] *Because of my informational assumptions I have to pay careful attention to the mechanism for implementing the contract. However, there is no asymmetric information between the active (inside) parties. Thus my model does not directly fit the literature on the general design of mechanisms. See e.g. Myerson (1989).*

[3] *The ex post constraints introduced here are familiar from the fix price literature, and they are there known as voluntary exchange constraints, see e.g. Malinvaud (1977).*

of the contract studied in this paper. Those constraints must be imposed on the parties because of the lack of precommitment on employment and the one-period nature of the relationship between the firm and the worker.

2. *The model and the characteristics of the first-best contract*

This model has three economic agents; a worker, a firm, and a third party. The worker and the firm will enter into a labor contract defined over the wage rate and the hours worked. Uncertainty comes from a random shock to firm's production function. This stochastic shock is observed by both inside parties. The third party observes only the wage rate and the state of the world.

The worker derives utility from consumption c, and labor hours l. Consumption is equal to the real wage bill. So the utility function can be written as

$$(1) \quad u = u(c, l) \equiv u(wl, l)$$

where w is the real wage rate. The function u is increasing in c, decreasing in l, strictly concave, and continuously twice differentiable. In addition the following assumptions are made[4]

$$(2) \quad wu_{11} + u_{21} < 0 \text{ and } wu_{12} + u_{22} < 0$$

$$(3) \quad u_1 + u_{11}wl + u_{21}l > 0.$$

Assumption (2) guarantees that consumption and leisure are not inferior goods. Assumption (3) together with (2) makes sure that the labor supply curve is upward-sloping. This labor supply curve is $l = g(w)$ of which we know that $g' > 0$. Furthermore I make the following assumption that will guarantee interior solution i.e. $0 < g(w(s)) < \bar{l}$, $\forall s$, and $w(s) > 0$, $\forall s$, where $\bar{l}$ is the endowment of leisure and s denotes the state of the world:

$$\lim_{l \to \bar{l}} \left[- \frac{u_2(wl, l)}{u_1(wl, l)} \right] = \infty$$

and

$$\lim_{l \to 0} \left[- \frac{u_2(wl, l)}{u_1(wl, l)} \right] = 0.$$

The owner of the firm is assumed to be risk-neutral. That assumption allows one to concentrate on the profit maximizing behavior of the firm. The output of the firm in the ith state, y_i, is given by the production function $s_i f(l_i)$, which furthermore has the following properties

$$(4) \quad \text{a) } f'(l) > 0, \; f''(l) < 0$$

$$\lim_{l \to \infty} f(l) = \infty, \; \lim_{l \to \infty} f'(l) = 0 \text{ and}$$

$$\lim_{l \to 0} f'(l) = \infty$$

So the production function is strictly concave and satisfies Inada conditions. The uncertainty in this model enters as a shock, s, to the production function. The random variable s is assumed to have a discrete non-degenerate distribution with n different values i.e. $s_i \in \{s_1, s_n\}$. The probability of s_i is denoted by p_i. Denote by S the set of all states

$$(5) \quad S = \{s \mid s_1 \leq s \leq s_n, \; s_i < s_{i+1}, \; \forall_i\}.$$

Both the worker and the firm know S and the probabilities p_i. $\sum_{i=1}^{n} p_i = 1$ holds, and furthermore $p_i > 0$, $\forall i$.

Next the first-best contract is characterized. We define the contract δ as a mapping from S to R^2_+ as follows

$$(6) \quad \delta = \{w_s, l_s\}.$$

The first-best contract is a solution to the following problem:

$$(\text{P.1}) \quad \max_{\{\delta\}} E_s \left[\tau u(w_s l_s, l_s) + (1-\tau)(sf(l_s) - w_s l_s)\right]$$

$$\text{s.t. } w_s \geq 0, \; l_s \geq 0, \; l_s \leq \bar{l}, \; \forall s.$$

Here τ is a constant number in [0, 1] which determines essentially what minimum expected utility or profit the contract must give to each party, and E_s denotes the mathematical expectation with respect to s. By varying τ we pick points on the expected utility possibility

[4] *Subscripts refer to partial derivatives, e.g.* $u_{12} = \partial^2 u / \partial c \partial l$.

