

THE CHOICE BETWEEN OWNER'S WAGES AND DIVIDENDS UNDER THE DUAL INCOME TAX*

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Tax-motivated shifting of income between different tax bases erodes tax revenues, confuses income statistics, and makes the effects of tax reforms unpredictable. Few studies have been able to use micro data to investigate this phenomenon. Using a rather unique data set, this study shows that the choice of type of payout from corporations to owners is strongly, but not uniquely, motivated by taxes. There are indications that the personal tax system is more clearly perceived by the owners than is the system of corporate taxes and regulations, and that own wage payments are motivated by rights to social security benefits. (JEL: H25, H26)

1. Introduction

Tax-motivated shifting of income between different tax bases erodes tax revenues, confuses income statistics, and makes the effects of tax reforms unpredictable. Few studies have been able to use micro data to investigate the mechanisms behind this phenomenon.

The particular type of income shifting we have studied is between two forms of payout from corporations to owners, as wages or as dividends. To our knowledge this has not been studied previously. One reason may be that under many tax systems capital gains dominate both of these forms. Capital gains are, however, of limited interest to many owners of small or

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new businesses. Another reason may be that under many tax systems one of the two payout forms dominates, so there is little cross-sectional variation in the choice. But there are exceptions.

The term “dual income tax” was introduced by Sørensen (1994a) to describe a tax system which distinguishes between labor income on one hand and all forms of capital income on the other.¹ Such systems were introduced in the 1990’s in Denmark, Finland, Norway and Sweden, with a constant marginal tax rate on capital income (such as interest, dividends),² and progressive rates on labor income, exceeding the rate on capital income at the margin for most taxpayers. One will then not only expect cross-sectional variation, but interior solutions: Taxes will often be minimized by paying some wages and some dividends.

We have concentrated on corporations with single owners so that the choice could have been motivated by taxes only. In addition to this, other possible motives are investigated. The data set consists of 225 pairs of single owners and their corporations in Norway, for whom we have both the personal and the corporate tax returns for 1991.

These data are rather unique, but still limited. We have not been able to estimate a structural model, but have made an exploratory investigation based on the available data. We find that the choices are strongly, but not uniquely motivated by tax minimization. There were links between rights to social security benefits and reported wage income, and these seem to motivate a higher wage payout. Tax perception seems to be stronger for the personal tax system than for the system of corporate taxes and regulations.

There is a small, but growing literature on how taxes affect the way income is reported and the way firms are organized. Gordon and MacKie-Mason (1995) give an overview, and stress the importance of income shifting for the design and analysis of tax policy. They argue that individuals easily can lower tax payments

by shifting their form of pay, and point out that income shifting involves non-tax considerations and real costs. They claim that research on the extent and the efficiency costs of income shifting would be highly valuable.

Gordon and MacKie-Mason (1994) report evidence that the tax system affects the decision whether to incorporate or not. Others report movements in the aggregate personal and corporate tax bases in accordance with the trends in relative tax rates (for example Poterba and Auerbach (1987)). Such movements between tax bases could be explained by income shifting, but this is not the only plausible explanation. To examine the extent of income shifting, studies based on micro data seem essential.

In the literature on tax perception most attention has been on the difference between perceived and actual income tax schedules. The econometric evidence is mixed.³ We consider a different problem, namely, whether the perceptions of the layers differ in a two-layer tax (and regulation) system.

Section 2 presents a model of the owner’s optimization problem. Section 3 describes how taxes and corporate regulations determine the choice set of the owner. Section 4 gives the empirical specification of the model with the hypotheses to be tested. Section 5 describes the data and the calculated budget sets. Section 6 presents the empirical results. Section 7 summarizes. An appendix gives more details about the tax system and calculation of the budget sets.

2. A model

This section presents a very simple model which identifies two motives. One is to minimize taxes in the current year. But there is also a trade-off between the current year’s after-tax income and the rights to future pension and so-

¹ See also Nielsen and Sørensen (1997).

² This was combined with different arrangements for capital gains taxation.

³ Arrazola, Hevia and Sanz (2000) find large differences for Spanish married men, and so do König et al (1995) for German married women. Brännäs and Karlsson (1996), however, find small differences for Swedish married men, and so do Fujii and Hawley (1988) for US American households.

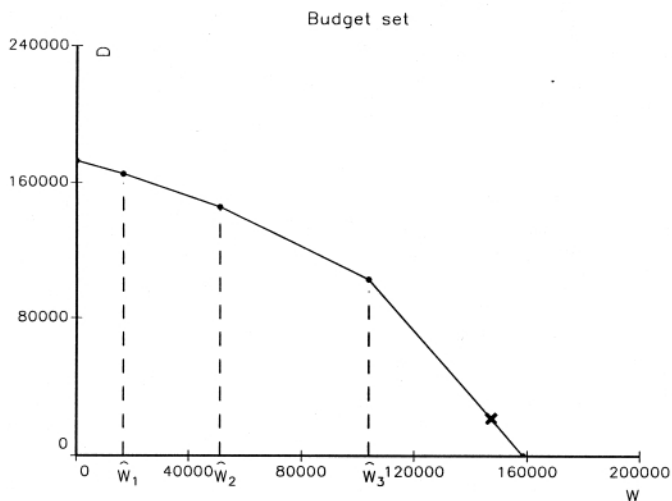


Figure 1. A budget set with four segments.

cial security benefits. We call this the social security motive.

Consider a corporation with only one shareholder. This owner may withdraw cash in the forms of dividends and/or own wages. We model the choice between these two forms conditional on a given total payout. Because the forms are taxed differently, a more precise statement is that we model the choice conditional on corporate profits before deduction of taxes and owner’s wages, Π , and on the retained part of these profits, R .

The corporation’s payout budget constraint is determined by the equation

$$(1) \quad \Pi - W_g(1 + \alpha) \equiv Y = T_c + G + R,$$

where W_g is gross wage payments to the owner, α is a payroll tax rate, Y is book profits⁴ before taxes, T_c is corporate taxes (apart from αW_g), and G is gross dividends (i.e., before personal taxes). This equation holds as an identity in our data.

⁴ Norway had a system of “uniform reporting,” see Cummins, Harris, and Hassett (1995), Kanninen and Södersten (1995), or Sørensen (1994b). This implied that the difference between book profits and the corporate income tax base was restricted by regulations. We have allowed for the difference, but regarded it as an exogenous constant for each corporation. To simplify the presentation the difference is set to zero except in the appendix.

