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OLD AGE CONSUMPTION AND PENSION POLICY IN A TWO-TIER DEVELOPING ECONOMY*

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In a number of developing countries, an important part of the economy is informal both in terms of production and of social protection. In this paper we consider introducing a universal pension system in the formal sector. It is shown to have two main effects: first, it makes the formal sector more attractive to migration and second, it affects capital accumulation in a way that depends on the type of social security introduced, PAYG or funded, and its induced effect on private saving. (JEL: H55, J61)

1. Introduction

In this article, we investigate how the introduction of a pension scheme will affect a *developing* economy. It is clear that the issues that social security systems face in developing countries stand quite in contrast to those faced by Western (i.e., developed) countries. This is due to numerous reasons (see, e.g., E. James in Börsch-Supan *et al.*, 1999). For one, the ageing process seen in many Western countries is currently not as important an issue in developing countries due largely to a relatively high (in comparison) birth rate, but also, unfortunately, to a lower life-expectancy in the latter. In fact, there seems to be a positive correlation between ageing and dependency indicators and the development of the economy (see Table 1, al-

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though we are of course not suggesting any causality).[1]

Furthermore, there is undoubtedly an insufficient amount of investment in these countries (which is in part why they are underdeveloped). This is largely due to the lack of political stability and financial credibility, resulting in, e.g., distrust in financial and public institutions and hence not only to capital flight of wealthy local citizens but also to reluctance of donors, global enterprises and international institutions to provide funds.

Another common feature of developing countries, the feature we will be focussing on, is that the economy usually consists of two sectors, a formal one and an informal one. The former often covers only a very small proportion of the labour force, this being largely the public sector, state enterprises and a small private (modern) sector, and is, in terms of importance of its contribution to total GDP, fairly small. By contrast, the informal sector is considerably larger in developing countries: Table 2 suggests that the larger the informal sector, the poorer the economy is. It consists of rural workers and of 'self-employed'. The aged in this sector lack access to capital markets and a formal old-age retirement system. Therefore they have to rely on a micro-support system where they are supported by their offspring and/or their local communities.[2] Hence, although cultural and traditional arrangements exist, the people in the informal sector suffer from a particularly high risk of not being able to have access to resources in times of need as they have to rely on the benevolence of their cohabitants and offspring. As a result, they work up to a very high age and do not significantly enjoy the fruits retirement has to offer.[3]

A further feature characterizing developing countries is that social security systems in the formal sector (which are largely public, although small private schemes do exist) are not only mainly embryonic in such that they only cover the public sector and foreign enterprises, hence a small part of the formal sector (see Table 3), but also embryonic as due to the abovementioned political instability, faulty institutional design, lack of incentives for boards to improve performance and other severe governance shortcomings, rates of return are often dismal and administrative costs are high (see World Bank, 1994b). However, one of the benefits of having a small formal sector in proportion to the informal sector is that even where retirement systems are based on pay-as-you-go (PAYG), they have not resulted in the copious 'free lunch' that has poisoned the retirement systems in industrial countries.

We therefore take as a reference an underdeveloped country (such as Peru or the Ivory Coast) where the informal sector is two-fold: economically informal (workers are lowskilled and receive wages in-kind that are to a large extent constant over time) and socially informal (the extended family provides for the social protection of all aged persons). We start off from an empirically relevant situation (see Table 2) where the formal sector is small compared to the informal sector. In the former there are two types of individuals: skilled workers who save and unskilled workers who do not (we shall explain later why this is the case). In the informal sector the salary that unskilled workers earn is low, aged persons here are looked after by their family. The absence of a retirement system implies that unskilled workers are left isolated and poor when aged. We suppose that in this economy there is at the beginning an interior migration equilibrium where a fraction of the unskilled workers work in the formal sector.

Although we do not discuss *how* a retirement scheme is introduced (i.e., we do not dwell on steps necessary to obtain financial credibility

[1] *Thus, whereas Europe is faced with an ageing of its population, Africa is rather witnessing the opposite and will only reach dependency levels similar to those observed in Latin America today in the 2040s.*

[2] *E.g., in 'tontines' in West Africa, co-operative arrangements exist to help provide medicaments and medical services as informal social security arrangements. See Barbone and Sanchez (1999).*

[3] *Again, taking Africa as an example, over 60% of elderly work once they are over 60 year of age. This is not only significantly higher in absolute terms than the respective proportion in OECD countries (in Belgium, e.g., less than 20% for males over 60 still work), but even more so, taking into account the differences in life-expectancy.*

and, e.g., political reliability) we consider what will happen when one is introduced to the formal sector, be it a PAYG scheme or a fully-funded (FF) scheme. We think that introducing a retirement system to the informal sector is unfeasible as the informal sector suffers from 'technical' difficulties inherent in to this sector: e.g., rural agents can rarely rely on a steady stream of income (their income is affected by weather, natural catastrophes etc.) and hence cannot commit to a steady stream of contributions to a system; also because most workers are self-employed, barely maintaining a decent standard of living, taxing may not only be difficult (due to, e.g., income measurement problems) but maybe even morally questionable.