frontier. The higher the value of τ the more expected utility for the worker the contract must guarantee. The first-order conditions give the well-known optimal insurance condition (7) and productive efficiency condition (8).

$$(7) \quad u_1(w_s l_s, l_s) = (1-\tau)/\tau, \ \forall s$$

$$(8) \quad \tau\,[u_1(w_s l_s, l_s)\, w_s + u_2(w_s l_s, l_s)] + (1-\tau)\,[sf'(l_s) - w_s] = 0, \ \forall s.$$

The indifference curves and the iso-profit curves are tangent to each other on the optimal insurance frontier $u_1 = (1-\tau)/\tau$. It is plausible that the first-best contract might sometimes underemploy (point A in Figure 1) and sometimes overemploy (point B in Figure 1) the worker.

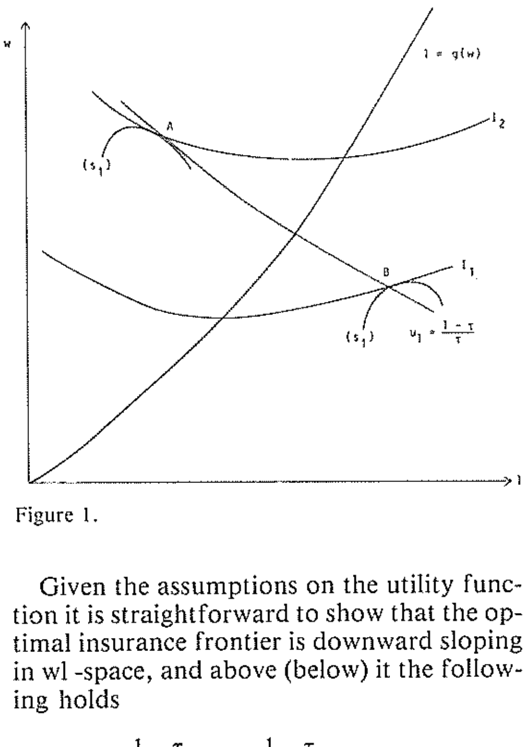

Figure 1.

Given the assumptions on the utility function it is straightforward to show that the optimal insurance frontier is downward sloping in wl -space, and above (below) it the following holds

$$(9) \quad u_1 < \frac{1-\tau}{\tau} \ [u_1 > \frac{1-\tau}{\tau}].$$

Furthermore, the curve shifts upwards (downwards) when τ is increased (decreased).

In this contracting framework firm cannot usually equalize the marginal product of labor to the wage rate. The reason is that, besides being an employer, the firm also acts as an insurance company. Because the firm operates in two capacities it cannot at the same time pay the wage rate that is equal to the marginal rate of substitution and the marginal product of labor, and provide the insurance. In an Arrow-Debreu economy workers are guaranteed constant marginal utility of consumption, and they are paid their marginal product. This is because the capacities of an insurer and an employer are distinct.

3. The implementable contract

The model has three economic agents: the worker, the firm and the so-called third party. The worker and the firm observe everything, but the third party observes only the wage rate and the state of the world. However, it does not observe the hours worked. If the worker and the firm disagree about the specification of the contract after the state is revealed there must be the third party enforcement. But because the third party cannot observe hours it cannot enforce the contract with respect to hours. That is why the contract itself must be finetuned to take care of this enforcement issue.

The first-best contract can specify the wage rate and the hours in some state in such a way that the worker must work more hours than he would like at that wage. This contract cannot be observed under the informational assumptions made above, because worker will refuse ex post to work more than is optimal for him. He can do that because the third party observes only the wage rate and the state, and thus cannot enforce the hours. The same type of argument holds for the firm. The contract cannot stipulate the firm to demand more labor than is optimal for it ex post.

These ex post rationality constraints are introduced formally in (10) and in (11) for the worker and for the firm respectively:

$$(10) \quad w_s \geq -\frac{u_2(w_s l_s, l_s)}{u_1(w_s l_s, l_s)}, \ \forall s,$$

$$(11) \quad w_s \leq sf'(l_s), \ \forall s.$$

Definition

The worker is rationed if (10) holds as a strict inequality, and the firm is rationed if (11) holds as a strict inequality.