The budget constraint is traced out as the maximum G for each value of W_g , for given observed values of (Π, R) , and given the tax rules. Through (1) this is equivalent to the maximum G for each value of Y (since α is a constant for each corporation). A high Y corresponds to a low value of W_g . The size of the budget differs between corporations because of differences in Π and R . In addition the shape of the constraint differs because taxes are affected by various pre-determined variables, such as loss carry-forwards and regulated accumulation of funds, which in a given year differ between corporations.

The payouts of W_g and G are subject to personal taxes and transformed into the after-tax variables W and D . Personal tax rates are in the interval $(0, 1)$, so W is increasing in W_g , and likewise for dividends. The owner’s personal budget constraint will be a downward sloping curve in the (W, D) diagram, giving the maximum D for any value of W . A typical budget constraint is shown in figure 1. The X shows the observed (W, D) point.

Without taxes the constraint would have been a 45 degree straight line. Progressive taxes on labor income, but not on dividend income, are sufficient to make it concave. This also implies that the owner’s incomes from other sources affect the budget constraint. We consider the choice between W and D conditional on these other incomes. The most important of these is

W^e , pre-personal-tax wage income from other sources.

An obvious short-run motive of the owner may be to maximize $W+D$, the after-tax payout, given Π and R , which amounts to minimizing the owner's personal and corporate taxes that year. In addition the wage payments affect the owner's rights to future national old age pensions and to social security benefits in case of disability, illness, maternity, and unemployment. The recording unit for these rights is known as a "pension point."

One hypothesis is that these benefits contribute to explain why some owners preferred more own wages and less dividends even if this led to more taxes. Private pensions and insurance were optional, but social security and national old age pensions applied to all, additional to and irrespective of any private pension and insurance they might have. The question is thus not the existence of, but the individuals' valuations of these benefits.

Let $v(W_g + W^e)$ be pension points earned in 1991 as a function of the owner's wage income in that year. This was a continuous, non-decreasing function with $v' = 0$ for $W_g + W^e > W_{g12} \equiv \text{NOK } 420,392$. (After exceeding this limit no more pension points were earned, even though no marginal tax rate decreased.)

The relationship between pension points and benefits is quite complicated. The benefits are uncertain, and it is not the case that in each possible state of the world benefits are proportional to accumulated pension points. The individual valuation of pension points is only partly observable, through variables that would lead to differences in expected benefits, such as age. We denote it by λ , and assume that it may vary across individuals. As a good approximation we assume it is constant for each individual for the year in focus, since it depends on lifetime characteristics which are almost invariant to decisions in that year.

Let $\psi(W, D; W^e) = 0$ be an implicit equation for a budget constraint of the type depicted in figure 1. The maximization problem of the owner is modelled as

$$(2) \quad \max_{W, D} [W + D + \lambda v(W_g + W^e)] \\ \text{s.t. } \psi(W, D; W^e) = 0.$$

Using T_w for the personal labor income tax function, we can relate W_g , the gross wage income from own corporation, to W by

$$(3) \quad W_g = W + T_w(W_g + W^e) - T_w(W^e).$$

We shall first treat T_w , ψ and v as if they were differentiable functions, with $T_w' \geq 0, T_w'' \geq 0, \partial\psi/\partial W > 0, \partial\psi/\partial D > 0, v' \geq 0$. Form the Lagrangian function

$$(4) \quad \mathcal{L}(W, D, \mu) = W + D + \lambda v(W_g + W^e) + \mu \psi(W, D; W^e),$$

where μ is the multiplier on the budget constraint. Denoting T_w' as m_w , we find the first order conditions with respect to W and D ,

$$(5) \quad \frac{\partial \mathcal{L}}{\partial W} = 1 + \frac{\lambda v'}{1 - m_w} + \mu \frac{\partial \psi}{\partial W} = 0,$$

and

$$(6) \quad \frac{\partial \mathcal{L}}{\partial D} = 1 + \mu \frac{\partial \psi}{\partial D} = 0.$$

Define now

$$(7) \quad \delta \equiv \frac{\partial \psi / \partial W}{\partial \psi / \partial D}.$$

This is the absolute value of the slope of the budget constraint. With this definition, the two first order conditions imply,

$$(8) \quad 1 + \frac{\lambda v'}{1 - m_w} = \delta.$$

The interpretation of (8) is straightforward: Without the willingness to pay for pension points, we have $\lambda = 0$, the fraction on the left-hand side is zero, and we seek a point with slope (minus) unity. Define (W^m, D^m) as the point which maximizes $W + D$. When the budget constraint is differentiable and strictly concave, this happens exactly when the slope is minus unity. When λ is strictly positive, however, (8) indicates that a point on the budget constraint with a steeper slope is chosen, i.e., a higher W , except if $v' = 0$, which occurs when $W_g + W^e$ exceeds W_{g12} .

3. Taxes, regulations, and the budget constraint

This section gives some details of the tax system and some features of the budget constraints. In particular we draw attention to the sources of cross-sectional variation and to the distinction between the personal taxes and the system of corporate taxes and regulations.

The personal tax system in Norway in 1991 consisted of a progressive tax on labor income, and proportional taxes on dividend income, net interest income, and capital gains. The separation of labor and capital incomes had been in place since 1987. There was also a personal wealth tax.

The system of corporate taxes and regulations was extremely complicated. This complexity was part of the motivation for a major tax reform in 1992. More details are given in the appendix.

One tax paid by the corporations was a proportional payroll tax at rate α , applicable at any wage level, deductible in the base for the corporate income tax. Before we give more details, we derive the price of owner's wages in terms of dividends at the margin, for a small change in wages. Consider an owner who gives up one unit of wages in year 1, after tax, which means that the wages given up before tax are $1/(1-m_w)$.⁵

Reduced wage payments show up in the corporate accounts as increased profits. The corporation pays

$$(9) \quad \Delta Y = \frac{1 + \alpha}{1 - m_w}$$

less in wages and payroll tax. The increased profits in (9) will be used for taxes and dividend distribution.