It is clear that introducing a social security system in the formal sector will make the latter more attractive for unskilled workers. If the scheme is furthermore FF, the total level of savings in the economy will increase, allowing for development. However, we shall see that, given our set-up, there are a variety of social security schemes that could be introduced.

Our analysis is applied to a model of overlapping generations *à la Diamond* (1965). In the second section we present the set-up of our model. The third section presents the stationary state if there is no social security system in the formal sector. In the fifth section we introduce a pay-as-you-go retirement system that only covers the unskilled in the formal sector, in the fourth section we discuss different retirement systems and calculate the new steady states. Section six dwells upon 'winners and losers' of the different social security systems, Section seven summarizes the results and Section eight concludes as well as gives a few possible extensions.

Table 1. Ageing and Dependency Ratios by Geographical Region, 1995.

	60+/Total (%)	60+/15–59 (%)
Sub-Saharan Africa	4.7	9.3
Middle East & No. Africa	6.0	11.1
South Asia	6.7	11.9
Latin America and Caribbean	7.4	12.6
East Asia and Pacific	8.8	13.9
Europe and Central Asia	14.6	24.0

Source: Barbone and Sanchez, (1999), World Bank (1997).

Table 2. Importance of the informal sector vis-a-vis the formal sector.

Country	%
Argentina	41
Bolivia	58.2
Chile	44.7
Mexico	54.0
Ivory Coast	53
Peru	51
Tanzania	67

Source: Key Indicators of the Labour Market, ILO, 1999.

Table 3. Retirement systems in a selection of countries. Importance and coverage.

Country	Importance as % of GDP	Rate of coverage (in %)
Bangladesh	0.0	3.5
Niger	0.2	2.8
Chad	0.0	1.1
Mali	0.7	2.5
Mozambique	0.0	0.5
US	6.5	68
Denmark	9.9	100
UK	9.5	94.2

Source: World Bank (1994a).

2 The model

2.1 Agents

The economy we are considering consists of a large number of agents. These agents are split into two different types: skilled and unskilled. Alternatively, we could interpret these workers as being poor and rich. In any case, the type of skill an agent has is decided upon at birth by chance and cannot be influenced by an agent's behaviour. As the probability of being born an (un)skilled agent is assumed to be constant over time, due to the law of large numbers, the proportion of unskilled to skilled agents will remain constant over time.

Each agent lives for 2 periods. In the first he works full-time, in the second he 'enjoys' leisure full-time. Agents have the same utility function, irrespective of skill, which only depends on consumption in the two periods of their lives, denoted by c_t and d_{t+1}. Throughout this paper, we assume that a loglinear utility

function can describe the preferences of individuals:

$$(1) \quad u_t = \log c_t + \beta \log d_{t+1}$$

where β (< 1) is the discount factor.[4]

What distinguishes skilled from unskilled agents is that skilled agents work exclusively in a formal sector whereas unskilled may work in the formal or in the informal sector (more on these sectors later). Furthermore, skilled workers earn a higher wage rate than unskilled agents, as they are h-times more productive ($h > 1$) as unskilled ones. In addition, we assume that whereas skilled agents can save, unskilled in the informal sector are unable to do so. This assumption could be justified by assuming that unskilled workers have no access to capital markets; hereby we would capture a feature of developing economies in that there are no capital markets for certain individuals, not only in the informal sector but also in the formal one. The number of skilled L_s and of unskilled L_u is constant.

This brings us back to the reasons why social security systems were introduced in many countries just after World War II. Thus, e.g., Diamond (1985) cites inaccessibility to capital markets along with a certain degree of myopia. To a large extent, we use the same factors to explain why some agents do not save for their retirement in LDCs where capital markets are typically underdeveloped.

• The behavior of skilled workers

Each of them maximizes his utility (1) subject to the budget constraints: $w_t h = c_t + s_t$ and $(1 + r_{t+1})\, s_t = d_{t+1}$, where w_t is the wage rate and r_{t+1} is the rate of interest. Optimal saving is thus given by:

$$(2) \quad s_t = \frac{\beta}{1+\beta}\, w_t h$$

• The behavior of unskilled workers in the formal sector

They earn w_t when young. Given our loglinear utility function, we have to rule out zero consumption for unskilled retired in the formal sector. Hence, we assume that unskilled workers in the formal sector can count on one unit when old, so that their utility in retirement is zero.[5] Utility for these agents is hence given by the following equation:

$$(3) \quad u_t^0 = \log (w_t - 1)$$

By assumption the scale of w_t is such that this unit is relatively small. We now describe the informal sector before turning to the production sector of the formal economy.