So worker is rationed if the wage rate exceeds his marginal rate of substitution between consumption and labor. Firm is rationed if the marginal product of labor is strictly greater than the wage rate. Thus, they are rationed if pushed outside their respective supply and demand curves.

Now we have enough machinery to go on describing a contract which can be implemented under our informational assumptions. The implementable contract $\delta = \{w_s, l_s\}$ is a solution to the following problem

$$\text{(P.2)} \quad \max_{\{\delta\}} E_s\,[\tau u\,(w_s l_s, l_s) + (1-\tau)\,(sf\,(l_s)) - w_s l_s)]$$

$$\text{s.t.} \quad \text{(i)}\ w_s \geq -\frac{u_2\,(w_s l_s, l_s)}{u_1\,(w_s l_s, l_s)},\ \forall s,$$
$$\text{(ii)}\ w_s \leq sf'\,(l_s), \quad \forall s$$
$$\text{(iii)}\ w_s \geq 0,\ 0 \leq l_s \leq 1,\ \forall s.$$ [5]

Unfortunately P.2 is a non-convex problem given our assumptions. However the optimal contract can be characterized in terms of the necessary conditions. From non-linear programming theory we know that if there exists a solution $\delta = \{w_s, l_s\}$ of P.2 then there exist non-negative multipliers $\{\mu, \alpha\}$ such that $\{w, l, \mu, \alpha\}$ satisfy the Kuhn-Tucker condition associated with the following Lagrangean function

$$\text{(12)} \quad L = \sum_{j=1}^{n} p_j\,[\tau u\,(w_j l_j, l_j) + (1-\tau)\,[s_j f\,(l_j) - w_j l_j]] + \sum_{j=1}^{n} \mu_j\,[w_j + \frac{u_2\,(w_j l_j, l_j)}{u_1\,(w_j l_j, l_j)}] + \sum_{j=1}^{n} \alpha_j\,(s_j f'\,(l_j) - w_j).$$ [6]

[5] *Interior solution is guaranteed by the assumptions about the utility and production functions. Furthermore, the constraint qualification is assumed to hold. In addition it is also assumed that when one constraint holds as an equality the respective multiplier is strictly positive. This assumption can be defended because the cases where those multipliers are zeroes are 'rare'. For a precise treatment of this problem of degeneracy consult Spingarn and Rockafellar (1979). Because of the assumptions guaranteeing an interior solution one need not worry about the constraints (iii).*

[6] *See Mangasarian (1979) p. 97—110. Especially see Kuhn-Tucker stationary-point necessary optimality theorem. w is a shorthand notation for $\{w_1, \ldots, w_n\}$ and so on.*

The Kuhn-Tucker conditions are

$$\text{(13)} \quad \text{(a)}\ L_{l_i} = p_i\,[\tau\,(u_1 w_i + u_2) + (1-\tau)\,(s_i f'\,(l_i) - w_i)] + \mu_i\,[\frac{u_{21} w_i + u_{22}}{u_1} - \frac{u_2\,(u_{11} w_i + u_{12})}{u_1^2}] + \alpha_i s_i f''\,(l_i) = 0,\ \forall i,$$

$$\text{(b)}\ L_{w_i} = p_i\,[\tau u_1 l_i - (1-\tau)\,l_i] + \mu_i\,[1 + \frac{u_{21} l_i}{u_1} - \frac{u_2\,(u_{11} l_i)}{u_1^2}] - \alpha_i = 0,\ \forall i,$$

$$\text{(c)}\ L_{\mu_i} = w_i + \frac{u_2}{u_1} \geq 0,\ \forall i,$$

$$\text{(d)}\ L_{\alpha_i} = s_i f'\,(l_i) - w_i \geq 0,\ \forall i,$$

$$\text{(e)}\ \mu_i L_{\mu_i} = 0,\ \forall i,$$

$$\text{(f)}\ \alpha_i L_{\alpha_i} = 0,\ \forall i.$$ [7]

With the Kuhn-Tucker conditions in mind, we will now proceed to the core of this study where it will be proved that generally the firm will be rationed in high states, and the worker will be rationed in low states.