The increased gross dividends allowed by ΔY is

$$(10) \quad \Delta G = \frac{dG}{dY} \Delta Y.$$

⁵ Personal wealth taxes are neglected throughout the analysis, except so far as a particular tax limitation comes into effect, limiting the sum of income and wealth taxes.

The fraction dG/dY is determined by the corporate taxes and regulations and will be discussed in the next subsection. Let m_d denote the marginal personal tax rate on dividend income. The owner is left with $(1-m_d)\Delta G$ after personal taxes, and the result is that the marginal price of wages in terms of dividends, is

$$(11) \quad \delta = \frac{(1 + \alpha)(1 - m_d)}{1 - m_w} \frac{dG}{dY}.$$

There was substantial variation in the parameters.⁶

3.1 The corporate budget set

The corporate budget constraint in the (W_g, G) space is defined above in connection with equation (1). That equation shows that, for a given Π , the slope dG/dW_g of the corporate budget constraint is related to the derivative dG/dY of equations (10) and (11) through

$$(12) \quad \frac{dG}{dW_g} = - \frac{dG}{dY} (1 + \alpha).$$

The derivative dG/dY is determined by a system of simultaneous equations. Here we provide one simple (and simplified) example of a set of equations. We then explain some of the reasons why alternatives must be considered. There are many alternative sets of equations, valid for different sets of exogenous variables and for different intervals of Y values. The borders between these intervals give rise to kinks in the budget constraint, i.e., discontinuous changes in dG/dY and thus in dG/dW_g . These kinks are of interest because they give rise to new explanatory variables. More details are in the appendix.

The corporations paid proportional income taxes both to the national and to municipal authorities, at rates $c_n=27.8$ percent and $c_m=23$ percent, respectively. Both corporate income tax bases allowed depreciation schedules for capital equipment, and deductions of net inter-

⁶ As an attempt to promote businesses in rural areas, α varied geographically between 0 and 0.167, and m_d varied geographically between 0.145 and 0.195. m_w varied progressively with the tax base between 0 and 0.578. dG/dY could take on positive values between 0.2531 and unity.

est payments, both here implicitly subtracted in Y . There were carry-forward provisions for (nominal) losses. Corporate taxes were

$$(13) \quad T_c = c_m(Y - L - \Delta F) + c_n(Y - L - \Delta F - G),$$

where L is losses, if any, carried forward from previous years.

ΔF represents a tax incentive for retained earnings. Up to $\phi = 23$ percent of a year's book profits could be retained tax free, as an allocation to a "consolidation fund." For a corporation reaching this limit, the allocation is

$$(14) \quad \Delta F = \phi(Y - L).$$

In order to avoid double taxation of dividends, dividends paid were deductible in the base for the national income tax. Roughly this made sense since the rate of the national corporate income tax was about the same as the rate of the personal dividend tax, but see Lund (1986).

The solution to the three equations (1), (13), and (14) in T_c , G , and ΔF , is

$$(15) \quad \frac{dG}{dY} = \frac{1 - (c_m + c_n)(1 - \phi)}{1 - c_n} \approx 0.843.$$

The appendix shows that this particular value is just one of many possibilities. We mention a few here: The corporation may be out of tax position for one or both of the corporate income taxes due to losses carried forward. For the national tax it may instead be due to high dividends. The latter implies that choosing high dividends (and low owner's wages) results in an equation different from (13) being relevant at the margin. Thus it induces a kink in the budget constraint in the (W_g , G) plane. Similarly, choosing higher dividends and lower wages implies that book profits increase. The given retained earnings could become less than 23 percent of profits, thus resulting in ΔF being determined residually instead of through (14).

We calculated the budget set for each corporation, which means to solve for the maximum G (and thus D) for each value of Y (and thus W_g , and thus W) from some high value for

which $W = 0$, down to some lower value for which W is so high that D goes to zero.

In the calculation of the budget sets, many variables had to be considered exogenous, while they are really choice variables for the owners of the corporations, in particular in a longer perspective. Examples are the losses carried forward from the previous year, the corporation's debt, and the owner's wage income from other sources. The values of the exogenous variables determine the different shapes of the budget sets for different observations.

The simplifying assumption of exogeneity of these variables was necessary in order to focus on the particular choice between two forms of payout from the corporation to the owner. Since both forms are available, and both are used, it is interesting to consider this choice separately. Keeping everything else constant, such as Π and R , the purpose of this study is to find the motives behind the choice. While this is an appropriate method for a theoretical discussion, the econometric study may suffer from an endogeneity problem. We return to this in section 6.

4. Empirical specification

We attempted to estimate the model using an econometric method developed for piecewise-linear budget sets. (A plausible hypothesis would be linear indifference curves in the (W , D) space, since both goods are money.) The estimation failed due to lack of convergence. One reason may be the unusual lack of curvature of the indifference curves. Any unexplained variation in their slopes will cause lots of noise in the regression.

Instead we have estimated a linear equation,

$$(16) \quad W = \beta_0 + \beta_1(W^m + D^m) + \beta_2 W^m + \beta_3 W^e + \text{other variables.}$$

The left-hand side, W , is the observed, chosen after-tax wage payout to the owner. If minimization of the year's taxes had been the only motive, one would have $\beta_2 = 1$, and no other significant coefficients. An alternative is that the choices were not influenced by tax considera-

tions. In order to have a plausible model for this case, we could have used such data which are ordinarily used to explain wages. We have few of those, and instead include the total payout to the owner, measured by the maximum total payout $W^m + D^m$. This is also necessary to avoid that W^m picks up pure size differences in the corporation's payout budget.

We test three different groups of hypotheses. One is that taxes matter for the choice of dividends versus owner's wages, which means that β_2 is significantly greater than zero. The other is that the social security motive matters. The third is that there is a difference in tax perception between the personal tax system and the system of corporate taxes and regulations.

4.1 The social security motive

The inclusion of W^e leads to a test of whether there is a social security motive. That motive for payout in the form of owner's wages should disappear when $W_g + W^e$ is high enough to bring v' down to zero. More precisely, define the gross-of-tax function for W , given W^e , as $W_g \equiv g(W; W^e)$, implicitly given by (3). There may be no social security motive at the tax-minimizing point, W^m : If the sum $W^e + g(W^m; W^e)$ exceeds W_{g12} , then $v'(W^e + g(W^m; W^e)) = 0$, and (8) is satisfied at $W = W^m$. This is the prediction for observations with $W^e + g(W^m; W^e) > W_{g12}$, of which there are 20 in our sample. Among these there should be no effect of W^e on the chosen W if the model describes the whole truth, i.e., if taxes and the social security motive are all that matters. An alternative hypothesis is that the owner's wage income from own corporation actually reflects work effort, in which case W and W^e are likely to be substitutes, resulting from alternative ways to spend the owner's work effort. Under this alternative, even those 20 would have a negative effect of W^e on W .