• Utility of agents in the informal sector

Agents earn a low wage when working in the informal sector. We denote this by $\hat{w}$ and assume that it is constant over time, hence no time index is introduced. In the informal sector, familial solidarity ensures that the aged have means for consumption. There is in fact an important literature on private transfers to the elderly in traditional societies. It generally focuses on the dichotomy of altruism versus exchange for explaining child-to-parent transfers. The altruistic explanation requires some 'ascending' altruism. Children assist their aged parents out of filial love. One can have two-sided altruism – ascending and descending – explaining education, bequests as well as old age security. The exchange explanation raises the question of the mechanisms that sustain and enforce a two-way transaction. Among the existing possibilities, there are explicit economic incentives such as the threat of disinheritance (Bernheim *et al.*, 1985) or also the legal power of the courts and the state. There is also mutual altruism as in Kotlikoff and Spivak (1981) or the so-called preference shaping mechanism whereby parents inculcate a sense of guilt for misbehaviour in their children (Cox and Stark, 1998).

There is another explanation for child-to-parent transfers when there is no exchange *per se* but just a commitment: 'I help my parent with the expectation that when retired, my child will help me'. The sustainability of such an arrangement has been explored by a number of authors including Hammond (1975).

What really matters for our purpose is that regardless of the line of explanation, these family arrangements aimed at supporting aged parents at retirement imply an implicit form of op-

[4] *The rate of time preference is thus equal to* $\frac{1-\beta}{\beta}$.

[5] *One could interpret this unit as 'savings under the mattress'.*

timal pay-as-you-go scheme. This means that if the active have an income of $\hat{w}$ and that there is no capital market, the child-to-parent transfer θ will be defined by the optimality condition for

$$Max_\theta \ln(\hat{w} - \theta) + \beta \ln \theta = \overline{v_0}$$

2.2 Production

In the formal sector, firms produce using labour L_t and capital K_t as inputs in a Cobb-Douglas technology:

$$(4) \qquad Y_t = F(K_t, L_t) = AK_t^\alpha L_t^{1-\alpha}$$

Total depreciation after one period is assumed.

Producers maximize their profits. They thus have to solve the problem:

$$(5) \qquad \max AK_t^\alpha L_t^{1-\alpha} - w_t L_t - (1 + r_t)K_t$$

which gives

$$(6) \qquad 1 + r_t = \alpha A k_t^{\alpha-1} \text{ and } w_t = (1 - \alpha) A k_t^\alpha$$

where

$$k_t = K_t / L_t.$$

It is important to realize that w_t is the wage rate per efficiency unit which implies that the actual wage of an unskilled worker is w_t and that of a skilled worker is hw_t. We here assume that this is perfectly observable.

At equilibrium, $L_t = hL_s + m_t L_u$, where m_t denotes the fraction of unskilled working in the formal sector, h (> 1) denotes the productivity (efficiency) of skilled relative to that of unskilled workers (normalized to one), L_t denotes the labour force working in the formal sector, which consists of the skilled population and the proportion of unskilled individuals, denoted by L_s and L_u respectively. L_t is thus expressed in efficiency units. K_{t+1} denotes capital and S_t represents total savings.

$$(7) \qquad K_{t+1} = S_t = L_t s_t.$$

3. Stationary equilibrium without formal social security

In this section we describe the economy outlined above where we assume that there is no social security. We show that one can have three migration equilibria: either complete migration (all unskilled work in the formal sector), partial migration or no migration. We compute the steady state stock of capital for the case where there is no migration in Section 3.3 and show how it is affected if migration is allowed. We then investigate in Section 4 the effects of introducing a PAYG social security scheme that covers only the unskilled in the formal sector before turning to other possible social security schemes (Section 5).

3.1 Capital

With the loglinear utility function, skilled workers save

$$s_t = \frac{\beta}{1 + \beta} w_t h.$$

The capital accumulation equation in an economy with no pension scheme where only the skilled save is therefore given by

$$(8) \qquad K_{t+1} = \frac{\beta}{1 + \beta} w_t L_s h$$

With $k_{t+1} = K_{t+1} / L_{t+1}$

$$(9) \qquad L_{t+1} k_{t+1} = \frac{\beta}{1 + \beta} (1 - \alpha) A k_t^\alpha L_s h$$

or, in *per capita* efficiency units:

$$(10) \qquad (1 + m_{t+1}\lambda) k_{t+1} = \frac{\beta}{1 + \beta} (1 - \alpha) A k_t^\alpha,$$

where λ denotes the proportion of unskilled workers relative to the skilled in efficiency units:

$$(11) \qquad \lambda = L_u / L_s h$$

Hence in the steady state this economy without any pension scheme is defined by

$$(12) \qquad k^* = \left(\frac{\beta}{1 + \beta} \frac{(1 - \alpha) A}{1 + m^* \lambda} \right)^{\frac{1}{1-\alpha}} = k^*(m^*)$$

$$\text{and } L^* = hL_s(1 + m^*\lambda)$$

where $0 \leq m^* \leq 1$.