The following proposition states the result that constraints (i) and (ii) in P.2 cannot hold as equalities at the same time in any state, except possibly in one.

Proposition 1:

There is possibly only one state where the worker and the firm are not rationed. Otherwise a particular solution in any other state is such that either the worker or the firm, but not both, is rationed.

Proof:

Suppose that there is a state, say r, such that the labor supply and demand curves intersect at the optimal insurance curve (a measure zero event in the case of a discrete state space). There cannot be more than one such state, because labor supply curve is upward sloping and the other curves are downward sloping. From Kuhn-Tucker conditions (b) it follows that

[7] *Note that complementary slackness conditions are not needed for (a) and (b) because of the assumptions that guarantee interior solution.*

$$\alpha_r = \mu_r \left[\frac{u_l + u_{21}l_r + u_{11}w_rl_r}{u_l}\right].$$

Plugging this into (a) we get

$$\mu_r \left[\frac{u_{21}w_r + u_{22}}{u_l} - \frac{u_2(u_{11}w_r + u_{12})}{u_l^2}\right]$$

$$= -s_rf''(l_r)\left[\frac{u_l + u_{21}l_r + u_{11}w_rl_r}{u_l}\right]\mu_r$$

The term in the brackets on the left hand side is negative and the terms multiplying μ_r on the right are positive. The only way that the Kuhn-Tucker conditions can hold is then that $\mu_r = \alpha_r = 0$. But then all the other KT-conditions hold, too. Thus we can have one state where nobody is rationed.

Suppose now that there exists a state, m, where the constraints (i) and (ii) in P.2 hold as equalities, i.e. nobody is rationed. For this particular state, m, the Kuhn-Tucker condition (a) can be written as

$$\frac{\mu_m}{\alpha_m} = -\frac{s_mf''(l_m)}{\frac{u_{21}w_m + u_{22}}{u_l} - \frac{u_2(u_{11}w_m + u_{12})}{u_l^2}}.$$

But according to our assumptions about the production and utility functions the numerator and the denominator are negative. Hence, if the above equation is to hold, one of the multipliers must be negative. But that is a contradiction for w_m and l_m to be a solution of P.2 in state m. Hence both constraints cannot hold as equalities at the same state.

Now suppose that the constraints (i) and (ii) hold as inequalities in some state, i.e. both parties are rationed. Then the multipliers for both constraints are zero. Thus the Kuhn-Tucker conditions are the same as in the first-best problem. But the solutions to P.1 (see Figure 1) lay on the optimal insurance frontier, and hence the solutions outside the labor supply curve fulfilled either $sf' < w$ or $u_lw + u_2 < 0$. So points like A or B in Figure 1 could not be solutions to P.2 because they violated either the constraint (i) or the constraint (ii). Thus both parties cannot be rationed in a particular solution in any state.

Q.E.D.

This proposition has a nice intuition behing it. Consider a case with only three states.

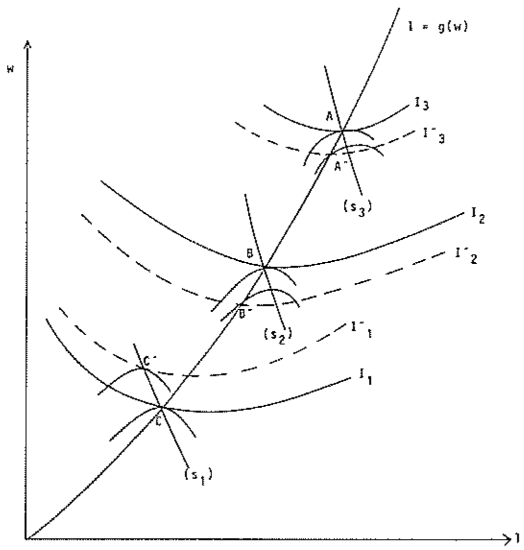

Figure 2.