If the sum $W^e + g(W^m; W^e)$ does not exceed W_{g12} , the prediction is simply that the coefficient of W^e should be between minus unity and zero. When the social security motive causes the owner to choose a higher W than W^m , there is substitution between W^e and W , which should show up in a negative regression coefficient. This results from the (discontinuously) declin-

ing marginal accrual of "pension points" (the v function).⁷

Because of the piecewise linearity of the budget constraint, the preferred point may be (W^m, D^m) even when the left-hand side in (8) is strictly greater than unity. Thus we predict that W^m has a strong explanatory power for W , and that many observations have W at, or very close to, W^m .

4.2 Perception of taxes and regulations

Due to the complexity of the system of taxes and regulations, it is hard to believe that all corporate owners had the full understanding of what the budget set looked like. The system of corporate taxes and regulations was particularly complex. We did a simple test of the perception of the rules by introducing the following indicator variables: $\chi_1^m = 1$ and $\chi_2^m = 0$ if the tax-minimizing kink point (W^m, D^m) , originates from the personal tax system, i.e., if it represents the move into a higher personal income tax bracket. If, however, (W^m, D^m) represents a change in the value of dG/dY , then $\chi_1^m = 0$ and $\chi_2^m = 1$. There are 28 observations with $\chi_1^m = 1$ and 132 observations with $\chi_2^m = 1$.⁸ The hypothesis is that when $W^m \chi_1^m$ and $W^m \chi_2^m$ replace W^m as explanatory variables, their coefficients will be significantly different from each other.

5. The data and the calculated budget sets

The data come from the Income and Wealth Survey for Corporations (Statistics Norway) and cover the accounting year 1991. The Income and Wealth Survey contains accounts, balance sheets and tax forms for each corporation in the sample, classified by 5-digit industry code. In 1991, the survey was extended to include ownership forms from closely held cor-

⁷ The v function had a constant slope for wage income between zero and $W_{g6} \equiv \text{NOK } 210,196$. There was another constant slope, a third of the first one, between this level and $W_{g12} = 2W_{g6}$. Above that the slope dropped to zero.

⁸ This is not an exhaustive list, since there are 60 observations with $W^m = 0$ ($\delta > 1$ everywhere) and 5 observations with $D^m = 0$ ($\delta < 1$ everywhere).

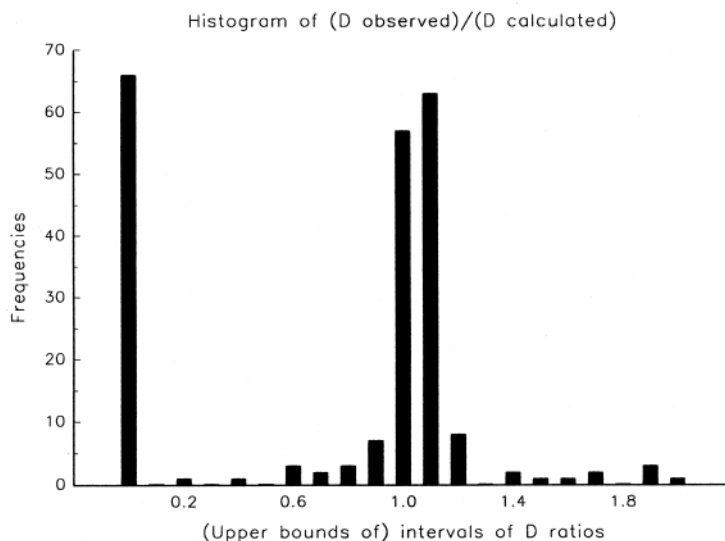


Figure 2: The ratios of observed to calculated dividends.

porations (defined as firms with 30 owners or less). The ownership forms identify the owners of the firm and report each owner's total number of shares. Ownership forms are linked to the employer-employee register and the tax register, by person code.

From the combined ownership and employer-employee data one can observe the wages owner-employees received from their own firm, and from the combined ownership and accounts data one can observe the dividends received. The tax register contains for every individual taxpayer the tax base, tax credits and taxes paid for each type of tax in the tax code. It is possible to calculate each owner-employee's marginal tax rates on different sources of income. One also observes the wages and dividends received from other sources. The tax register also reports age, sex and the taxpayer's municipality code, which determines the payroll tax rate. The 1991 extended Income and Wealth Survey thus represents a unique opportunity to study the combined effects of taxes on the corporate and personal level.

The total number of corporations in the Income and Wealth Survey is 6632. The sample is stratified, to be representative for firms by industry and size. Closely held corporations are drawn from a separate stratum. The number of firms from which ownership forms are collected

is 4830 and the total number of ownership forms is 9913 (gross). The estimations are based on a sample of 632 firms, those with only one shareholder.

From our sample of 632 firms, 13 violated some of the restrictions on the exogenous variables, and 3 were owned by married owners over the age of 70, for whom we did not have sufficient tax data, since they are taxed jointly with their spouses. This leaves 616 observations. Of these, 388 were excluded because they could not legally pay dividends.⁹ 3 more observations were excluded because of the occurrence of a negative dG/dY . In order to save space, we do not explain this anomaly here.

Of the 225 remaining observations, 139 paid both wages and dividends to the owner, 46 paid only wages, 20 paid only dividends, while 20 neither paid wages nor dividends, although they could have done so legally. Between the 225, we observed 242 different values of the slopes of the budget constraint, ranging (in absolute value) from 0.2194 to 8.947. There are 160 out of the 225 observations for which the value goes from less than unity to more when owner's wages increase.

⁹ They could not legally pay dividends because of negative free equity at the end of the year. 261 of these nevertheless paid wages to the owners.

Table 1. Frequency distribution of observed versus net-payout-maximizing net wage.