It is interesting to see that the equilibrium wage rate depends on m^*. Indeed, we have:

$$(13) \quad w^* = \left(\frac{\beta}{1+\beta}\right)^{\frac{\alpha}{1-\alpha}} (1 + m^*\lambda)^{\frac{-\alpha}{1-\alpha}} ((1-\alpha)A)^{\frac{1}{1-\alpha}} \equiv W(m^*)$$

$W(m^*)$ is a decreasing function of m^*.

3.2 Migration

In our framework we observe migration to the formal sector if the utility in the latter is greater than the utility unskilled workers have in the former. We define a reservation wage, w_R, as the value of w_t which implies equal levels of utility in both sectors for unskilled workers:

$$(14) \quad \log(w_R - 1) = \bar{v}_0$$

Hence, migration only takes place to the formal sector if

$$(15) \quad w_t > w_R.$$

In the appendix we present the dynamics of this model.

3.2.1 Three types of equilibria

In our model we can have three different types of long-term migration equilibria. We can represent each of these graphically. In the first case we have an interior solution of the proportion of unskilled working in the formal sector (Figure 1). In the second and third case we have a corner solution: in the second case the utility of being in the informal sector is higher than being in the formal sector, hence all unskilled remain in the informal sector (Figure 2). In the third case the inverse if true: all unskilled are in the formal sector (Figure 3).

Analytically, the interior equilibrium is given by the equation:

$$(16) \quad w_R = W(m^*).$$

Starting from these three cases, we proceed with some simple comparative statics. An improved old-age protection in the formal sector or an erosion of family solidarity in the informal sector will obviously lower the horizontal relation between reservation wage and m^*. This modification will allow us to move from the second to the first case or from the first to the third. In other words, we will observe an increase in the equilibrium value of m^*.

On the other hand, if the capital market is (partially) open to unskilled, the stock of capital will increase and the $W(m)$ schedule will

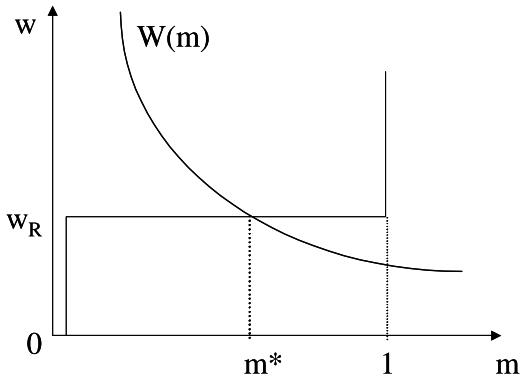

Figure 1. Interior solution.

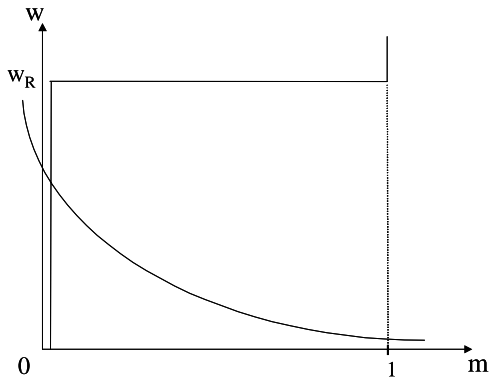

Figure 2. Corner solution: $m^* = 0$.

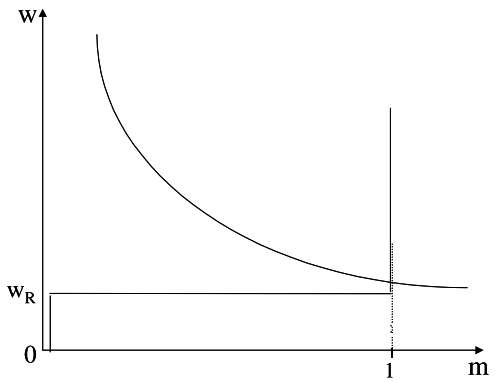

Figure 3. Corner solution: $m^* = 1$.

shift upwards, having the same effect as a reduction of the reservation wage.

3.3 No mobility

When there is no mobility between sectors, it is clear from (14) that the steady state stock of capital is equal to

$$(17) \quad k_a = \left[\frac{\beta}{1+\beta}(1-\alpha)\,A\right]^{\frac{1}{1-\alpha}} = k^*(0)$$

as $m_a = 0$; the subscript a denotes the steady state equilibrium value in a setting without any formal social security system and without migration.