This case is depicted in Figure 2. If the proposition is false, the contract will be indicated by points A, B and C. In those points nobody is rationed because $w = sf' = -(u_2/u_l)$. And the expected utility is $\sum_{i=1}^{n} p_iI_i = \bar{u}$. The argument is now that the firm can increase its expected profits by offering the contract {A', B', C'} which will guarantee the worker the same expected utility as before i.e. $\sum_{i=1}^{3} p_iI_i' = \bar{u}$. The reason is that the firm by moving 'southeast' on its labor demand curves in high states 2 and 3 can gain more than it will lose in expected profits by moving 'northwest' on its labor demand curve in low state 1. The conjecture from this is that in high states firm will be rationed. (e.g. $s'f' > w$ in point A') and in low states worker will be rationed. Indeed, this verbal-geometric argument will be proven to be correct.

Proposition 2:

1) Firm will be rationed in all those states, where the solution is above the optimal insurance curve, i.e. for all these states, s,

   $u_l(w_sl_s, l_s) < (1-\tau)/\tau.$

2) Worker will be rationed in all those states, where the solution is below the first-best

optimal insurance curve, i.e. for all these states, s,

$$u_1(w_s l_s, l_s) > (1-\tau)/\tau.$$

Proof:

1) Consider first the case when the firm is rationed. Then (ii) in P.2 holds as a strict inequality. By Proposition 1 (i) must hold as equality. Hence, parts (a) and (b) of the Kuhn-Tucker conditions for some such state k are

$$p_k\,[1-\tau)\,(s_k f'-w_k)] + \mu_k\,[\frac{u_{21}w_k + u_{22}}{u_1} - \frac{u_2(u_{11}w_k + u_{12})}{u_1^2}] = 0$$

$$p_k\,[\tau u_1 l_k-(1-\tau)\,l_k] + \mu_k\,[1 + \frac{u_{21}l_k}{u_1} - \frac{u_2 u_{11} l_k}{u_1^2}] = 0.$$

Solving for μ_k one obtains from the second equation

$$\mu_k = -\frac{p_k l_k \tau\,[u_1 - \frac{1-\tau}{\tau}]\,u_1}{u_1 + u_{21}l_k + u_{11}w_k l_k}.$$

The fact that $w_k = -[u_2\,(w_k l_k,\, l_k)/u_1\,(w_k l_k,\, l_k)]$ has been utilized above. The denominator is positive by assumptions about the utility function. To guarantee that μ_k is positive, which also means that $s_k f' > w_k$, one must have $u_1 < (1-\tau)/\tau$. So these states must be such that the solutions are above the $u_1 = (1-\tau)/\tau$ curve.

2) The proof of this part parallels that of the previous part. Consider now the case when the worker is rationed. Then (i) in P.2 holds as a strict inequality. So again by Proposition 1 (ii) must hold as equality. Kuhn-Tucker conditions (a) and (b) can now be written for such state, m, as

$$p_m\,[\tau\,(u_1 w_m + u_2)] + \alpha_m s_m f'' = 0,$$

$$p_m\,[\tau u_1 l_m-(1-\tau)\,l_m]-\alpha_m = 0.$$

Solving for α_m one obtains

$$\alpha_m = -\frac{p_m\tau\,(u_1 w_m + u_2)}{s_m f''} = p_m \tau l_m\,[u_1 - \frac{1-\tau}{\tau}].$$

To guarantee positive multipliers for these states, m, one must have $u_1 > (1-\tau)/\tau$ and $u_1 w_m + u_2 > 0$. So the worker is rationed in those states where the solution is below $u_1 = (1-\tau)/\tau$ curve.

Q.E.D.

Proposition 2 points out the critical role of the first-best optimal insurance curve in determining in how many states each party is rationed. Clearly, if the weight of the worker, τ, in the objective function is increased, the number of the states where worker is rationed will increase, and vice versa if τ is diminished. This is because an increase in τ shifts the optimal insurance curve upwards and to the right. Hence in the first-best contract worker will be able to get higher wages for each level of hours, when τ is increased. But this cannot happen in the implementable contract, because the firm is not willing to exceed its demand curve. When τ is increased worker gains more power, but he still cannot force the firm to do whatever he wishes the firm to do.