	24	48	72	96	120	144	168	192	216	240	264	288
24	54	0	0	0	0	0	0	0	0	0	0	0
48	5	3	0	0	0	0	1	0	0	0	0	0
72	1	2	2	0	0	0	0	0	0	0	0	0
96	7	1	3	8	0	0	0	0	0	0	0	0
120	6	0	2	5	4	0	0	0	0	0	0	0
144	5	2	1	2	8	11	2	0	0	0	1	0
168	3	0	1	2	3	6	11	0	0	0	0	0
192	2	0	1	0	0	4	2	6	0	0	2	0
216	0	0	0	0	1	3	2	4	7	1	0	0
240	3	0	0	0	0	0	0	1	1	1	0	0
264	2	0	0	1	0	1	0	0	1	2	4	0
288	0	0	0	0	0	0	1	0	0	0	0	2

Observed net wage, W , along vertical axis, net wage at 45 degree kinkpoint, W^m , along horizontal axis, both in thousand NOK, head showing the upper limits of intervals. 225 observations.

In the budget set calculations for each observation, the observed W and D play no role. A good check on our calculations is based on the ratio between the observed net dividend and the maximal net dividend we calculate for the observed net wage. The histogram in figure 2 shows the frequency of these ratios for the 225 non-empty sets.

There is a strong cluster around unity, as there should be. There is also a high frequency, 66/225, of non-dividend-paying observations, with a ratio of zero. According to our assumptions, these could have paid dividends at the W they have chosen, but instead paid too much in taxes.

The explanation for this may be transaction costs of paying dividends or saving taxes, not included in our model. It turns out that the average calculated maximal D (at the observed W) for these is as low as NOK 6235 (about USD 1000), while the average calculated maximal D (at the observed W) for the other 159 is as high as NOK 168628, or about 27 times as much. We find this difference reassuring: While our budget set model is not strictly true for these observations, the deviation is very small. The 66 corporations may not have bothered to pay a low dividend because of some minor transaction cost.¹⁰

¹⁰ There were some situations in which a taxpayer's budget set was non-convex, found for 117 observations in our sample. There was a slight non-convexity below an annual wage income of NOK 22304 for all taxpayers (approx. USD

6. Descriptive statistics and linear regressions

We present descriptive statistics and linear regressions, trying to explain the observed (W , D) in relation to the calculated budget set.

It is clearly interesting to try to explain deviations from (W^m , D^m). Table 1 shows the frequency distribution of the observed wage relative to the net-payout-maximizing wage. The numbers inside the table sum to 225, our number of remaining observations. Most deviations between the two variables, 113 to be precise, are less than the width of the intervals, NOK 24 000. That is, there are 113 pairs on the main diagonal with observed $W \approx W^m$. There are 7 pairs above the diagonal, with a net-payout-maximizing net wage which exceeds the observed net wage by more than the width of the

3400). For people above 69 years of age, and for the disabled, there was a special tax credit which gave a somewhat more important non-convexity, although it was only slight. More important was a non-convexity for those with a high personal wealth: There was a limitation on the personal tax liability, which includes a personal wealth tax, not to exceed eighty percent of taxable personal income, creating a high marginal income tax for those with low income and high wealth. Empirically, however, the non-convexities turned out to be negligible. Convexifying the 117 non-convex sets increased the area of the set by less than 0.4 percent for 87 percent of the observations, and by less than 4 percent for 97 percent of the observations. Furthermore, the convexified sets have the same payout-maximizing points as the original sets. In our verbal discussions we thus treat the budget sets as if they were convex.

Table 2. Linear OLS regression.

	const.	$W^m + D^m$	W^m	W^e	$W^m \chi_1^m$	$W^m \chi_2^m$	$W^e \chi_1^e$	$W^e \chi_0^e$	R^2 (R^2 adj)
2.1	64.9 (9.166) [2.423]	0.058 (4.100) [1.428]	0.556 (12.111) [2.926]	-0.165 (-4.136) [-2.557]	-	-	-	-	0.575 (0.569)
2.2	65.5 (9.812) [2.745]	0.056 (4.164) [1.477]	-	-0.166 (-4.378) [-2.881]	0.988 (10.153) [6.947]	0.509 (11.491) [2.786]	-	-	0.618 (0.611)
2.3	67.4 (9.196) [2.378]	0.057 (4.011) [1.380]	0.546 (11.775) [2.798]	-	-	-	-0.123 (-2.361) [-2.913]	-0.213 (-3.925) [-2.226]	0.578 (0.571)
2.4	68.1 (9.875) [2.707]	0.055 (4.068) [1.428]	-	-	0.979 (10.066) [6.737]	0.499 (11.169) [2.674]	-0.120 (-2.439) [-3.127]	-0.216 (-4.217) [-2.555]	0.622 (0.613)

Dependent variable: W^* (observed net wage)
 225 observations, t -statistics and R^2 adj. in parentheses
 Heteroskedasticity-consistent t -ratios in square brackets
 All variables measured in thousand NOK

intervals. There are 105 below the diagonal. Almost all large deviations are thus in the direction of a greater net wage than the one which maximizes the net payout. This may be explained by the social security system. Also, in some instances the tax authorities may have demanded that part of the payout be regarded as wages. On average, among those 185 with an observed strictly positive net wage, the ratio of the net-payout-maximizing net wage to the observed net wage is 70 percent.

The regression results are given in tables 2 and 3.

There may well be heteroskedasticity in the regressions, but we do not know its form. Thus we present OLS coefficient estimates, but we present both the standard t -statistics and heteroskedasticity-consistent t -ratios.¹¹

Consider first line 2.1 in table 2. The coefficient of $W^m + D^m$ is (here and elsewhere) less than 0.09. This means that increasing the budget size itself does not lead to much more wage income being paid to the owner. Allowing for heteroskedasticity one cannot reject that the coefficient is zero.

¹¹ The heteroskedasticity-consistent t -ratios are of the HC_3 type discussed in Davidson and MacKinnon (1993) p. 554. We also tried to divide through the equations with the size of the total payout, but this did not remove the heteroskedasticity, and we do not report these regressions.

The constant term is quite high and significant. Single owners tend to earn wage income from their corporations irrespective of the total payout they get from the corporations.

The coefficient of W^m is around 0.5, and significantly different from zero. This shows that the owners are clearly aware of some aspects of the shapes of their budget sets. Even though they deviate from W^m , this net-payout-maximizing point strongly influences how much is paid out as wage income to the owner.