Even if we now allow for mobility between sectors, it will not occur if we have

$$W(0) < w_R$$

If instead we have the inequality sign '>', there will be migration. In this case, there are two possibilities: $m_b = 1$ or $m_b < 1$ where the subscript b denotes the steady state equilibrium with migration but without a formal pension system.

We have $m_b = 1$ if

$$W(1) \geq w_R$$

Note that with a larger labour force and an unchanged capital stock, workers expect next period capital stock (per efficiency unit) and hence the wage rate to go down.

$$k = k^*(1) < k_a .$$

The other possibility is $0 < m_b < 1$. This will occur when

$$W(1) < w_R < W(0) .$$

We will assume that this will be the case and that the steady state capital stock is

$$(18) \quad k_b = k^*(m_b) < k_a .$$

4. Steady-state equilibrium with a formal pension system

Until now we have not yet introduced a pension scheme in our model. We now introduce a pay-as-you-go pension scheme in the formal sector that only covers the unskilled agents; i.e., these are the only agents that contribute to the scheme and that receive a pension. This amounts to imposing a payroll tax levy, T, on the unskilled workers and to transfer to them when they are retired the amount T. This does not modify savings: $K_{t+1} = L_s s_t$, but migration modifies the labor force and thus modifies the capital intensity k_t. We consider an interior steady state with partial migration in the absence of PAYG system: $m^* = m_b$ is the solution of $W(m_b) = w_R$, where $0 < m_b < 1$. The PAYG system is assumed to increase utility: $T > 1$ and determines a new reservation wage smaller than w_R. Indeed if $\ln(w_R - T) + \beta \ln T > \bar{v}_0$, the modified reservation wage w_T such that $\ln(w_T - T) + \beta \ln T = \bar{v}_0$ is lower than w_R : $w_T < w_R$. Thus $m_c > m_b$.

We thus have again two possible equilibria: $m_c = 1$ or $1 > m_c > m_b$, where the index c denotes the steady state equilibrium with a PAYG social security scheme for the unskilled workers.

If $w_T < W(1)$, then $m_c = 1$ and $k_c = k^*(1) < k_b$. If $w_T > W(1)$ then m_c satisfies $m_b < m_c < 1$ and $W(m_c) = w_T$. The corresponding capital stock is $k_c = k^*(m_c)$ and satisfies $k_c < k_b$. To sum up, the introduction of the PAYG scheme makes the formal sector more attractive to the unskilled workers even though it implies a decline in the capital stock in efficiency units and thus a drop in the wage rate.

It is clear that with technical progress (Harrod neutral), the outcome would be less gloomy. It is also clear that if the unskilled could be induced to save part of their earnings, the decline of capital accumulation would be mitigated.

When the pension system is introduced, one can consider that the contributions collected which represent a free lunch can be given to the retirees in the formal sector. Instead they could be used either to compensate the aged in the informal sector who cannot count anymore on their children's assistance or to build a social security trust fund.

5. Other Possible Pension Schemes

Introducing a PAYG social security scheme that only covers the unskilled working in the

formal sector is of course only one possible scheme that could be introduced. In this section we consider what happens to the new steady state if the social security scheme is different. In particular, we consider a PAYG scheme imposed on both types of individuals and a fully funded (FF) scheme.

5.1 FF scheme

Suppose that instead of transferring the contributions T from the workers to the retirees of the same period, it is invested such that one period later one receives a gross yield equal to $T(1 + r)$. Note that this scheme could be extended to the skilled workers. As long as their contribution is lower than their saving without social security, this will have no effect on the overall saving effort.

Compared to the case studied in the previous section, migration is even more attractive for unskilled workers than the PAYG scheme as for the same contribution individuals can expect a higher return (note, we make the assumption that $r > n = 0$). Choosing the index d for this regime, we hence have $m_d > m_c$.

With a FF scheme

$$K_{t+1} = L_s s_t + m_t L_u T$$

and at the steady-state

$$(1 + m_d\lambda)\, k = \frac{\beta}{1+\beta}(1-\alpha)\, Ak^{\alpha} + m_d\lambda T.$$

Assuming that the interest rate is positive, the reservation wage $w_T(r)$ is such that

$$\log\left(w_T(r) - T\right) + \beta \log\left((1-r)T\right) = \bar{v}_0.$$

It is smaller than $w_T(0) = w_T$ the PAYG reservation rate. Thus again two possibilities occur: $m_d = 1$ or m_d satisfies $m_c < m_d < 1$.