We notice that in Problem P.2 the maximization of the expected value of the weighted sum of the worker's utility and the firm's profit is the same problem as that of maximizing the weighted sum of the utility and the profit state by state subject to constraints (i)—(ii). The slope of the social indifference curve i state s is given by

$$(13)\quad \frac{dw}{dl} = -\frac{\tau\,(u_1 w_s + u_2) + (1-\tau)\,(sf'-w_s)}{\tau u_1 l_s-(1-\tau)\,l_s}.$$

Thus below the optimal insurance curve and in the feasible set (defined by the constraints (i)—(iii)) the slope is negative, and above the curve it is positive. The curvature of the indifference curve is ambiguous. Figure 3 describes some of these indifference curves.

In the sequal I will make the following assumption about the slopes of the neoclassical demand curves for labor and the optimal insurance curve:

$$\text{A.1}\quad -\frac{u_{11}w + u_{12}}{u_{11}l} > sf'',\ \forall s.$$

So it is assumed that labor demand curves are steeper than the optimal insurance curve. If this assumption is not made, one state might possibly admit multiple solutions where firm

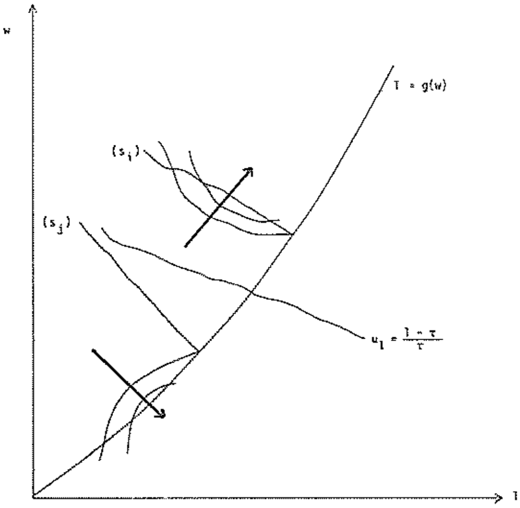

Figure 3.

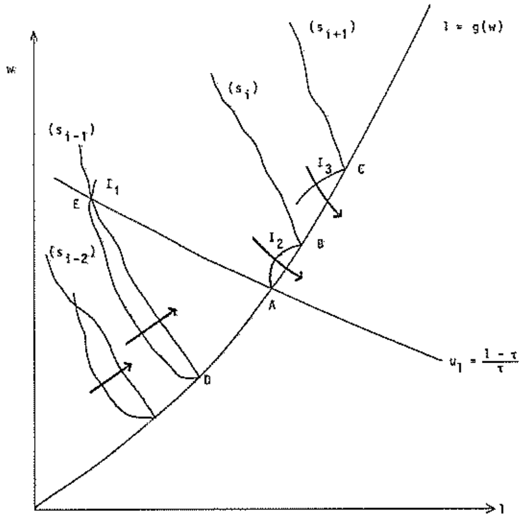

Figure 4.

might be rationed in one solution and worker in another.[8] Assumption A.1 rules out these types of multiple solutions. In the case of a separable utility function A.1 amounts to the assumption that the elasticity of labor demand with respect to the wage rate should be between 0 and 1.

Proposition 3:

Given assumption A.1, the solution to the problem P.2 is such that, if worker is rationed in some state, he will also be rationed in all lower states; and if firm is rationed in some state, it will also be rationed in all higher states.

Proof:

This proof relies on Proposition 2, Kuhn-Tucker conditions, and Figure 4. According to Proposition 2 a solution which is above the optimal insurance curve must lie on the labor supply curve, and a solution which is below the optimal insurance curve must lie on some labor demand curve. Consider first the state i, which is the first state 'above' the optimal insurance curve. Clearly the solution must be between the points A and B, because at A the social indifference curve has slope infinity ($u_1 = (1-\tau)/\tau$) and at B zero ($s_i f' = w_i$ and $u_1 w_i - u_2 = 0$). By the same argument the solution in state i + 1 must lie between A and C. Kuhn-Tucker conditions in these cases can be written as

$$-\frac{(1-\tau)\,(sf'-w)}{\tau u_1 l-(1-\tau)\,l} = -\frac{u_1 + u_{11}wl + u_{21}l}{w\,(u_1 lw + u_{12}) + u_{21}w + u_{22}}.$$

The slope of the social indifference curve equals the slope of the labor supply curve. So if the firm is rationed in some state, s_i, it will definitely be rationed in all higher states s_{i+k}, $k > 0$.