We are also interested in the effect of the wage income from other sources, W^e . The coefficient estimate is negative, as expected, and is significantly different from zero. The coefficient is around -0.17. Thus there is substitutability between various sources of wage income, but only to some extent.

The picture that emerges from this is that dividends fluctuate more than wage incomes. In this cross-section those corporations which pay more to their owners, do so mainly in the form of dividends, but to some extent also in the form of wage income. The taxpayers are clearly aware of some aspects of the shapes of their budget sets, as there is a strong effect of the net-payout-maximizing net wage on the actually observed net wage.

Consider now lines 2.2–2.4, in which the variables W^e and W^m have been split through multiplication with indicator variables. This

amounts to dividing the sample in two parts according to the indicator variable, but restricting the coefficients of the other variables to be the same across the whole sample.

The χ_1^m indicates a kink-point from the personal tax system. There is a striking difference between the coefficients of $W^m \chi_1^m$ and $W^m \chi_2^m$. A χ^2 -test of the difference tells that it is clearly significant. We interpret this as a result of differences in tax perception: $W^m \chi_1^m$ has an estimated coefficient of about unity, which is what the theoretical model predicts when the tax motive is the only motive. Thus the tax motive works very directly when the marginal personal tax on labor income causes the payout-maximizing kink. But when the kink is caused by the system of corporate taxes and regulations, the effect is very much smaller, with an estimated coefficient of about 0.5. This may indicate that the owners do not observe these kinks as precisely.

There may be several reasons why the corporate taxes and regulations have a weaker effect. It may be difficult to communicate with the accountants in order to find the tax-minimizing point. In addition the system of corporate taxes and regulations is very much more complicated than the personal tax system. One could also speculate that there is a timing problem: Wage income is normally paid out during a year, while corporate accounts may be quite uncertain until late December.

In the subsequent regressions W^e is multiplied with χ_1^e , which is one when $W^e + g(W^m; W^e) > W_{g12}$ and zero otherwise, and χ_0^e which is defined the other way around. Only 20 of 225 observations have $\chi_1^e=1$. If behavior is motivated solely by (2), then there should be no effect of W^e on observed W among those for whom $\chi_1^e=1$, as long as W^m is included as an explanatory variable. But there is, with an estimated coefficient of about -0.12 . The coefficient of W^e for those with a clear social security motive is much larger in absolute value, however.

Even those without any social security motive at the tax-minimizing point show a trade-off between wages from own corporation and wages from other sources, given the total payout from own corporation. The higher wage income from other sources, the less is paid out as

wage income from own corporation, although the coefficient -0.12 is small. This cannot be explained by our model in section 2. It may be an indication that wages actually reflect work effort, so that more time spent working in one place reduces the time spent in another place.

One may worry that there could be a sample selectivity problem in the regressions. Out of 616 observations, 388 were excluded because they could not legally pay dividends. The model is not intended to explain the owner's wages per se, however. It is a conditional model, explaining these wages conditional on there being a choice between the two types of payout. Then there is no sample selectivity problem, cf. Davidson and MacKinnon (1993), p. 531.

There are reasons to suspect an endogeneity problem. The explanatory variables are decision variables on part of the owner, and may be affected by almost any unobserved heterogeneity in the data.

Because most of the variables behind this endogeneity are unobservables, we cannot run regressions based on a more elaborate model with several equations. We are left with the instrumental variable technique to overcome the endogeneity. None of the explanatory variables are included among the instruments. Instead we have included the year's profits and some variables which were determined at the beginning of the year, some for the corporation and some for the owner.¹²

We use the H2SLS estimator as defined in Davidson and MacKinnon (1993), p. 599 and p. 612. Results are shown in table 3, reproduc-

¹² *The instruments on the corporate side: Book value of assets, four different elements of book equity, the loss carry-forward from previous years (for tax purposes), L. These are predetermined balance sheet variables. If decisions are not serially correlated, they are exogenous. Profits of the current year, Π , are included because it is strongly correlated with the total payout. However, we believe it is mainly a result of previous years' decisions and exogenous factors, and determined before the payout-versus-retention decision is made.*

The instruments on the owner side: Two taxes on gross income with different bases give rise to the difference as instruments. The main element is net capital income apart from dividends, but there will also be various other tax deductions. They are correlated with W^m because they influence the shape of the budget set. Furthermore: The owner's sex and age, which may be correlated with W^e .

Table 3. Linear H2SLS regression.

	const.	W^m+D^m	W^m	W^e	$W^m \chi_1^m$	$W^m \chi_2^m$	$W^e \chi_1^e$	$W^e \chi_0^e$
3.1	37.4 (3.045)	0.088 (4.496)	0.710 (7.303)	-0.081 (-0.903)	-	-	-	-
3.2	59.2 (4.471)	0.060 (2.902)	-	-0.318 (-2.454)	1.447 (5.202)	0.637 (6.744)	-	-
3.3	121.1 (4.960)	0.030 (1.188)	0.369 (2.909)	-	-	-	-0.132 (-1.183)	-1.014 (-3.539)
3.4	10.0 (5.821)	0.021 (0.836)	-	-	1.205 (4.017)	0.445 (4.293)	-0.135 (-2.887)	-0.971 (-4.237)

Dependent variable: W^* (observed net wage)

225 observations, heteroskedasticity-consistent t -ratios in parentheses

Instruments: Book assets, Π , $\Pi \chi_i^m$ ($i=1, 2$), four equity categories (beg. of year), loss carry-forwards, personal income tax base gross minus net (two alt.), age, sex

All variables measured in thousand NOK

ing the combinations of variables in table 2. In all cases the signs of the coefficient estimates are the same as in the OLS regressions.

Some coefficient estimates are quite different from the OLS estimates. This can be taken to confirm the endogeneity problem, although we have not done any formal Durbin-Wu-Hausman type tests. The H2SLS estimates should then be considered the more reliable. We warn the reader, however, that the combination of a small sample and poor correlation between instruments and explanatory variables may make the H2SLS estimates less reliable.