With a payroll tax: $T_t = \tau w_t$ we have:

$$(1 + m_{t+1}\lambda)\, k_{t+1} = \frac{\beta}{1+\beta}(1-\alpha)\, Ak_t^{\alpha} + m_t\lambda\tau(1-\alpha)\, Ak_t^{\alpha}.$$

In the steady state:

$$(1 + m_d\lambda)\, k = \left[\frac{\beta}{1+\beta} + \tau m_d\lambda\right](1-\alpha)\, Ak^{\alpha}.$$

Note that for $\tau = \dfrac{\beta}{1+\beta}$ we have

$$k_{d'}^{1-\alpha} = \frac{\beta}{1+\beta}(1-\alpha)\, A = k_{\alpha}^{1-\alpha}.$$

In this particular case denoted by subscript d' the social security contribution rate corresponds to the saving rate that is optimal from the viewpoint of the individual's lifetime utility.

5.2 Universal PAYG scheme

We can now assume that the PAYG scheme is imposed on both types of workers. As one can expect this will have a depressing effect on the saving effort by the skilled workers. The value of m will be the same as for the partial PAYG scheme, i.e., $m_c = m_e$ where the index term e indicates the values for the universal PAYG scheme. What is going to change is the equation for the capital accumulation, which is now:

$$K_{t+1} = \left[\frac{\beta(1-\alpha)}{1+\beta}\, AK_t^{\alpha}L_t^{-\alpha} - \frac{T}{1+\beta}\left(\beta + \frac{1}{\alpha}AK_{t+1}^{1-\alpha}L_{t+1}^{\alpha-1}\right)\right] hL_s$$

where $L_t = hL_s + m_t L_u$. As the skilled now save $s_t = \dfrac{\beta}{1+\beta}\, w_t\lambda - \dfrac{T_{t+2}}{1+\beta}\left(\beta + \dfrac{1}{1+r_{t+1}}\right)$ per efficiency unit, we get with constant $m_t = m_e$:

$$(19)\quad k_{t+1} = \left[\frac{\beta(1-\alpha)}{1+\beta}\, Ak_t^{\alpha} - \frac{T}{1+\beta}\left(\beta + \frac{1}{\alpha}Ak_{t+1}^{1-\alpha}L_e^{\alpha-1}\right)\right]\frac{1}{1+m_e\lambda}.$$

To obtain the value of k_e, the steady state equilibrium value of capital per efficiency unit, one solves the above equation for $k_t = k_{t+1}$. It is clear that we now have a lower level of capital stock than in the case where the PAYG social security scheme was restricted to the unskilled, as extending the PAYG to skilled reduces their savings effort. Also, one easily checks that $\dfrac{\partial k_e}{\partial \mathrm{T}} < 0$.

As we will later compare steady state stocks of capital that arise in the different schemes, let us assume that in the universal PAYG a proportional tax, τ, is levied instead of the per unit tax. In this case, the equation for capital accumulation is given by

$$K_{t+1} = \left[\frac{\beta}{1+\beta} (1-\tau) w_t - \frac{\tau}{1+\beta} \frac{w_{t+1}}{1+r_{t+1}} \right] hL_s.$$

Simple manipulation gives

$$k_{t+1} = \left[(1-\tau)\, k_t^{\alpha} \frac{\beta}{1+\beta} (1-\alpha)\, \mathrm{A} - \frac{\tau}{1+\beta} \frac{1-\alpha}{\alpha} k_{t+1} \right] \frac{1}{1+m_e\lambda}$$

which yields in the steady state

$$(20)\quad k_e = \left[\frac{1-\tau}{1+m_e\lambda} \frac{\beta}{1+\beta} (1-\alpha) A \frac{1+\beta}{(1+\beta)\alpha + \tau \frac{1-\alpha}{1+m_e\lambda}} \right]^{\frac{1}{1-\alpha}}.$$

5. *Winners and losers*

So far, we have only concentrated on the steady state values, whilst ignoring which generations are affected by the social security scheme. First of all, it is clear that upon introduction of a social security scheme and hence migration to the formal sector, the old remaining in the informal sector whose children migrated will lose as the implicit generational contract is assumed to be broken.

In comparison to this, there will, however, be several winners according to which pension scheme is introduced. If a PAYG scheme is introduced (be it a partial one covering only the unskilled in the formal sector or covering both types of workers in the formal sector), the winners will be the old at time of introduction, who receive a pension ('free lunch') without having had to contribute (i.e., the old unskilled if a partial PAYG and all old if a universal PAYG is introduced). Further winners will in any case be the old skilled workers at time of introduction, regardless if they are covered by the PAYG scheme or not: due to the migration, the *per capita* level of capital per efficiency unit decreases, which increases the rate of return on capital. Hence, the old skilled at time of introduction receive a higher-than-expected return on their savings. As noted above, these 'free lunches' could be used to compensate the $(m_i - m_b) L_u$ old (for $i = c, d, d', e$) who would lose in the informal sector.