Now consider state s_{i-1} in Figure 4. According to Proposition 2 solution in this state cannot lie on that portion of labor demand schedule which is above the optimal insurance curve. It must lie between points D and E, because at D the slope of the indifference curve is zero ($s_{i-1}f' = w$, $u_1 w_{i-1} - u_2 = 0$) and at E it is infinity ($\tau u_1 l-(1-\tau)l = 0$. By the same argument the solution for the state i—2 must be on the labor demand curve for that state. Kuhn-Tucker conditions in these states reduce to

$$-\frac{\tau\,({}_1w + u_2)}{\tau u_1 l-(1-\tau)l} = sf'',$$

[8] *One should notice that this situation does not violate Proposition 1, because that only describes a specific solution saying nothing about the multiplicity of solutions. Proposition 1 states that a solution must lie either on the labor demand curve or on the labor supply curve.*

which says that the slope of the social indifference curve equals the slope of the labor demand curve. So clearly, if worker is rationed in some state s_{i-1}, he will be rationed in all states s_{i-1-j}, $j > 0$.

Q.E.D.

By Proposition 2 solution for state i—1 must lie between points D and E and for state i between points A and B. The points in question cannot be solutions. For example, suppose that point D is the solution for state i—1. Then it is the case that $u_1 w_{i-1} + u_2 = 0$, and from Kuhn-Tucker conditions it follows that the multiplier should be zero, and thus $\tau u_1 l_{i-1} - (1-\tau)\, l_{i-1} = 0$ which is a contradiction to the fact that the solution is below the optimal insurance curve.

Because the proof of Proposition 3 is partly based on the geometry of indifference curves, corner solutions must be ruled out. Clearly the assumptions about the indifference curves guarantee that the labor supply approaches $\bar{l}$ when w is increased. So $\bar{l}$ cannot be a solution in any state. And the solution where l would be zero cannot be optimal either, because that would mean that utility is u (0,0) and profits are negative. So by increasing l a little utility and profits can be increased.

How do the wage rate and employment level behave with respect to the changes in the state of the world? Proposition 4 gives a partial answer to this question.

Proposition 4:

In those states where the firm is rationed the implementable contract $\delta = \{w_s, l_s\}$ stipulates the wage rate and the employment level to be increasing functions of the state.

Proof:

Consider some state, i, where the firm is rationed but the worker is not. The slope of the indifference curve at that point for state i is

$$-\frac{(1-\tau)\,(s_i f' - w_i)}{\tau u_1 l_i - (1-\tau) l_i} = \left.\frac{dw}{dl}\right|_{s_i}$$

The slope of the indifference curve going through that same point for state i + 1 is

$$-\frac{(1-\tau)\,(s_{i+1} f' - w_i)}{\tau u_1 l_i - (1-\tau) l_i} = \left.\frac{dw}{dl}\right|_{s_{i+1}}.$$

So it is obvious that the solution in state i + 1 lies strictly to the 'northeast' of the solution in state i because of the monotonicity of the labor supply curve, and because

$$\left.\frac{dw}{dl}\right|_{s_{i+1}} > \left.\frac{dw}{dl}\right|_{s_i}.$$

Q.E.D.

So in those states where the firm is rationed the wage rate and the employment level are increasing functions of the state. In states where the worker is rationed one cannot say much about the dependence of the wage rate and the employment level on the state. In those states the contract picks points on the demand curves for labor. But there is nothing to say that the tangency points of the social indifference curve and the respective labor demand curves stipulate that the wage rate and the employment level are increasing functions of the state.

Propositions 1—4 state the main results of the paper. We have established that in almost every state at least one inside party is rationed. There is excess demand for labor in high states, and excess supply of labor in low states.

Implementable contract possesses similar characteristics to those obtained in fix price models. However, I did not make any assumption of sticky prices to generate rationing.

I did not touch the question of efficiency in these implementable contracts. An interesting problem in this second-best world is if the contracts are constrained efficient, i.e. is there a tax scheme government can impose on agents to improve the employment allocation given that government has the same information as the third party. As of now this is an open question.

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