The good news is that the coefficients not only have the same signs as with OLS, but they are all significantly different from zero. Moreover, the differences between the coefficients of those variables that include alternative indicators, are significant. In line 3.4 the χ^2 for testing $\beta_2 = \beta_3$ (the coefficients of $W^m \chi_1^m$ and $W^m \chi_2^m$) is 7.211, with a p value of 0.0072. Thus the null hypothesis that the two coefficients are equal, is clearly rejected. The similar values for the two final coefficients of line 3.4 is $\chi^2 = 15.431$ and $p = 0.0001$, and again equality is rejected.

We also tried to add other explanatory variables to the regression in an ad hoc way. A high possibility of bankruptcy might explain a high wage payout, since dividends will be legally restricted. One might also expect the wage payout to be increasing in the number of

employees. None of these variables were significant.¹³

7. Conclusion

For a corporation with a single owner, we have defined the budget constraint for the decision on how to pay out cash to the owner, in the form of wage income or in the form of dividends. The net payouts are affected by taxation and regulations. An interior solution with both forms of payout may minimize taxes if, e.g., one form is taxed progressively while the other is taxed proportionally at the personal level.

From data for Norway in 1991 we have been able to calculate these budget constraints for a sample of 225 pairs of owner and corporation. The choice of type of payout is strongly, but not uniquely, motivated by the minimization of taxes. There were also other significant explana-

¹³ These and other variables (than those presented in the tables), are likely to influence the wage payments. Apart from those mentioned, we lack data. We ran RESET tests for the specification of the model, including fourth-degree polynomials of the independent variables (without cross terms). (See, e.g., Kmenta (1986).) We can reject the combined hypothesis that the coefficients of all higher-order terms are zero, even though no single higher-order term had a significant coefficient. For instance in equation 2.4 the χ^2 test statistic with 15 degrees of freedom is 73.3121. Thus we cannot regard our model as an ultimate truth about what determined the wage payouts.

tory variables. A preference for wage income may be explained by the fact that rights to social security benefits depend on wage income. The data support this indirectly, since the negative effect of the owner's wage income from other sources is stronger when rights to social security benefits may be earned at the margin.

There is a striking difference between the effect of tax considerations when these originate from the personal part of the tax system and when they originate from the corporate part. This seems to indicate a difference in tax perception.

The decision whether to incorporate or not, which has had considerable attention in the income shifting literature, involves non-tax considerations and real costs. This means that income shifting represents not only a tax avoidance problem, but also an efficiency problem. On the other hand, the real costs involved will probably reduce the amount of income shifting through choice of organizational form. In contrast, shifting of (corporate) income between wages and dividends is relatively cheap, and thus represents an attractive alternative. Future research on corporate income shifting should have this alternative in mind.

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Appendix: Calculation of budget sets

The personal budget constraint in the W, D plane is a simple transformation of the corporate budget constraint in the W_g, G plane, applying personal tax rates, possibly progressive on wage income. The corporate budget constraint is defined as the maximum gross dividends, G , for each level of gross wages, W_g , and differs between corporations because it depends on their profits, retentions, and various predetermined variables.

This appendix explains how we made the computer calculate each budget constraint exactly, based on tax rules and corporate regulations.¹⁴ An alternative method would have been to use numerical optimization methods to maximize G for each W_g . There would have been two important drawbacks. First, the maximization would have been done for an arbitrary grid of W_g values, resorting to approximations in between, and without identification of kinks in the budget constraint. Second, the numerical calculations would not contribute to checking the logical consistency of the model. With the chosen method, on the other hand, it is highly unlikely that a logically inconsistent model would result in 225 budget constraints of connected, but non-overlapping intervals. In fact some inconsistencies were discovered along our way, and new (rather unlikely) cases added.

Each segment of the budget constraint results from the solution of a system of five simultaneous equations in the five variables $G, T_c, \Delta F, \Delta V$ and ΔE . ΔV is allocation to a “reserve fund,” a legally required part of retained earnings. ΔE is allocation to free equity, that part of retained earnings which are neither tax deductible (ΔF) nor legally required (ΔV). The accumulated free equity at the beginning of the year is denoted E_b .

The solutions express the five variables as functions of Y and various variables which are predetermined for each corporation, or exogenous parameters such as tax rates. For the budget constraint itself, only the solution for G matters. By varying Y (and thus W_g , cf. (1)), the

budget constraint is traced out by this equation. One example of the first derivative of such a solution is given in equation (15).

There are 288 alternative linear sets of five equations. These must be identified in order to solve the sets. We discuss the equations briefly before describing how to sort out which sets apply where.

A.1 First equation

The first equation is

$$(A.1) Y = T_c + G + R^*,$$

where R^* is the observed value of R .

A.2 Second equation

The second equation is, in shorthand,

$$(A.2) T_c = T_e + T_m + T_n,$$

where T_e is the corporate wealth tax, which we have assumed to be exogenous. Norway had a two-tier system of corporate income taxes. The municipal corporate income tax is

$$(A.3) T_m = c_m \max(0, Y - Y_\Delta - L_m - \Delta F),$$

where c_m (=0.23 in 1991) is the rate of the tax, and L_m is loss carry-forward in the base for this tax.¹⁵ The national corporate income tax is

$$T_n = c_n \max\{0, Y - Y_\Delta - L_n - \Delta F$$

$$(A.4) -\min[G - G_r, Y - \Delta F - \max(0, T_c + \Delta V^+ - \max(E_b - \Delta V^-, 0))]\},$$

where c_n (=0.278 in 1991) is the rate of the tax, and L_n is loss carry-forward in the base for this tax. G_r is dividends received by the corporation, here taken as exogenous. Also, $\Delta V^+ \equiv \max(\Delta V, 0)$, and $\Delta V^- \equiv \min(\Delta V, 0)$.

This requires some explanation. Dividends paid are deductible, and dividends received tax-

¹⁴ More details are available at <http://folk.uio.no/dilund/wagediv>

¹⁵ Y_Δ is the difference mentioned in footnote 4.

able, for T_n . The deduction for dividends is limited to $Y - \Delta F$ minus “taxes and reserve fund allocations to the extent that they are not covered by free equity,”¹⁶ i.e., limited to

$$(A.5) \quad Y - \Delta F - \max(0, T_c + \Delta V^+ - \max(E_b - \Delta V^-, 0)).$$

Since T_m has two linear variants and T_n has six linear variants, there are altogether twelve linear variants of the second equation.