Whether in the new steady state workers in the formal sector (skilled or unskilled) are worse off depends on the steady state stock of capital (*per capita* per efficiency unit). If it is higher than in the initial steady state without social security (i.e., k_b) , they are better off; if it is lower, they are worse off. Table 4 outlines which generation alive at time t will be better off (+), which will be worse off (–) and which is unaffected (0) given that each scheme induces complete migration. We also show how in the

Table 4. Welfare change.

Cohort	Reform	Informal Sector	Formal skilled	Sector unskilled
Old at time of the reform	Partial PAYG (case c)	–	+	+
	Fully Funded (case d)	–	+	0
	Fully Funded (case d')	–	+	0
	Universal PAYG (case e)	–	+	+
Young at time of the reform	Partial PAYG (case c)		–	+
	Fully Funded (case d)		–	+
	Fully Funded (case d')		+	+
	Universal PAYG (case e)		–	+
Young in new stationary state	Partial PAYG (case c)		–	–
	Fully Funded (case d)		–[1]	–
	Fully Funded (case d')		0	+
	Universal PAYG (case e)		–	–

[1] Assuming that the contribution rate is not too large.

new steady state the generations will be affected according to the respective scheme.

To summarize our results, the choice we face between the different retirement schemes yields the classic results. In terms of capital accumulation, migration of unskilled agents who do not save or save less than skilled agents has a depressing effect on the capital stock per efficiency unit and hence on the level of wage. A fully funded system is most preferable given that we are in an economy that displays low amounts of capital. Introducing a PAYG on the other hand will reduce capital accumulation and this may be highly undesirable for a developing economy given that this has, by definition, a low level to start off with. Table 5 summarizes the results obtained regarding the new steady state stock of capital (in *per capita* per efficiency units). Of course, the steady state capital stocks reported in Table 5 are shown for an identical level of migration (in particular, we have assumed complete migration for the table).

One can compare the values k_i, where $i = a, ..., e$ denotes each of the respective cases in Table 4. We obtain $k_a = k'_d > k_d$ (if $t < \frac{\beta}{1+\beta}$). Furthermore, it is clear that $k_c > k_e$ for all $t > 0$. We also have $k_c < k_b < k_a$. Hence, we see that in all cases the workers in the formal sector are not better off in the new steady state.

7. Conclusion

Most theoretical work on social security is based on evidence from the US and to a somewhat lesser extent on the European Union. The circumstances of these regions are, however, fundamentally different from the situation that applies to the rest of the world, in particular to the very poor countries. Initial conditions, demographics and the political economy there differ from the scenarios generally assumed in the literature.

We have therefore studied in this article the case of an economy with an important informal sector both in terms of production and of social protection. In the formal sector only high wage workers have access to capital markets; low wage workers are without resources when retired. We thus consider introducing a mandatory social security system in the formal sector. Such an introduction has two effects: first, it makes moving from the informal sector to the formal sector more attractive to the low wage workers; second, it influences capital, depending on the system chose.

With a PAYG system (be it partial, i.e., covering only the unskilled workers, or universal, i.e., covering the entire formal sector), capital accumulation will decrease as will welfare. One has to note, however, that when workers move

Table 5. Steady state capital stock.

	Regime	$k^{1-\alpha}$
Case a	No retirement scheme, no migration	B
Case b	No retirement scheme, migration	$\frac{B}{1+m_b\lambda}$
Case c	Partial PAYG and migration ($m_c = 1$)	$\frac{B}{1+\lambda}$
Case d	FF and migration ($m_d = 1$)	$B + (\tau(1-\alpha) - B)\frac{\lambda}{1+\lambda}$
Case d'	FF and migration ($m_d = 1$), contribution mimics opt. savings	B
Case e	PAYG for all ($m_e = 1$)	$(1-\tau)\frac{B}{1+\lambda}\frac{(1+\beta)\alpha}{(1+\beta)\alpha+\tau\frac{1-\alpha}{1+\lambda}}$
	where $B = \frac{\beta}{1+\beta}(1-\alpha)A$ and $\lambda = \frac{L_u}{hL_s}$	

away from the informal sector, they breach an implicit 'Generational Contract' between themselves and their parents. Clearly, some of them could remit part of their income but there is no guarantee that they will do so. The interest of a PAYG system, even a minimal one, is that in the short term the introduction of such a scheme generates a free lunch which could be used by the government to compensate the elderly people in the informal sector for the departure of their children.

From a long-term point of view our model simply underlies the common consensus today (see World Bank, 1994b, or Valdes-Prieto, 1997): the introduction of a FF scheme is more desirable than the introduction of a PAYG scheme from the point of view of longrun welfare. In fact, this is the only case where in our economy one could achieve a Pareto-improvement: if the 'free-lunch' obtained by the old skilled upon introduction suffices to compensate the old in the informal sector who lose upon introduction, all can be made better off.