A.3 Third equation

The third equation gives the reserve fund allocations. The rules are too complicated to be included here.¹⁷ There are five different linear variants when $\Delta V > 0$. When the required reserve fund is obtained, no allocation is needed, which is a sixth variant. There is also the possibility of a withdrawal from the reserve fund, $\Delta V < 0$, when the fund exceeds the target. There are two linear variants for this. There are altogether eight linear variants of the third equation.

A.4 Fourth and fifth equation

While the first three equations are determined by rules, the fourth and fifth are determined by tax-minimizing behavior. There are three (or rather, four) linear candidates for the fourth and fifth equation

$$(A.6) \quad \Delta F + \Delta E + \Delta V = R^*,$$

$$(A.7) \quad \Delta F = \phi(Y - Y_\Delta - L_m) \text{ or } \Delta F = 0,$$

¹⁶ See Zimmer (1988) p. 44f and Norwegian Tax Directorate (1991) p. 40. If $\Delta V > 0$, it is clear from the sources that this magnitude and T_c should be deducted from $Y - \Delta F$ in order to find the maximum deduction for dividends, but only to the extent that $T_c + \Delta V$ are not covered by a positive E_b . If $\Delta V < 0$ because the reserve fund exceeds its target, we assume that $|\Delta V|$ can be treated like E_b , and if the sum of these is positive, T_c can be covered by it in whole or in part. Observe also that the maximum limitation (A.5) is based on book profits $Y - \Delta F$, while the tax itself is based on taxable profits $Y - Y_\Delta - L_n - \Delta F$. This has the implication that the tax base in some cases is reduced to $-Y_\Delta - L_n$ if this is positive.

¹⁷ More details are available at <http://folk.uio.no/dilund/wagediv>

and

$$(A.8) \quad \Delta E = -E_b.$$

The following restrictions on the variables are relevant:

- $0 \leq \Delta F \leq \phi \max(0, Y - Y_\Delta - L_m)$, and
- $\Delta E \geq -E_b$, a legal requirement¹⁸ for $G > 0$.

The owner will try to allocate as much as possible to the consolidation fund in order to save taxes. Equations (A.6) and (A.7), but not (A.8), obtain when a full consolidation fund allocation, $\phi \max(0, Y - Y_\Delta - L_m)$, is made, while ΔE is determined residually.

But if $R^* - \Delta Y - \phi \max(0, Y - Y_\Delta - L_m) < -E_b$, then maximizing ΔF while allowing dividend payments does not lead to $\Delta F = \phi \max(0, Y - Y_\Delta - L_m)$. Instead a maximum reduction of free equity is made, while ΔF is determined residually. Equations (A.6) and (A.8), but not (A.7), obtain.

Equation (A.7) (in its $\Delta F = 0$ version) and (A.8), but not (A.6), may also obtain, but this conflicts with our definition of the corporate payout budget constraint. In this case a maximum reduction of free equity is made, and nothing is allocated to the consolidation fund, but the required ΔV still exceeds $R^* + E_b$.

Equation (A.7) has two linear variants. When combined with (A.6), both are relevant. But this does not add to the total number of equations, since the zero variant applies if and only if the zero variant of T_m applies. In our calculation of the total number of cases, we may thus say that there are altogether three linear variants of the union of the fourth and fifth equation.

¹⁸ Since $G = Y - T_c - \Delta F - \Delta V - \Delta E$, the requirement can be written as $G \leq \max(0, Y - T_c + E_b - \Delta F - \Delta V)$. Observe that we allow the use of previously retained earnings, E_b , to pay dividends. This is relevant in a detailed and short-run model, but often neglected in long-run models, e.g., explicitly in Sinn (1987), cf. his footnote 3, p. 75.

A.5 Total number of equation systems, and solution method

With twelve alternative linear expressions for the taxes and 24 for the allocations, there are 288 alternative sets of five simultaneous linear equations, which should be solved in order to find the five endogenous variables expressed in terms of the exogenous ones. Based on this, the crucial variable dG/dY might take as many as 288 values.¹⁹ It turns out that many of these are equal, reducing the number of different theoretical values to 57.

For each observation we now ask: For the interval of relevant Y values (which allow both $G > 0$ and $W_g > 0$), which set of equations is valid for each Y ? There will be a number of Y -subintervals, and in each, only one of the 288 systems will be valid. The method for finding which of the 288 are relevant for each observation is by trying them all, and eliminating those which are not.

For each set of equations, we know the inequalities defining when that set of linear equations is valid. However, as given by the tax and accounting rules, the inequalities are expressed partly in the endogenous variables, G , T_c , ΔF , ΔV , and ΔE . After solving each equation system, we must therefore rewrite the inequalities relat-

ing to that system in terms of exogenous variables (including Y).

We need inequalities which are linear, in order to solve for a unique Y value which is the boundary for the validity of each inequality, given a particular equation system. This will be a value expressed in terms of the other exogenous variables.

Connected to each equation system, there are a number of inequalities to check. Altogether there are 1920 inequalities. For each observation we solve for the unique Y value which is the critical one for each of these inequalities. For each observation and each of the 288 equation systems, we then determine the Y interval for which all of the inequalities connected to that equation system are satisfied. Most of these are empty intervals.

For each observation we collect those non-empty intervals which are found.²⁰ It turns out that for every observation this gives a sequence of between one and eight connected, but non-overlapping intervals from a lower bound to plus infinity. The lower bound is always a Y value for which G goes to zero. When plugging this into the budget set calculation, there will also be an upper cut-off point, where W_g goes to zero.

¹⁹ Some of the 288 cases are unlikely to occur, but cannot be ruled out a priori. This has to do with the difference Y_Δ between a year's taxable and book profits, and the different principles for carrying forward losses in books and in tax bases. It is possible, but not very likely, that a corporation in one year has high book profits but negative taxable profits, or vice versa, or that it has high book profits after deduction of previous years' uncovered book losses, but negative taxable profits after carrying forward tax deductible losses, or vice versa. The unlikely combinations are allowed by our equations and inequalities, and the data for exogenous variables will determine what cases are actually relevant.

²⁰ When there are groups of intervals with identical lower and upper bounds and identical values for dG/dY , we reduce the number of these intervals to one. Such groups occur because some of the equation systems happen to be identical for some values of the exogenous variables, e.g., when $G_r=0$, but they may perhaps also occur because we have not been able to detect that some systems are always identical.