We have made several assumptions which need to be revisited here. The first concerns under-accumulation of capital (i.e., $r > n$). Our results would necessarily be reversed if the opposite were true. We think, however, that this is a realistic assumption particularly as we are talking about a developing country.

We have also constantly assumed the introduction of a social security system where benefits are related to pension contributions. If instead we were to have assumed a system with uniform benefits (in the case of the FF scheme or the universal PAYG scheme), our conclusion would be affected. Although there will be more intra-generational equity between skilled and unskilled workers – such redistribution may be deemed necessary if there is too large a gap between the earning of the two types of workers – it would affect the capital accumulation in the economy as the skilled will save less, lowering the steady state stock of capital.

Finally, long-run capital accumulation will depend on the tax rates. In our specification, there are no allocative tax distortions; this would no doubt change if we endogenised the labour supply of agents in the formal sector.

In conclusion, we believe that the best policy is to introduce a universal social security system with a mixture of a PAYG that could be used to finance a flat rate pension and of an actuarially fair funded system. The so called 'free lunch' generated by the PAYG part would be used to finance the elderly people in the informal sector. These proposed reforms should not be costly as in many of those countries the existing PAYG systems are at most very small, immature and cover only a minority of workers.

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Appendix: Dynamics of our model

We provide here a rather rigourous treatment of the dynamics of the model studied in this paper. We limit ourselves to the transition from autarky to an open economy without social security. The other cases can be readily analyzed:

The two basic equations are:

$$\text{(A1)}\quad (1+m_{t+1}\lambda)k_{t+1} = \frac{\beta}{1+\beta}(1-\alpha)Ak_t^{\alpha} \equiv Bk_t^{\alpha}$$

$$\text{(A2)}\quad m_{t+1} = m_t + \varphi(\ln(w_{t+1}-1) - \bar{v}_0)$$

with $w_{t+1} = w(k_{t+1}) = (1-\alpha)Ak_{t+1}^{\alpha}$, $\varphi(0) = 0$, $\varphi'(0) > 0$ and finite.

One rewrites (A2) so that:

$$1+m_{t+1}\lambda = 1+m_t\lambda+\lambda\varphi(\ln(w_{t+1}-1)-\bar{v}_0)$$

and then substituting $1 + m_{t+1}\lambda = Bk_t^{\alpha}k_{t+1}^{-1}$ yields

$$-Bk_t^{\alpha}k_{t+1}^{-1} + Bk_{t-1}^{\alpha}k_t^{-1} + \lambda\varphi(\ln(w(k_{t+1})-1) - \bar{v}_0) = 0.$$

One now shows that there exists a unique, stable, interior solution with $\ln((1-\alpha)A\hat{k}-1) = \bar{v}_0$ and

$$1 + \lambda\hat{m} = B\hat{k}^{\alpha-1}.$$

By assumption $0 < \hat{m} < 1$.

Here are the necessary and sufficient conditions for this result. We start by linearizing (A3) around $k = \hat{k}$.

$$dk_{t+1}\left(B\hat{k}^{\alpha-2} + \lambda\varphi'(0)\frac{(1-\alpha)A\alpha\hat{k}^{\alpha-1}}{(1-\alpha)A\hat{k}^{\alpha}-1}\right) - dk_t\left(\alpha B\hat{k}^{\alpha-2} + B\hat{k}^{\alpha-2}\right) + d\,k_{t-1}B\alpha\hat{k}^{\alpha-2} = 0$$

Dividing by $B\alpha\hat{k}^{\alpha-2}$, we have:

$$dk_{t+1}(1+\Delta) - (1+\alpha)\,dk_t + \alpha\,dk_{t+1} = 0,$$

where $\Delta = \lambda\varphi'(0)\dfrac{(1-\alpha)A\hat{k}/B}{A(1-\alpha)\hat{k}^{\alpha}-1}$.

We can now write the characteristic equation:

$$P(x) = x^2(1+\Delta) - (1+\alpha)x + \alpha = 0.$$

This equation has two roots x_1 and x_2 (real or complex) that are stable.

Proof: The product of the roots is $x_1 x_2 = \delta$, with $0 < \delta < 1$, $\delta = \dfrac{\alpha}{1+\Delta}$.

With complex roots, $|x_1| = |x_2| = \sqrt{\delta} < 1$.

With real roots, $x_1 > 0$, $x_2 > 0$ and $0 < x_1 \leq \sqrt{\delta} \leq x_2$.

One also has $P(1) > 0$ which implies that both roots are either larger or smaller than 1. Hence $0 < x_1 \leq \sqrt{\delta} \leq x_2 < 1$.